

Design of Experiments – Variance Component Designs

Summary

The *Variance Component Designs* selection on the *Create Design* menu creates experiments designed to quantify the sources of variability in a process. The runs are arranged in a hierarchical manner, with factors nested one within another. The primary output of such an experiment is determination of the relative contribution of different components to the overall variability in a response variable.

Example:

Box, Hunter and Hunter (1978) describe an experiment designed to determine the relative contribution of three components to the variability in the moisture content of a pigment paste. The study consisted of:

1. Selecting 15 different batches of pigment paste.
2. Taking 2 samples from each batch.
3. Measuring the moisture content of each sample twice.

From this design, the experimenters could estimate the variability from batch to batch, the variability of samples within a batch, and the repeatability of the measurement process. The relative magnitude of these 3 components was expected to provide important information about where the majority of the observed variability was being introduced.

Unlike the *Multi-factor Categorical* experiments, the factors in a *Variance Components* design are nested within each other. In the example, samples are nested within batches, while tests are nested within samples. When specifying the factors, it is important that they be specified in the order of nesting, from the topmost factor down.

Sample StatFolio: *doe varcomp.sgp*

Design Creation

To create a variance components design, select *Create Design* from the design of experiments menu and complete the dialog boxes as described below.

Dialog #1 – Design Type

The first dialog box displayed during design creation specifies the type of design to be created:

- **Design Class:** type of design to be created.
- **No. of Response Variables:** the number of response variables Y that will be measured during each experimental run. This number may range from 1 to 16.
- **No. of Experimental Factors:** the number of experimental factors X to be studied. This number may range from 2 to 8.
- **Comment:** a comment to appear on the output from the analysis procedures.

Dialog #2 – Experimental Factor

The second dialog box requests information about each experimental factor to be tested:

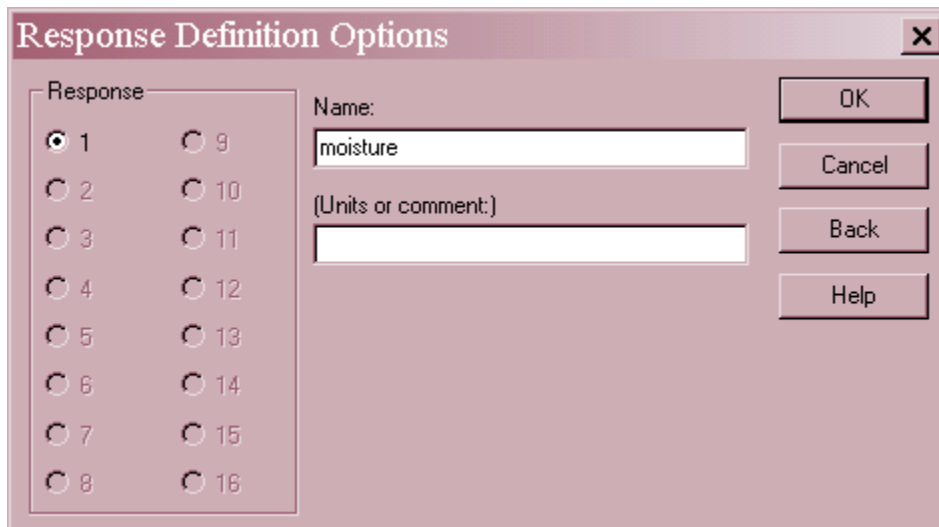
- **Name:** enter a name for the factor containing up to 32 characters. A column will be created in the datasheet for each factor.
- **No. of levels:** number of levels of the factor at which experiments will be performed.
- **Units or comment:** an optional label or comment of up to 64 characters to be included on the experiment worksheet.
- **Labels button:** Press this button to enter identifiers for each level:

Except for factor A, each factor is assumed to be nested in the factor immediately above it. For the current example, the information needed to complete the dialog boxes is shown below:

<i>Name</i>	<i>No. of levels</i>	<i>Labels</i>
batch	15	1, 2, ..., 15
sample	2	1, 2
test	2	1, 2

Dialog #3 – Response Variables

The third dialog box requests information about each of the response variables:

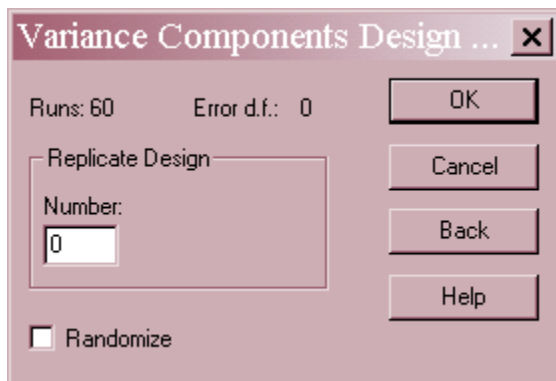


Click on the numbers 1, 2, 3, ..., one at a time and enter the following information for each response variable in the experiment:

- **Name** - a name for each response containing up to 32 characters.
- **Units or comment** - an optional label or comment of up to 64 characters to be included on the experiment worksheet.

Dialog #4 – Design Options

The fourth dialog box is used to specify the number of replicates at each combination of the factor levels and whether the order of the runs should be randomized:



- **Replicate Design:** number of additional runs at each combination of the factor levels (not counting the first run).
- **Randomize:** check to put the runs in random order within each set of replicates.

The above dialog box requests a total of 60 runs, 1 each at the $15 \times 2 \times 2 = 60$ combinations of the factor levels.

Design Attributes

Once the dialog boxes have been completed, the *Design Attributes* window will be displayed:

<u>Variance Components Design Attributes</u>		
Design class: Variance Components		
File name: <Untitled>		
Comment: Pigment paste example		
Base Design		
Number of experimental factors: 3		
Number of responses: 1		
Number of runs: 60		
Randomized: No		
<i>Factors</i>	<i>Levels</i>	<i>Units</i>
batch	15	
sample	2	
test	2	
<i>Responses</i>	<i>Units</i>	
moisture		

It contains a summary of the generated design. To save the design, select *Save Design File* from the main *File* menu. The example is saved in the *pigment paste.sfx* file.

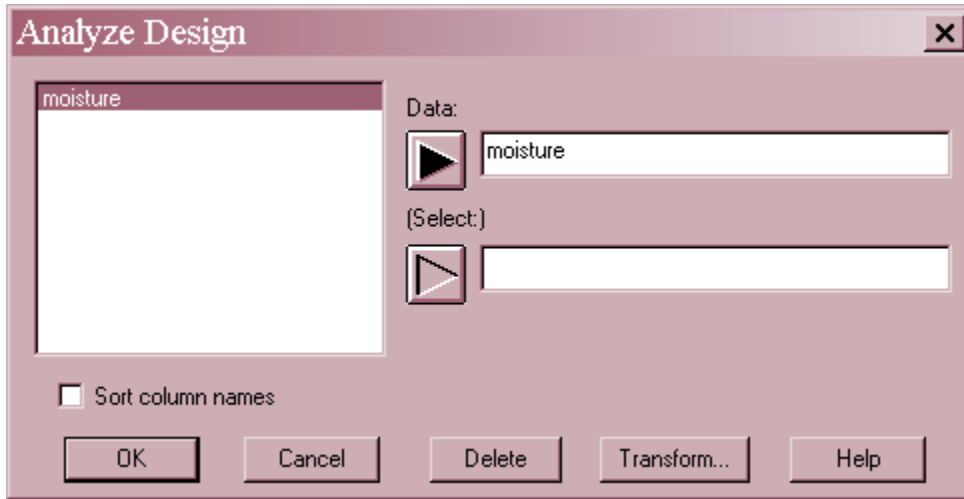
Worksheet

The experimental runs are entered into the datasheet automatically. They may also be displayed in the *Worksheet*, a portion of which is shown below:

Worksheet for pigment paste.sfx - Pigment paste example				
<i>run</i>	<i>batch</i>	<i>sample</i>	<i>test</i>	<i>moisture</i>
1	1	1	1	
2	1	1	2	
3	1	2	1	
4	1	2	2	
5	2	1	1	
6	2	1	2	
7	2	2	1	
8	2	2	2	
9	3	1	1	
10	3	1	2	
11	3	2	1	
...	

Analyzing the Data

After the results of the experimental runs have been entered, *Analyze Data* can be selected from the DOE menu. A dialog box will be displayed requesting the column containing the response to be analyzed:



- **Data:** column containing the values of the response variable to be analyzed.
- **Select:** subset selection.

When OK is pressed, the program will invoke the *Variance Components Analysis* procedure. Full details of that analysis is contained in the *Variance Components Analysis* documentation.

Of particular interest in the current example are several tables and graphs:

Analysis Summary

The *Analysis Summary* displays the percentage contribution of each component to the overall variance of the response variable:

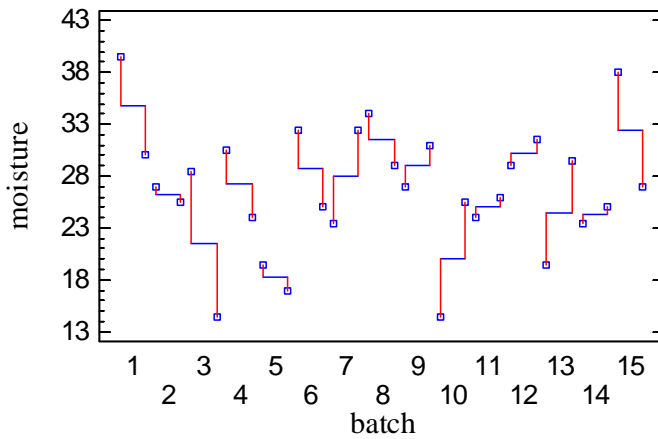
<u>Variance Components Analysis</u>					
Dependent variable: moisture					
Factors:					
batch					
sample					
test					
Number of complete cases: 60					
Analysis of Variance for moisture					
<i>Source</i>	<i>Sum of Squares</i>	<i>Df</i>	<i>Mean Square</i>	<i>Var. Comp.</i>	<i>Percent</i>
TOTAL (CORRECTED)	2108.18	59			
batch	1210.93	14	86.4952	7.12798	19.49
sample	869.75	15	57.9833	28.5333	78.01
test	27.5	30	0.916667	0.916667	2.51

In the example, the majority of the variability was introduced from samples within the same batch, indicating a possible problem with mixing.

Variance Components Plot

The *Variance Components Plot* displays the average value of each level of one factor when nested within another:

Variance Component Plot

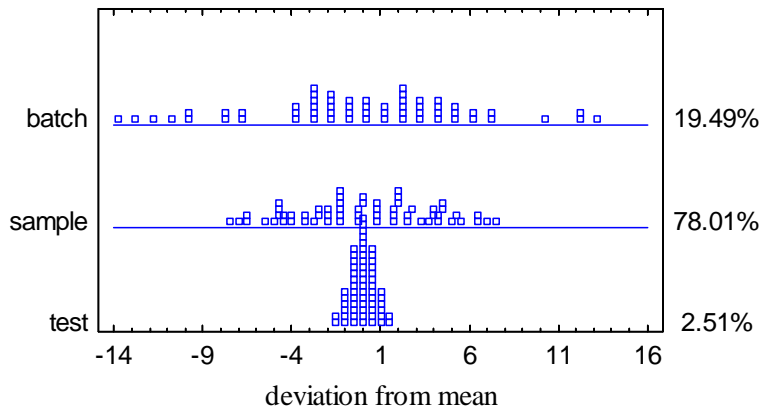


In the plot above, each point represents the average of the two tests run on each sample from each batch. Note the large differences between samples coming from the same batch.

Component Deviation Plot

The *Component Deviation Plot* displays the deviation of each observation from the mean of all observations at the same level of a selected factor:

Component Deviation Plot for moisture



The bottom line of the plot shows the deviation of each observation from the mean of all *tests* made on the same sample. The middle line shows the deviation of each observation from the mean of all *samples* taken from the same batch. The top line shows the deviation of each observation from the mean of all *batches* (the grand mean). It is easy to see that a large amount of variability is introduced at the middle level, from differences between samples taken from the same batch.