# DOE Wizard – Multilevel Factorial Designs

## Summary

The *Multilevel Factorial* designs are used to study the effects of q quantitative factors. The user begins by specifying a range over which each factor is to be varied and the number of different levels at which it will be studied. The program creates a datasheet containing all combinations of the different levels of the variables.

One important use of this type of design is to create a set of candidate runs from which an optimal subset can be selected using the *Select runs* feature of the DOE Wizard, which selects a subset of runs so as to achieve a D-optimal design.

## Example

As an example, suppose the experimenter wished to study 3 factors over the region shown below:

Factor	Low	High	Units
temperature	160	180	degrees
flow rate	50	80	liters/min
concentration	20	40	%

Unfortunately, rather than designing a well-balanced experiment from the beginning, he decided to try a few runs that he thought might work well. The runs performed are shown below, together with the measured response:

run	temperature	flow rate	concentration	yield
1	160	50	20	18.7
2	180	75	40	32.9
3	165	80	25	22.1
4	175	80	35	29.0
5	170	55	25	22.8
6	180	75	35	29.9

After 6 runs and little progress, he decided to stop and design a good experiment. He did not, however, want to throw away the runs that had already been performed, each of which was costly and time-consuming.

This document will demonstrate how the DOE Wizard can be used to help select additional experiments to augment the runs that have already been performed.

Sample StatFolio: doewiz multilevel.sgp

## **Design Creation**

To begin the design creation process, start with an empty StatFolio. Select DOE - Experimental Design Wizard to load the DOE Wizard's main window. Then push each button in sequence to create the design.

#### *Step #1 – Define Responses*

The first step of the design creation process displays a dialog box used to specify the response variables. For the current example, there is a single response variable:

		eated using a multilevel fac	ctorial							
	f responses: 1		A we have		C	Taurah	lana at (1 E)	0		
esponse 1	yield	Units	Analyze Mean	-	Goal Maximize 💌	Target	Impact (1-5) 3.0	Sensitivity	Minimum	Maximum
2	Var_2		Mean		Maximize 💌	0.5	3.0	Medium 💌	·	
3	Var_3		Mean	-	Maximize 💌	0.5	3.0	Medium 💌		
4	Var_4		Mean	-	Maximize 💌	0.5	3.0	Medium 💌		
5	Var_5		Mean	-	Maximize 💌	0.5	3.0	Medium 💌		
6	Var_6		Mean	-	Maximize 💌	0.5	3.0	Medium 💌		
7	Var_7		Mean	Ŧ	Maximize 💌	0.5	3.0	Medium 💌		
8	Var_8		Mean	-	Maximize 💌	0.5	3.0	Medium 💌		
9	Var_9		Mean	-	Maximize 💌	0.5	3.0	Medium 💌		
10	Var_10		Mean	7	Maximize 💌	0.5	3.0	Medium 💌		
11	Var_11		Mean	7	Maximize 💌	0.5	3.0	Medium 💌		
12	Var_12		Mean	<b>v</b>	Maximize 💌	0.5	3.0	Medium 💌		
13	Var_13		Mean	-	Maximize 💌	0.5	3.0	Medium 💌		
14	Var_14		Mean	<b>v</b>	Maximize 💌	0.5	3.0	Medium 💌		
15	Var_15		Mean	<b>v</b>	Maximize 💌	0.5	3.0	Medium 💌		
16	Var_16		Mean	-	Maximize 💌	0.5	3.0	Medium 💌	Γ	

- **Name**: The name for the variable is *yield*.
- Units: The units are not relevant in this example.
- Analyze: The parameter of interest is the *mean* yield.
- **Goal**: The goal of the experiment is to maximize the yield.
- Impact: The relative importance of each response (not relevant if only one response).
- Sensitivity: The importance of being close to the best desired value.
- Minimum and Maximum: Range of desirable values for the response.

#### *Step #2 – Define Experimental Factors*

The second step displays a dialog box on which to specify the factors that will be varied. In the current example, there are 3 factors:

Design	of Experiments Wiza	rd - Define Factors					×
Design	file: <untitled></untitled>						
Comme	nt: A fix-up design create	d using a multilevel factorial	l				
Numbe	r of controllable process fa	ctors: 3 📩 Numbe	er of controllable mi	xture components:		Number of n	oise factors: 0
Factor	Name	Units	Туре	Role	Low	High	Levels
A	temperature	degrees	Continuous 💌	Controllable	160	180	1,2,3,4
В	flow rate	liters/min	Continuous 💌	Controllable	50	80	1,2,3,4
С	concentration	%	Continuous 💌	Controllable	20	40	1,2,3,4
D	Factor_D		Continuous 💌		-1.0	1.0	1,2,3,4
E	Factor_E		Continuous 💌		-1.0	1.0	1.2.3.4
F	Factor_F		Continuous 💌		-1.0	1.0	1.2.3.4
G	Factor_G		Continuous 💌		-1.0	1.0	1,2,3,4
н	Factor_H		Continuous 💌		-1.0	1.0	1,2,3,4
I.	Factor_I		Continuous 💌		-1.0	1.0	1,2,3,4
J	Factor_J		Continuous 💌		-1.0	1.0	1,2,3,4
К	Factor_K		Continuous 💌		-1.0	1.0	1.2.3.4
L	Factor_L		Continuous 💌		-1.0	1.0	1,2,3,4
м	Factor_M		Continuous 💌		-1.0	1.0	1,2,3,4
Tota	l for controllable mixture co	mponents: 100.0			F	Factors A-M	Factors N-Z
	OK	Back	<	Cancel		H	felp

- Name Each factor must be assigned a unique name.
- Units Units are optional.
- **Type** Set the type of each factor to *Continuous*, since they can be set at any value within a continuous interval.
- The three factors are all *Controllable*.
- Low the lower level  $L_i$  for the factor.
- **High** the upper level  $U_i$  for the factor.

#### *Step #3 – Select Design*

The third step begins by displaying the dialog box shown below:

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Design o	f Expe	rimen	ts Wizard - S	ielect De	sign			×
Design fi								Robust Parameter Design
Commen	nt: <mark>A fix</mark> -	up desi	gn created using	; a multilev	el factorial			Combined array
		Segn	nent	Factors	Runs	Blocks	Design	C Crossed array
Optio	ons	Proce	ess factors	3	0	0	Press the Options button.	
Optio	ons	Mixtu	ire components	0	0	0		
Optio	ons			0	0	0		
		СОМ	BINED	3	0	0	Samples per run: 1	
	BLO	ГК	temperat	ure	flow	rate	concentration	
	020	UK .	degree		liters		%	
•								▼ 
			ОК		Cano	el	Rerandomize	Help

Since all of the factors are controllable process factors, only one *Options* button is enabled. Pressing that button displays a second dialog box:

Designs for Continuous or Two-Lev	el Factors 🛛 🔀
Design Class	OK
C Screening	Cancel
C Response Surface	
Multilevel Factorial	Help
Orthogonal Array	

Four general classes of designs are offered:

- 1. *Screening* designs intended to select the most important factors affecting a response. Most of the designs involve only 2 levels of each factor. The factors may be quantitative or categorical.
- 2. *Response Surface* designs intended to select the optimal settings of a set of experimental factors. The designs involve at least 3 levels of the experimental factors, which must be quantitative.

- 3. *Multilevel Factorial* designs involving different numbers of levels for each experimental factor. The factors must be quantitative.
- 4. *Orthogonal Array* a general class of designs developed by Genichi Taguchi. The factors may be quantitative or categorical.

If *Multilevel Factorial* is selected, a third dialog box will be displayed on which to indicate the number of levels at which each factor should be run:

Multilevel Factorial Des	ign Options		
Multilevel Factorial Des Factor temperature flow rate concentration	ign Options	Runs: 125 Error d.f.: 115 Replicate Design Number: 0	OK Cancel Back Help

- Levels the number of levels at which each factor should be run.
- **Replicate design** if a number other than 0 is entered, the entire design will be repeated the indicated number of times.
- **Randomize** check this box to randomly order the runs in the experiment. Randomization is generally a good idea, since it can reduce the effect of lurking variables such as trends over time. However, when replicating the examples in this documentation, **do not** randomize the designs.

For the current example, select 5 for each factor, resulting in a total of 5 x 5 x 5 = 125 runs at each combination of the factors.

The tentatively selected design is displayed in the *Select Design* dialog box:

#### STATGRAPHICS - Rev. 9/3/2009

Design	of Experi	ments Wizard - S	ielect D	esign			×
-	i file: <untit ent: A fix-u</untit 	led> p design created using	; a multile	vel factorial			Robust Parameter Design
		Segment	Factors	Runs	Blocks	Design	Crossed array
Opt	ions	Process factors	3	125	1	Multilevel Factorial	
-							
Upt	ions	Mixture components	0	0	0		
Opt	ions		0	0	0		
		COMBINED	3	125	1	Samples per run: 1	
	BLOCI	K temperat	ure	flow	rate	concentration	
		degree		liters	/min	%	
1	1	160.0		50.0		20.0	
2	1	165.0		50.0		20.0	
3	1	170.0		50.0		20.0	
4	1	175.0		50.0		20.0	
5	1	180.0		50.0		20.0	
6	1	160.0		57.5		20.0	
7	1	165.0		57.5		20.0	
8	1	170.0		57.5		20.0	
9	1	175.0		57.5		20.0	
10	1	180.0		57.5		20.0	
11	1	160.0		65.0		20.0	
12	1	165.0		65.0		20.0	
13	1	170.0		65.0		20.0	
14	1	175.0		65.0		20.0	-1
	1	180.0		65.0		20.0	
•							<b>&gt;</b>
		ок		Can	cel	Rerandomize	Help

If the design is acceptable, press OK to save it to the STATGRAPHICS DataBook and return to the DOE Wizard's main window, which should now contain a summary of the design:

器 Đ	cperin	mental I	Design W	izar	d													
4-1	Step	p 1:Define	e responses		Step 3:S	elect de	sign	Step 5:	Selecti	runs		Step 7	Save experi	iment	Step 9:0 pl	imize responses	s Step 11:Au	igment design
Ľ	Step	2:Define	exp. factors	:	Step 4:S	pecify m	odel	Step 6:Ev	/aluate	desig	n	Step	) 8:Analyze d	ata	Step 10	: Save results	Step 12:	Extrapolate
Ext	erim	nental I	)esign W	/izaı	rd													~
Sten	1 · Def	fina tha ra	sponse vari	ishler	z to he mi	herroee												
Man			Analyze	Goa		Target	Impact	Sensiti	vity	Lon	$H_{1}$	gh						
yiel	d	1	Mean	Max	cimize		3.0	Media	ım									
Sten	2. Def	fine the ev	merimental	facto	ors to be t	varied												
Man			Units		vpe	Roli	;	Low	High	1	levels	٦						
A:te	empera	ature	degrees		ontinuou		trollable	160.0	180.0	0								
B:fl	low rat	te	liters/min	Co	ontinuou	s Cor	ıtrollable	50.0	80.0									
C:ce	oncent	ration	%	Co	ontinuou	s Cor	trollable	20.0	40.0									
Step	3: Sele	ect the ext	perimental o	lesig	n													
	e of	Design		<u> </u>	 Centerpo	oints	Centerpoi	nt De	sign is		Numl	er of	Total	Total	Error			
	tors	Туре			Per Bloc		Placement		ndomiz		Repli	ates	Runs	Blocks	D.F.			
Pro	cess	Multile	evel factoria	1	0		Random	No			0		125	1	115	]		
Num	ber of	'samples:	1															
Sten	4: Sne	cify the i	nitial model	ltoh	ie fit to th	ie experi	mental res	ults										
	tors	Model	Coeffic			luded effi		and										
	cess	quadrat				u												
			imal subset	oftl	he runs (o	optional)												
125 1	runs se	elected																~

#### Step #4: Specify Model

The next step in the design selection process is to specify the model that will be fit to the response data. Pressing the fourth button on the DOE Wizard's toolbar displays a dialog box to make that choice:

DOE Wizard Model Options		
Process Factors Model	Mixture Components Model	ОК
C Mean	🖲 Mean	Cancel
C Linear (Main Effects)	C Linear	
C 2-Factor Interactions	C Quadratic	Help
Quadratic	C Special Cubic	
C Cubic	C Cubic	
Include: A:temperature B:flow rate C:concentration AA AB AC BB BC CC	Exclude:	

The default *quadratic* model includes main effects for each of the 3 experimental factors, 3 twofactor interactions (shown as two-letter combinations with different factors), and 3 quadratic terms (shown as two-letter combinations with identical factors). Selected terms could be excluded by double-clicking on them with the left mouse button.

#### Step #5: Select runs

The next step is to select a subset of the 125 runs to add to the 6 runs already performed. Before pressing the button for step 5, the original 6 runs need to be added to the datasheet. Click on the DataBook and scroll to the 126<sup>th</sup> row. Then add the information for the original 6 runs, including the block number and measured response values:

C:\Docl	)ata <b>16\m</b> ultilevel	2.sgx				
	BLOCK	temperature	flow rate	concentration	yield	c_
		degrees	liters/min	\$		
120	1	180.0	72.5	40.0		
121	1	160.0	80.0	40.0		
122	1	165.0	80.0	40.0		
123	1	170.0	80.0	40.0		
124	1	175.0	80.0	40.0		
125	1	180.0	80.0	40.0		
126	1	160	50	20	18.7	
127	1	180	75	40	32.9	
128	1	165	80	25	22.1	
129	1	175	80	35	29.0	
130	1	170	55	25	22.8	
131	1	180	75	35	29.9	
132						
133						-
HAPH	multilevel2 B	C/D/E/F/G	K H / I / J / K	•		

After entering the original 6 runs, return to the DOE Wizard and press the button labeled *Step 5*. The following dialog box will be displayed:

BLOCK	temperature	flow rate	concentration	yield	
BLUCK	degrees	liters/min	%	yieiu	-
1 1	160.0	50.0	20.0		-
2 1	165.0	50.0	20.0		-
3 1	170.0	50.0	20.0		
4 1	175.0	50.0	20.0		-
5 1	180.0	50.0	20.0		-
5 1	160.0	57.5	20.0		-
7 1	165.0	57.5	20.0		-
8 1	170.0	57.5	20.0		
9 1	175.0	57.5	20.0		
0 1	180.0	57.5	20.0		
1 1	160.0	65.0	20.0		
2 1	165.0	65.0	20.0		
3 1	170.0	65.0	20.0		
4 1	175.0	65.0	20.0		
5 1	180.0	65.0	20.0		
6 1	160.0	72.5	20.0		
7 1	165.0	72.5	20.0		
8 1	170.0	72.5	20.0		
9 1	175.0	72.5	20.0		
20 1	180.0	72.5	20.0		
21 1	160.0	80.0	20.0		
2 1	165.0	80.0	20.0		
					•
mber of runs de:	-i		rithm D-efficiency:		
	sirea: Seler	ct runs using forward algoi	rithm D-erriciency:		
	Select	t runs using backward alg	orithm A-efficiency:		
del coefficients:	10	ply exchange algorithm at	G-efficiency:		

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In the bottom left is a field where the desired number of runs should be specified. It is usually a good idea to select at least 3 more runs than there are coefficients in the selected model.

To select the runs, press either of the 2 buttons on the dialog box. Since the number of ways of choosing subsets of the candidate runs is too large to check all possibilities, STATGRAPHICS (like other programs) uses a selection algorithm to choose a subset. The *Forward* method begins with the runs that have already been performed (if any) and adds runs one at a time, adding at each step the run that adds the most to the D-efficiency of the experiment. The *Backward* method begins with all of the candidate runs and removes runs one at a time, removing at each step the run that adds the least to the D-efficiency of the experiment. In either case, once the desired number of runs has been selected, an *exchange* algorithm can be performed. This algorithm tests all pairs of runs consisting of one that has been selected and one that has not, making any exchanges that would increase the efficiency of the experiment. Exchanges continue until no further improvements can be made by switching one run that's been selected with one run that has not been selected.

	BLOCK	temperature	flow rate	concentration	yield	
		degrees	liters/min	%		
1 1		160.0	50.0	20.0		
2 1		165.0	50.0	20.0		
3 1		170.0	50.0	20.0		
4 1		175.0	50.0	20.0		
5		180.0	50.0	20.0		
6 1		160.0	57.5	20.0		
7 1		165.0	57.5	20.0		
3 1		170.0	57.5	20.0		
3 1		175.0	57.5	20.0		
0 1		180.0	57.5	20.0		
1		160.0	65.0	20.0		
2 1	1	165.0	65.0	20.0		
3 1	1	170.0	65.0	20.0		
4 1		175.0	65.0	20.0		
5 1		180.0	65.0	20.0		
6 1		160.0	72.5	20.0		
7 1		165.0	72.5	20.0		
8 1	1	170.0	72.5	20.0		
9 1	1	175.0	72.5	20.0		
0 1		180.0	72.5	20.0		
1 1	•	160.0	80.0	20.0		
2 1		165.0	80.0	20.0		
						<b></b>
umber of runs desired: Select runs using forward algorithm D-efficiency: 37,40%						
13		Select	runs using backward alg	orithm A-efficiency	: 16.69%	
del co	pefficients: 10		oly exchange algorithm at	end G-efficiency	G-efficiency: 32.23%	

When the algorithm is complete, the selected rows will be highlighted in red:

The efficiencies of the selected design will also be displayed. You can try another algorithm or press OK to accept the selection, at which point the rows of the datasheet will be reduced to the selected runs:

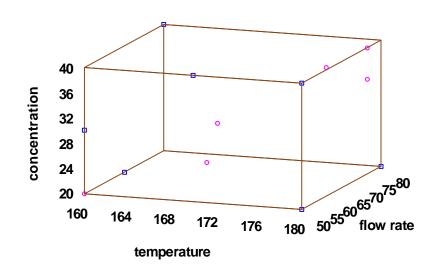
C:\DocData16\multilevel2.sgx									
	BLOCK	temperature	flow rate	concentration	yield	c_			
		degrees	liters/min	\$					
1	1	180	50	20					
2	1	160	65	20					
3	1	180	80	20					
4	1	160	50	30					
5	1	170	50	40					
6	1	180	50	40					
7	1	160	80	40					
8	1	160	50	20	18.7				
9	1	180	75	40	32.9				
10	1	165	80	25	22.1				
11	1	175	80	35	29.0				
12	1	170	55	25	22.8				
13	1	180	75	35	29.9				
14						-			
H   H multilevel2 / B / C / D / E / F / G / H / I / J / K									

The main DOE Wizard window will reflect the design:

🖺 Experimental Design Wizard													
Step 1:Define responses Step 3:Select design Step 5:Select runs Step 7:Save experiment Step 9:Optimize responses S	Step 11:Augment design												
Step 2:Define exp. factors Step 4:Specify model Step 6:Evaluate design Step 8:Analyze data Step 10: Save results	Step 12:Extrapolate												
Experimental Design Wizard													
Step 1: Define the response variables to be measured													
Name Units Analyze Goal Target Impact Sensitivity Low High													
yield Mean Maximize 3.0 Medium													
Step 2: Define the experimental factors to be varied													
Name Units Type Role Low High Levels	=												
Attemperature degrees Continuous Controllable 160.0 180.0													
B:flow rate liters/min Continuous Controllable 50.0 80.0													
C:concentration % Continuous Controllable 20.0 40.0													
Step 3: Select the experimental design													
Type of Design Centerpoints Centerpoint Design is Number of Total Total Error													
Factors Type Per Block Placement Randomized Replicates Runs Blocks D.F.													
Process Multilevel factorial 0 Random No 0 125 1 115													
Number of samples: 1													
Step 4: Specify the initial model to be fit to the experimental results													
Factors Model Coefficients Excluded effects													
Process quadratic 10													
Step 5: Select an optimal subset of the runs (optional)													
13 runs selected													

If the selection is acceptable, press *Step 7: Save experiment* to save the reduced number of runs.

You can also use the *Design Plot* to display the final design:



## A fix-up design created using a multilevel factorial

The original runs are shown as red asterisks, while the added runs are shown as blue squares.