

Cusum Charts (V-Mask)

Summary

Cumulative sum charts monitor variable data by plotting the cumulative sum of deviations from target. By so doing, they incorporate the past history of the process into the plotted points, achieving greater sensitivity and shorter run lengths than X-bar or X charts against small shifts in the process mean. STATGRAPHICS offers two procedures for creating cusum charts:

1. The *Cusum Charts (Tabular)* procedure, which plots two one-sided cumulative sums with horizontal control limits.
2. The *Cusum Charts (V-Mask)* procedure described in this document, which plots a single two-sided cumulative sum with a leading “V-Mask” to determine when the process goes out-of-control.

This procedure also create an MR(2), R or S chart to monitor process variability.

Sample StatFolio: *cusumvmask.sgp*

Sample Data:

The file *process shift.sgd* contains a sample of random numbers described by Montgomery (2005). The data consist of $m = 30$ observations, to be treated as individuals. A partial list of the data in that file is shown below:

<i>Sample</i>	<i>X</i>
1	9.45
2	7.99
3	9.29
4	11.66
5	12.16
6	10.18
7	8.04
8	11.46
9	9.20
10	10.34

The first 20 observations were randomly generated from a normal distribution with $\mu = 10$ and $\sigma = 1$. The last 10 observations were randomly generated from a normal distribution with $\mu = 11$ and $\sigma = 1$, representing a 1-sigma shift in the process mean. Montgomery uses this example to illustrate the properties of various types of time-weighted charts.

Data Input

There are two menu selections that create tabular cusum charts, one for individuals data and one for grouped data. In the case of grouped data, the original observations may be entered, or subgroup statistics may be entered instead.

Case #1: Individuals

The data to be analyzed consist of a single numeric column containing n observations. The data are assumed to have been taken one at a time.

- **Observations:** numeric column containing the data to be analyzed.
- **Date/Time/Labels:** optional labels for each observation.
- **LSL, Nominal, USL:** optional lower specification limit, nominal (target) value, and upper specification limit.
- **Select:** subset selection.

Case #2: Grouped Data – Original Observations

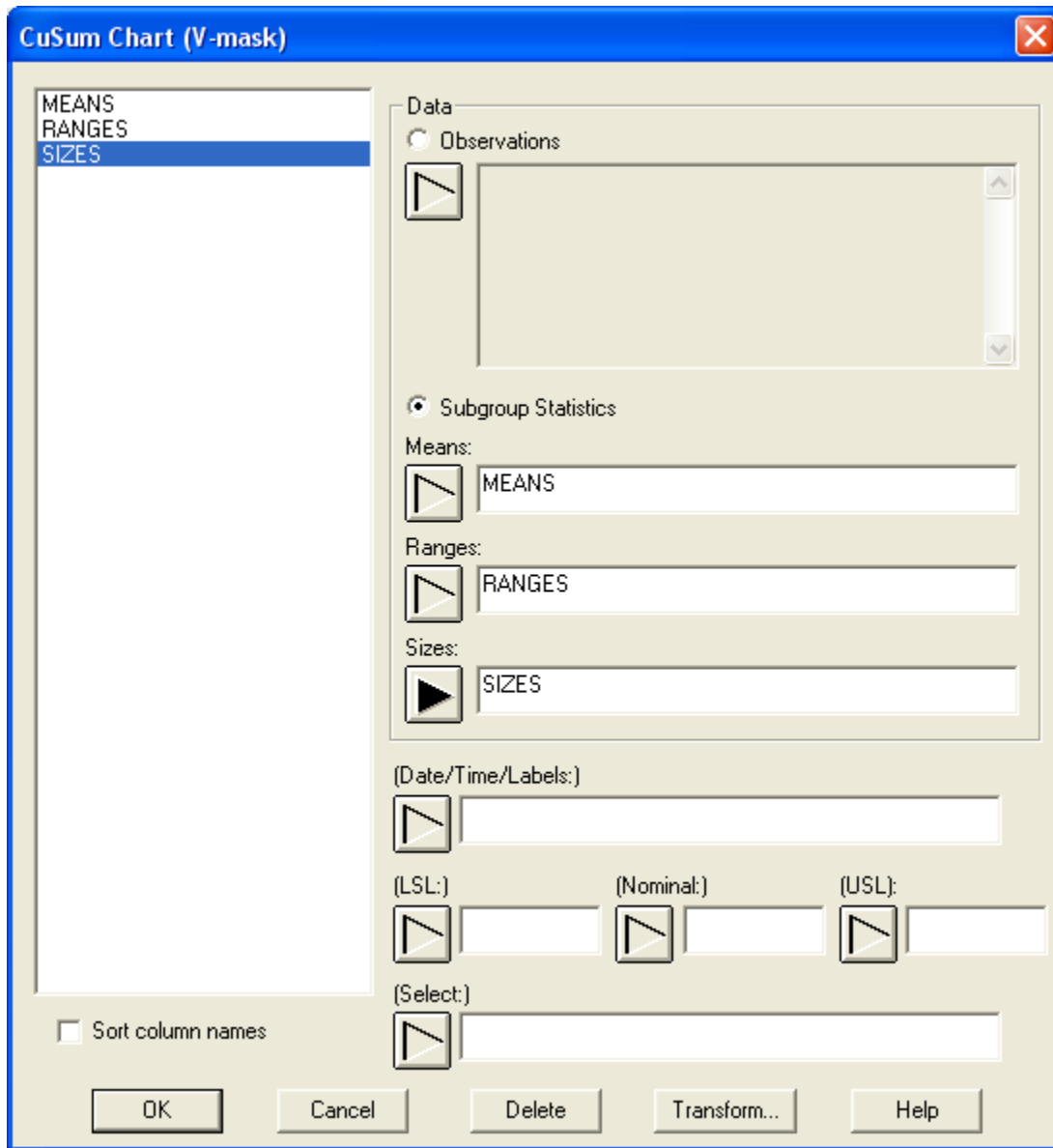
The data to be analyzed consist of one or more numeric columns. The data are assumed to have been taken in groups, in sequential order by rows.

- **Observations:** one or more numeric columns. If more than one column is entered, each row of the file is assumed to represent a subgroup with subgroup size m equal to the number of columns entered. If only one column is entered, then the *Date/Time/Labels or Size* field is used to form the groups.
- **Date/Time/Labels or Size:** If each set of m rows represents a group, enter the single value m . For example, entering a 5 as in the example above implies that the data in rows 1-5 form the first group, rows 6-10 form the second group, and so on. If the subgroup sizes are not equal, enter the name of an additional numeric or non-numeric column containing group identifiers. The program will scan this column and place sequential rows with identical codes into the same group.
- **LSL, Nominal, USL:** optional lower specification limit, nominal (target) value, and upper specification limit.
- **Select:** subset selection.

Case #3: Grouped Data – Subgroup Statistics

In this case, the statistics for each subgroup have been computed elsewhere and entered into the datasheet, as in the table below:

<i>Sample</i>	<i>Means</i>	<i>Ranges</i>	<i>Sizes</i>
1	10.11	4.17	5
2	9.844	3.42	5
3	10.098	2.44	5
4	9.932	2.32	5
5	10.924	2.96	5

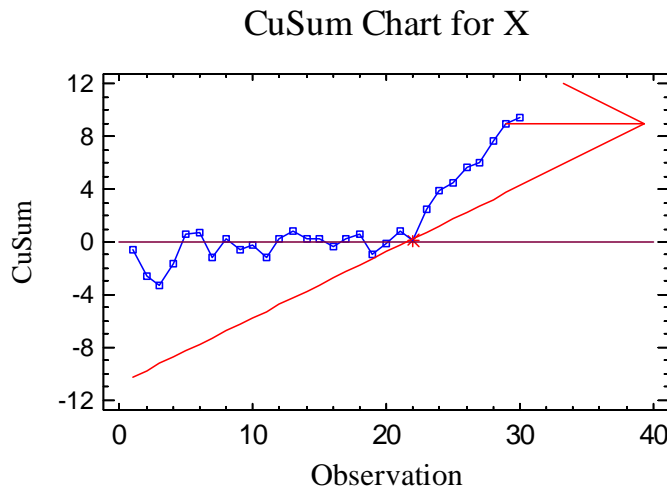


- **Subgroup Statistics:** the names of the column containing the subgroup means, subgroup ranges, and subgroup sizes.

- **Date/Time/Labels:** optional labels for each subgroup.
- **LSL, Nominal, USL:** optional lower specification limit, nominal (target) value, and upper specification limit.
- **Select:** subset selection.

CuSum Chart

The V-mask form of the cusum chart plots the cumulative sums of deviations from a target value, together with a V-mask to determine whether or not the process is in a state of statistical control.



Assuming that a process is designed to operate at a mean level μ , the *CuSum V-Mask* procedure plots the cumulative sum of deviations from the mean. For individuals data, the cusum is defined

$$C_j = \sum_{i=1}^j (x_i - \mu) \tag{1}$$

and for grouped data by

$$C_j = \sum_{i=1}^j (\bar{x}_i - \mu) \tag{2}$$

If the process is in a state of statistical control, the cusum should vary around 0, shown by a horizontal line.

To determine whether or not the process is in a state of statistical control at time j , a V-mask is placed at a distance d in front of C_j . The opening of the mask is drawn at an angle of $\pm \theta$ from the horizontal. If the entire history of the cusum lies within the opening of the mask, the process is deemed to be in a state of statistical control. If any point lies outside the mask, the process is deemed to be out-of-control. The chart above shows a V-mask positioned in front of the cusum at $j = 29$, where the point at time $j = 22$ is outside the mask. That is the first time at which the

chart would have indicated an out-of-control condition. Since the slope of the cusum is positive, the process mean appears to have shifted upwards.

The parameters d and θ impact the performance of the chart. They are related to the to the risk α of incorrectly generating an out-of-control signal when the process is in control (a false alarm or Type I error) and the risk β of not generating an out-of-control signal when the process shifts by an amount $\delta\sigma$ (a missed alarm or Type II error). The parameters are related to the risks by:

$$d = \frac{2}{\delta} \left[\frac{\sigma^2}{\delta} \ln \left(\frac{1-\beta}{\alpha/2} \right) \right] \tag{3}$$

$$\theta = \tan^{-1} \left(\frac{\delta}{2} \right) \tag{4}$$

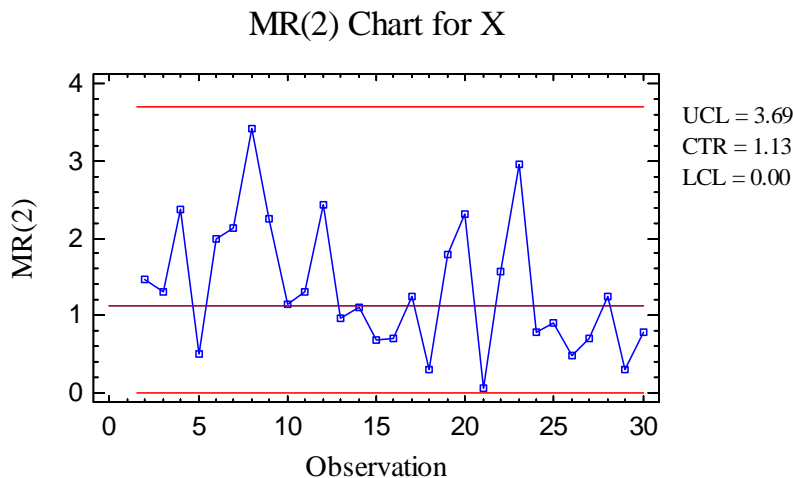
The cusum charts are most commonly used during Phase 2 to monitor a process against an established standard. In the current example, the parameters were set at $\mu = 10$ and $\sigma = 1$, corresponding to the distribution from which the first 20 rows were generated. If used during Phase 1, the parameters are replaced by estimates from the current data. In that case,

$$\hat{\mu} = \bar{x} = \frac{\sum_{j=1}^m n_j \bar{x}_j}{\sum_{j=1}^m n_j} \tag{5}$$

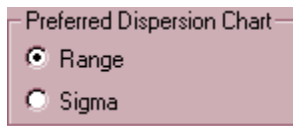
while σ is estimated using the method specified on the *Control Charts* tab of the *Preferences* dialog box.

MR(2)/R/S Chart

A second chart is also included to monitor the process variability.

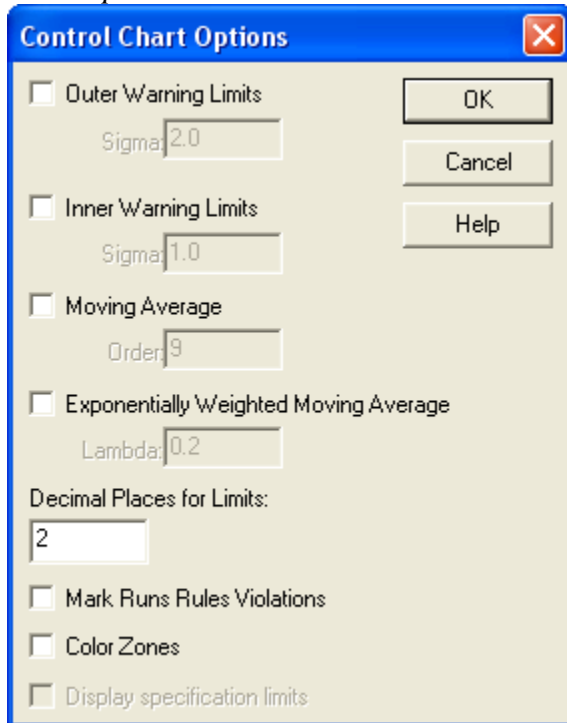


For individuals data, the chart displayed is an MR(2) chart, described in the *Individuals Control Charts* documentation. For grouped data, either an R chart or an S chart is plotted, depending on the setting on the *Control Charts* tab of the *Preferences* dialog box:



These charts are described in the *X-Bar and R Charts* and the *X-Bar and S Charts* documents.

Pane Options



- **Outer Warning Limits:** check this box to add warning limits at the specified multiple of sigma, usually at 2 sigma.
- **Inner Warning Limits:** check this box to add warning limits at the specified multiple of sigma, usually at 1 sigma.
- **Moving Average:** check this box to add a moving average smoother to the chart. In addition to the observations, the average of the most recent q points will also be displayed, where q is the *order* of the moving average. The default value $q = 9$ since the 1-sigma inner warning limits for the original observations are equivalent to the 3-sigma control limits for that order moving average.
- **Exponentially Weighted Moving Average:** check this box to add an EWMA smoother to the chart. In addition to the observations, an exponentially weighted moving average of the observations will also be displayed, where λ is the smoothing parameter of the EWMA. The

default value $\lambda = 0.2$ since the 1-sigma inner warning limits for the original observations are equivalent to the 3-sigma control limits for that EWMA.

- **Decimal Places for Limits:** the number of decimal places used to display the control limits.
- **Mark Runs Rules Violations:** flags with a special point symbol any unusual sequences or runs. The runs rules applied by default are specified on the *Runs Tests* tab of the *Preferences* dialog box.
- **Color Zones:** check this box to display green, yellow and red zones.

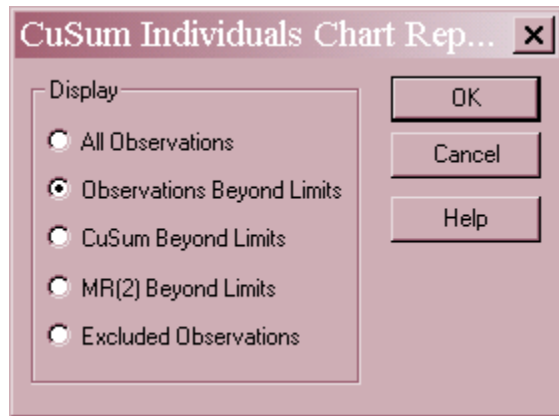
CuSum Chart Report

This pane tabulates the values plotted on the control charts:

CuSum Individuals Chart Report		
Observations Beyond Limits		
X = Excluded * = Beyond Limits		
Observation	CuSum	MR(2)
22	* 0.15	1.57

Out-of-control points are indicated by an asterisk. Points excluded from the calculations are indicated by an X.

Pane Options



- **Display:** specify the observations or subgroups to display in the report.

Analysis Summary

The *Analysis Summary* summarizes the data and the control charts.

<u>Cumulative Sum Individuals Chart (V-Mask) - X</u>		
Number of observations = 30		
0 observations excluded		
Distribution: Normal		
Transformation: none		
CuSum Chart		
<i>Period</i>	#1-30	
Shift to detect	1.0 sigma	
alpha	0.01	
beta	0.1	
V-mask at sample 29		
MR(2) Chart		
<i>Period</i>	#1-30	
UCL: +3.0 sigma	3.6855	
Centerline	1.128	
LCL: -3.0 sigma	0.0	
0 beyond limits		
Estimates		
<i>Period</i>	#1-30	<i>Standard</i>
Process mean	10.315	10.0
Process sigma	1.19987	1.0
Mean MR(2)	1.35345	
Mean deviation	0.315	
Sigma estimated from average moving range		

Included in the table are:

- **Subgroup Information:** the number of observations or subgroups m and the average subgroup size \bar{n} (if not individuals). If any observations or subgroups have been excluded from the calculations, that number is also displayed.
- **Distribution:** the assumed distribution for the data. By default, the data are assumed to follow a normal distribution. However, one of 26 other distributions may be selected using *Analysis Options*.
- **Transformation:** any transformation that has been applied to the data. Using *Analysis Options*, you may elect to transform the data using either a common transformation such as a square root or optimize the transformation using the Box-Cox method.
- **CuSum Chart:** a summary of the parameters used to position the V-mask. The parameters include the false-alarm probability α and the probability β of not detecting a shift of magnitude $\delta\sigma$.
- **MR(2)/R/S Chart:** a summary of the centerline and control limits for the dispersion chart.

- **Estimates:** estimates of the process mean μ and the process standard deviation σ . The methods for estimating the process sigma depending upon the settings on the *Control Charts* tab of the *Preferences* dialog box, accessible through the *Edit* menu. The options are described in the *Individuals Control Charts* and *X-bar and R Charts* documentation.
- **Mean MR(2), Mean Range, or Mean S:** the value of the statistic used to calculate the process sigma.
- **Mean Deviation:** the average deviation of the observations or subgroup means from the target value μ .

Analysis Options

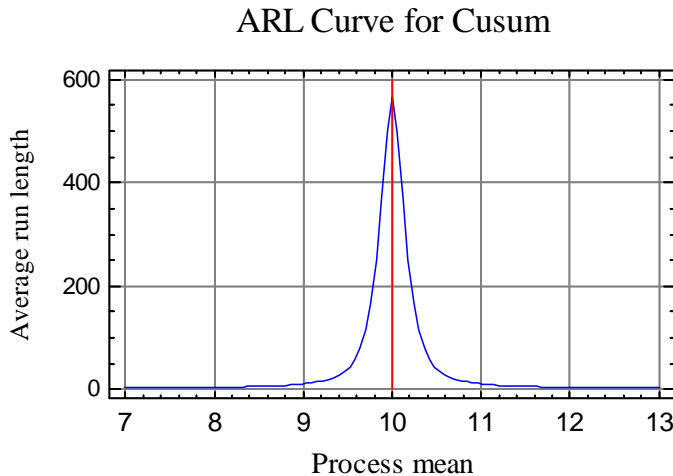
- **Type of Study:** determines how the control limits are set. For an *Initial Study* (Phase 1) chart, the limits are estimated from the current data. For a *Control to Standard* (Phase 2) chart, the control limits are determined from the information in the *Control to Standard* section of the dialog box.
- **Detect Sigma Shift:** multiple of sigma at which the Type II risk β (the risk of not getting a signal) is set.
- **Place Mask At:** value of j at which the V-mask is positioned.
- **CuSum Chart:** the false alarm probability α and the missed alarm risk β .
- **MR(2)/R/S Control Limits:** sigma multiple at which the control limits are positioned on the dispersion chart. Set to 0 to suppress a limit.
- **Control to Standard:** To perform a Phase 2 analysis, select *Control to Standard* for the *Type of Study* and then enter the established standard process mean and sigma (or other parameters if not assuming a normal distribution).

- **Exclude button:** Use this button to exclude specific subgroups from the calculations.
- **Transform Button:** Use this button to specify a transformation or non-normal distribution.

For a discussion of the *Exclude* and *Transform* features, see the documentation for *Individuals Control Charts*.

ARL Curve

The *ARL Curve* displays the performance of a Phase 2 chart.



The ARL curve plots the average run length (average number of values plotted up to and including the first point beyond the control limits) as a function of the true process mean. Assuming that the process mean suddenly shifts to a new value, the plot shows how long it takes on average until an out-of-control signal is generated. For very small shifts, it can take in excess of 400 points on average to detect the shift. At a shift to $\mu = 11$, the ARL is approximately 10.5.

Save Results

The following results can be saved to the datasheet, depending on whether the data are individuals or grouped:

1. *Cumulative sum* – the cumulative sum at each time period.
2. *Ranges, sigmas, or moving ranges* – the values used to estimate the process sigma.
3. *Sizes* – the subgroup sizes.
4. *Labels* – the subgroup labels.
5. *Process Mean* – the estimated process mean.
6. *Process Sigma* – the estimated process standard deviation.
7. *Included Observations* – a column of 0's and 1's for excluded and included observations, respectively. This column can then be used in the *Select* field on other data input dialog boxes.

Calculations**Equivalence to Tabular Cusum Chart**

The V-mask form of the cusum chart is equivalent to a tabular cusum chart with the following parameters:

$$k = \tan \theta \quad (6)$$

$$h = dk \quad (7)$$

ARL

The ARL of a two-sided cusum chart is calculated from the ARLs of the tabular cusums according to

$$\frac{1}{ARL} = \frac{1}{ARL_+} + \frac{1}{ARL_-} \quad (8)$$

where for $\Delta \neq 0$

$$ARL_+ = \frac{\exp(-2\Delta b) + 2\Delta b - 1}{2\Delta^2}, \quad \Delta = \delta - k \quad (9)$$

$$ARL_- = \frac{\exp(-2\Delta b) + 2\Delta b - 1}{2\Delta^2}, \quad \Delta = -\delta - k \quad (10)$$

and $b = h + 1.166$. If $\Delta = 0$, then ARL_+ or $ARL_- = b^2$.

Grouped Data

The formulas in this document assume that the data to be analyzed are individual measurements. If analyzing grouped data, substitute \bar{x}_j in place of x_j and σ/\sqrt{n} in place of σ .