Design of Experiments Wizard

Summary

The *Experimental Design* section of STATGRAPHICS contains a new wizard that assists users in constructing and analyzing designed experiments. It guides the user through twelve important steps. The first 7 steps are executed before the experiment is run:

Step 1: Defining the response variables.

Step 2: Defining the experimental factors.

Step 3: Selecting the appropriate experimental design.

Step 4: Defining the model to be fit to the data.

Step 5: Selecting an optimal subset of the experimental runs (if creating a D-optimal design).

Step 6: Evaluating the design.

Step 7: Saving the experiment that has been created.

The final 5 steps are executed after the experiment has been performed:

Step 8: Analyzing the results by constructing a statistical model for each response variable.

Step 9: Finding the setting of the experimental factors that optimize the responses.

Step 10: Saving the results.

Step 11: Augmenting the design if necessary by adding additional runs.

Step 12: Extrapolating the models beyond the experimental region to search for locations that may yield even better results.

This document describes the basic function of the DOE Wizard.

Invoking the DOE Wizard

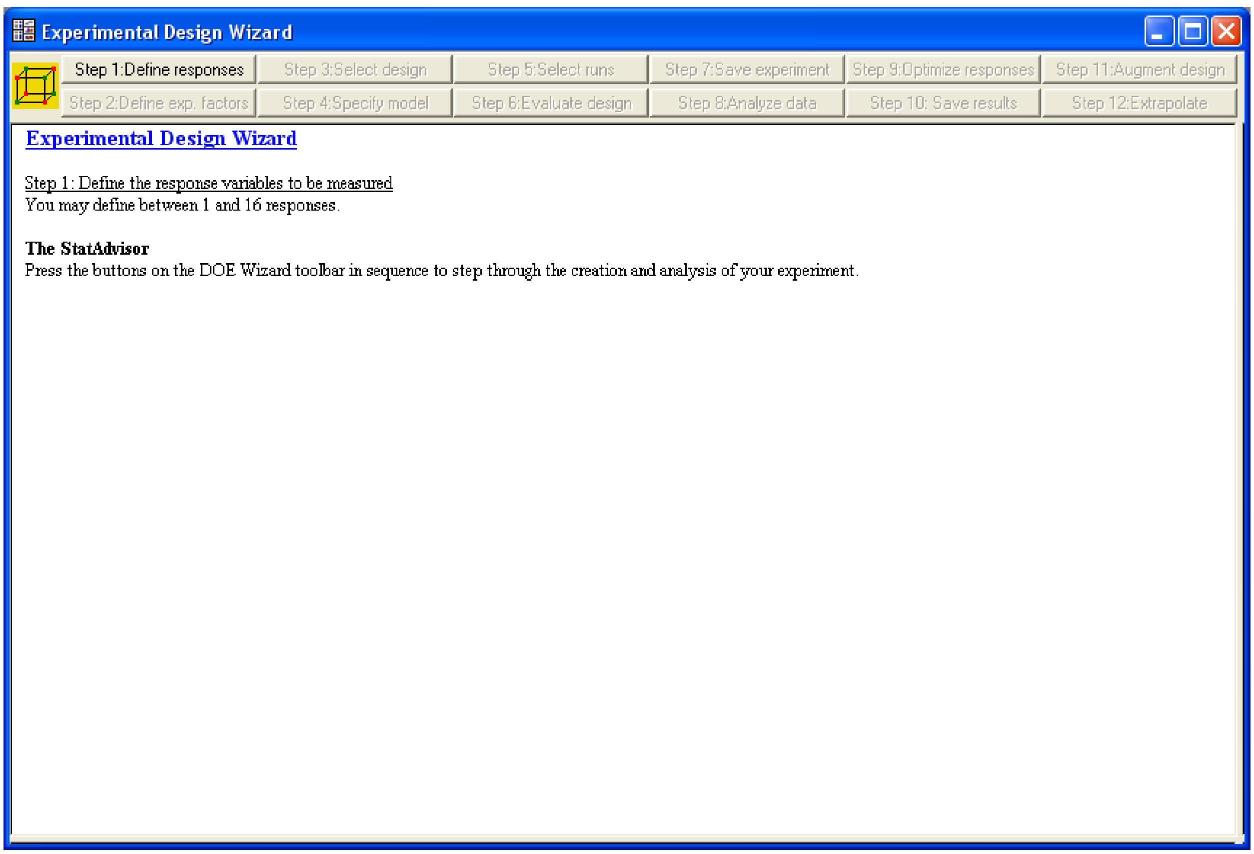
Before invoking the DOE Wizard, you must start with an empty StatFolio. Once the design is constructed, it will automatically be saved in sheet A of the STATGRAPHICS DataBook.

To start the wizard:

1. If using the classic STATGRAPHICS menu, select *DOE – Experimental Design Wizard* from the main menu.

2. If using the STATGRAPHICS Six Sigma menu, select *Improve – Experimental Design Wizard*.

When the wizard starts up, it will create the window shown below:



To construct an experiment, press each wizard button in sequence. As the design is constructed, information about the experiment will be added to the window.

<u>Example</u>

For illustration, we will consider the example from Myers, Montgomery and Anderson-Cook (2009, p. 298) of an experiment designed to study the effect of three factors on the strength of breadwrapper stock. The experimental factors are:

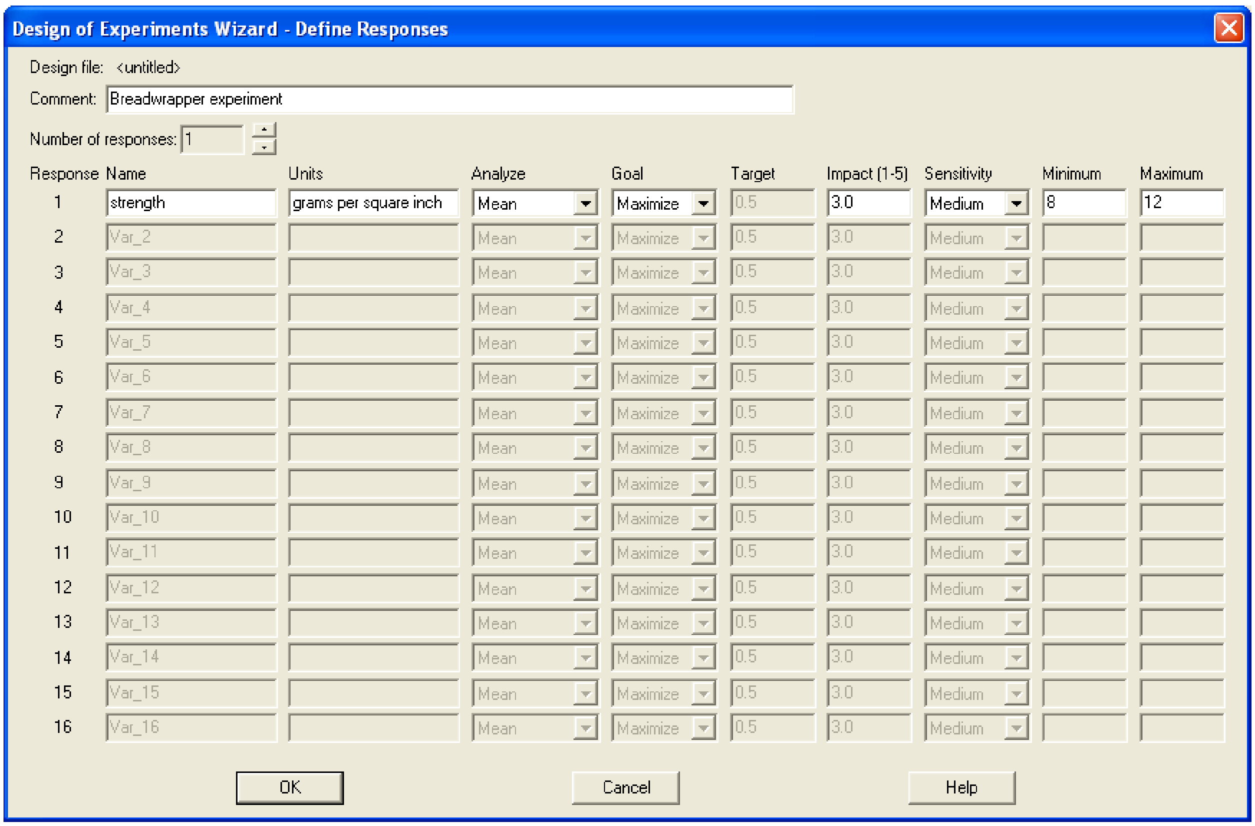
> X_1 – sealing temperature in degrees F
>
> X_2 – cooling bar temperature in degrees F
>
> X_3 – percent polyethylene additive

The response is:

> Y – strength in grams per square inch

Step 1: Define Responses

The first step in creating an experiment using the DOE Wizard is to define the response variables that will be recorded each time an experimental run is performed. This is accomplished by pressing the "Step 1: Define responses" button and completing the dialog box shown below:



The following information should be entered:

> *Comment* – a comment to appear at the top of the output when the results of the experiment are analyzed.

Number of responses – the number of response variables that will be recorded each time an experiment is performed, between 1 and 16.

Name – a name for each response. Since variable names will become column names in the datasheet when the experiment is constructed, they may have between 1 and 32 characters.

Units – the units of each response (if any).

Analyze – the statistic to be analyzed. This setting is only relevant if you will be collecting multiple samples during each run, or if you create a robust parameter design with crossed factors.

- **Mean** – creates a model for the mean response. If Y_{ij} equals the measurement obtained from the *i*-th sample collected during the *j*-th run and letting *m* be the number of samples collected during that run, then the mean is calculated from

$$\bar{Y}_j = \frac{\sum_{i=1}^{m} Y_{ij}}{m} \quad (1)$$

- **Std. deviation** – creates a model for the standard deviation of the response. The standard deviation observed during the *j*-th run is calculated from

$$s_j = \sqrt{\frac{\sum_{i=1}^{m} \left(Y_{ij} - \bar{Y}_j\right)^2}{m-1}} \quad (2)$$

- **C.V.** – the coefficient of variation, calculated from

$$CV_j = 100 \frac{s_j}{\bar{Y}_j} \% \quad (3)$$

- **SNR: -10 log s^2** – a "signal-to-noise ratio" developed by Prof. Genichi Taguchi for cases in which it is desired that the response equal a target value and the response mean and variance can be altered separately. The statistic is calculated from

$$SNRT1_j = -10 \log(s_j^2) \quad (4)$$

- **SNR: target** – a "signal-to-noise ratio" developed by Taguchi for cases in which it is desired that the response equal a target value and the response mean and variance cannot be altered separately. The statistic is calculated from

$$SNRT2_j = 10 \log\left(\frac{\bar{Y}_j^2}{s_j^2}\right) \quad (5)$$

- **SNR: larger –** a “signal-to-noise ratio” developed by Taguchi for cases in which it is desired that the response be maximized. The statistic is calculated from

$$SNRL_j = -10\log\sum_{i=1}^{m}\left(\frac{1}{mY_{ij}^2}\right) \tag{6}$$

- **SNR: smaller** – a “signal-to-noise ratio” developed by Taguchi for cases in which it is desired that the response be minimized. The statistic is calculated from

$$SNRS_j = -10\log\sum_{i=1}^{m}\left(\frac{Y_{ij}^2}{m}\right) \tag{7}$$

The selected statistic will automatically be calculated from the multiple samples taken during each run or from the runs in the outer array if a robust parameter design with crossed factors is constructed. For simple experiments with only one measurement taken during each experimental run, leave the statistic set to *Mean*.

Additional information should also be entered for use during response optimization:

Goal – the goal for each response statistic. You may select *Maximize, Minimize*, or *Hit target*. Note that all of Taguchi’s signal-to-noise ratios are defined so as to be *maximized*. The goal for the standard deviation or C.V. would usually be set to *minimize*.

Target – the desired target if *Goal* is set to *Hit target*.

Impact – a number between 1.0 and 5.0 that describes the relative importance of each response.

Sensitivity – how important it is to be close to the desired goal. This affects the shape of the desirability functions created during response optimization.

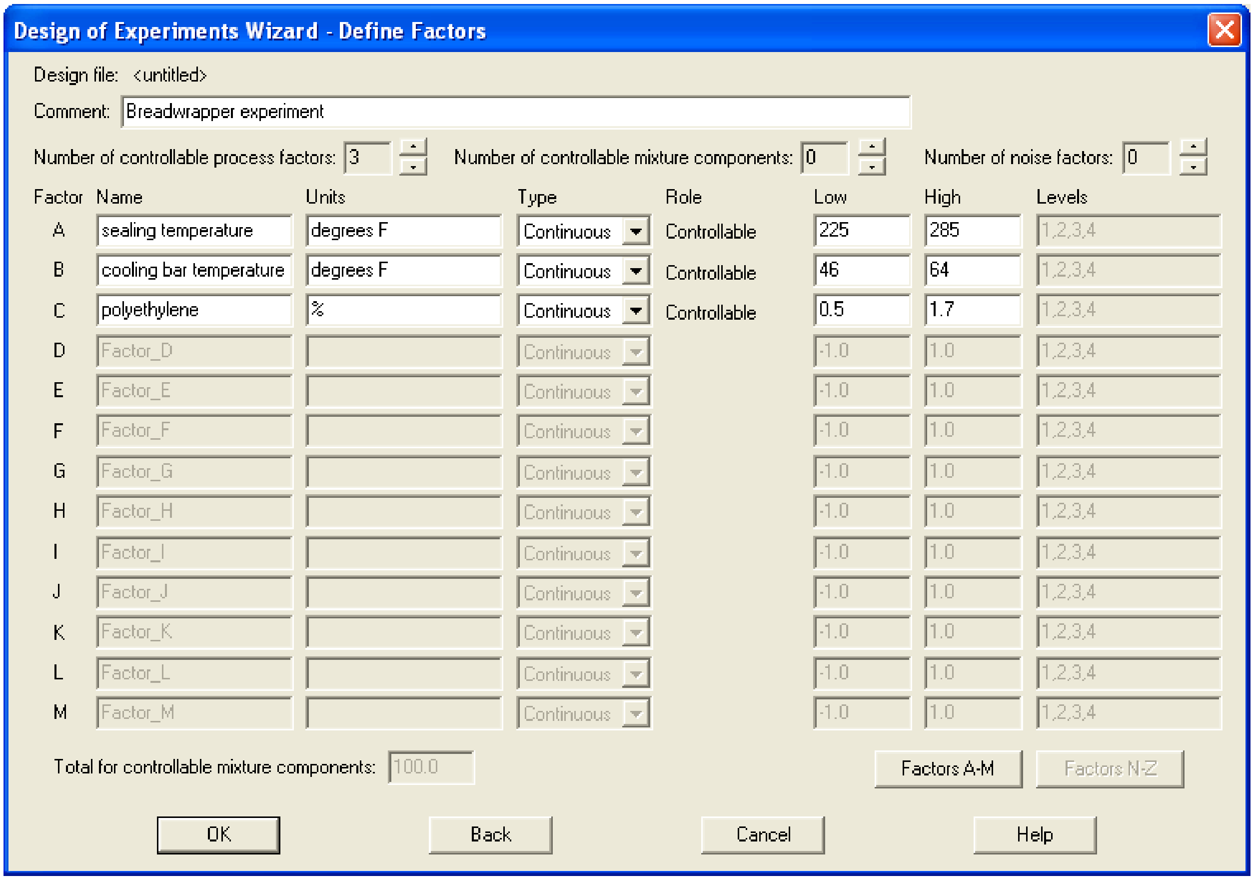
Minimum – the value below which the response is completely unacceptable (if *Goal* is set to *Maximize* or *Hit target*) or completely acceptable (if *Goal* is set to *Minimize*). If this value is left blank, it will be set equal to the smallest value observed during the experiment.

Maximum – the value above which the response is completely unacceptable (if *Goal* is set to *Minimize* or *Hit target*) or completely acceptable (if *Goal* is set to *Maximize*). If this value is left blank, it will be set equal to the largest value observed during the experiment.

For the current example, the mean value of *strength* is to be maximized. Any value below 8 grams per square inch is deemed completely unacceptable, while any value of 12 or greater is completely acceptable. As will be seen, setting *sensitivity* to “Medium” implies that the desirability of strength increases in a linear fashion between 8 and 12.

Step 2: Define Experimental Factors

The second step in creating an experiment is to define the experimental factors that will be varied during the experiment. This is accomplished by completing the dialog box shown below:



The following information should be entered:

Comment – a comment to appear at the top of the output when the results of the experiment are analyzed.

Number of controllable process factors – the number of controllable non-mixture factors that will be varied during the experiment, between 0 and 16.

Number of controllable mixture components – the number of controllable mixture components that will be varied during the experiment, between 0 and 12.

Number of noise factors – the number of normally uncontrollable factors that will be varied during the experiment, between 0 and 15. These factors are used in robust parameter studies to determine setting of the controllable factors with minimum process variation. Although normally uncontrollable, these factors must be controlled during the experiment.

Name – a name for each factor. Since factor names will become column names in the datasheet when the experiment is constructed, they may have between 1 and 32 characters. (Press the *Factors N-Z* button to enter the information for factors 17 through 26).

Units – the units of each factor (if any).

Type – the type of factor. Factors are divided into 3 types:

- **Categorical** – a factor that can only take a discrete set of values, such as *A* or *B*.
- **Continuous** – a factor that can be varied over a continuous range.
- **Mixture** – a factor that represents the amount of a component contained in a mixture.

Role – the role of each factor, either *Controllable* or *Noise*. Controllable factors are factors that can be controlled and whose effects are of primary interest. Noise factors are factors that cannot normally be controlled and whose effect on the response needs to be minimized.

Low – for continuous or mixture factors, the low end of the range at which the factor will be set during the experiment.

High – for continuous or mixture factors, the high end of the range at which the factor will be set.

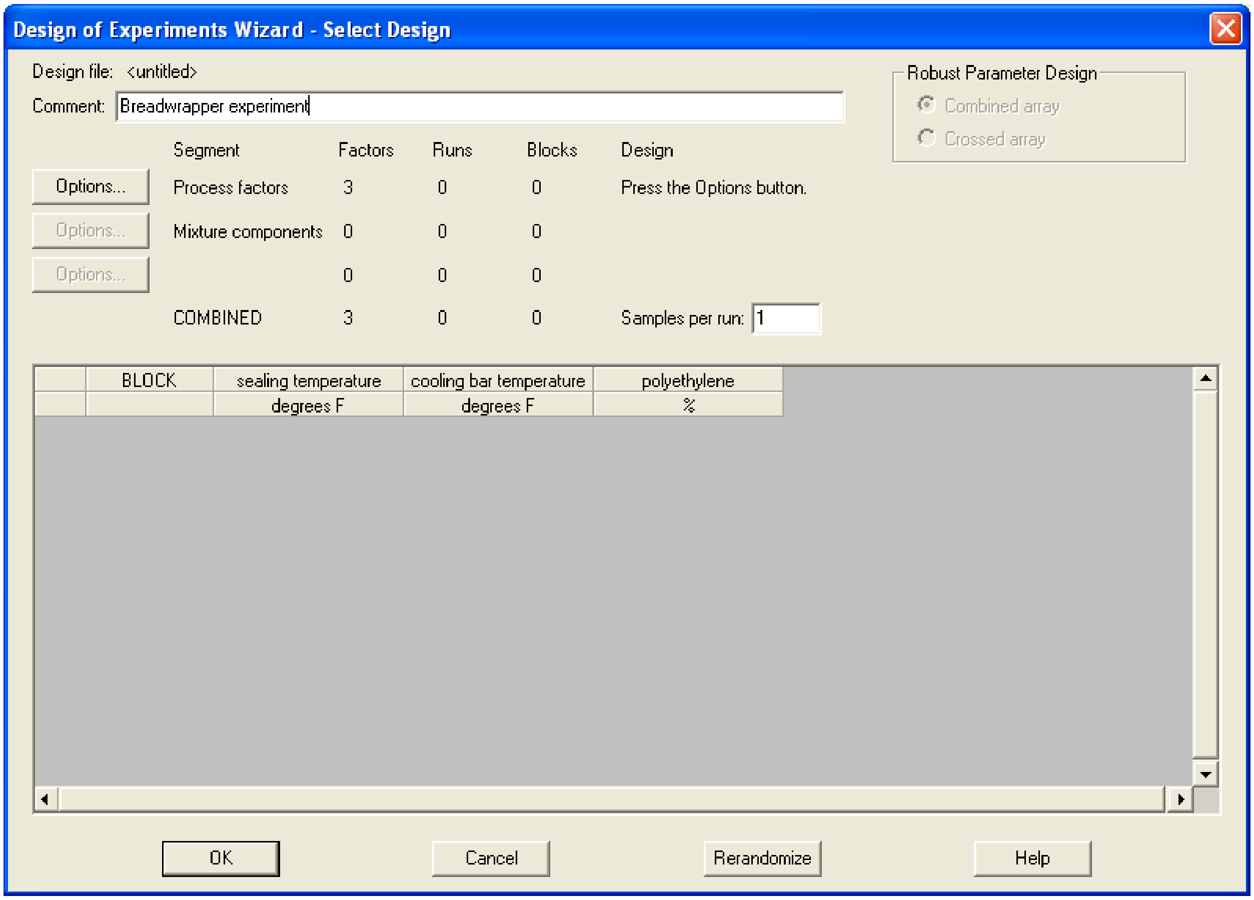
Levels – for categorical factors, the specific levels that will be used in the experiment. Separate each level with a comma (,).

Total for controllable mixture components - the total to which the controllable mixture components must sum.

The total number of factors cannot exceed 26. In addition, there must be at least 1 controllable process factor or 2 controllable mixture components.

Step 3: Select Design

The third step in creating an experiment is to select an experimental design. The first dialog box displayed in this step is shown below:



If the experiment involves both controllable and noise factors, the *Robust Parameter Design* (RPD) field will be active. In an RPD, the primary goal is to determine the settings of the controllable factors that achieve the desired response goals while minimizing the amount of variability induced by the noise factors:

1. *Combined array* – This approach constructs a single design for both the controllable and noise factors. It develops a single model for all of the factors and uses the estimated interactions between controllable factors and noise factors to determine the variability transmitted by the noise factors (see Montgomery et al., 2009).

2. *Crossed array* – This approach constructs separate designs for the controllable factors (inner array) and the noise factors (outer array). It directly estimates the variance caused by the noise factors using the multiple runs at each combination of the controllable factors (see Taguchi, 1987).

The final experiment will consist of up to 3 sections:

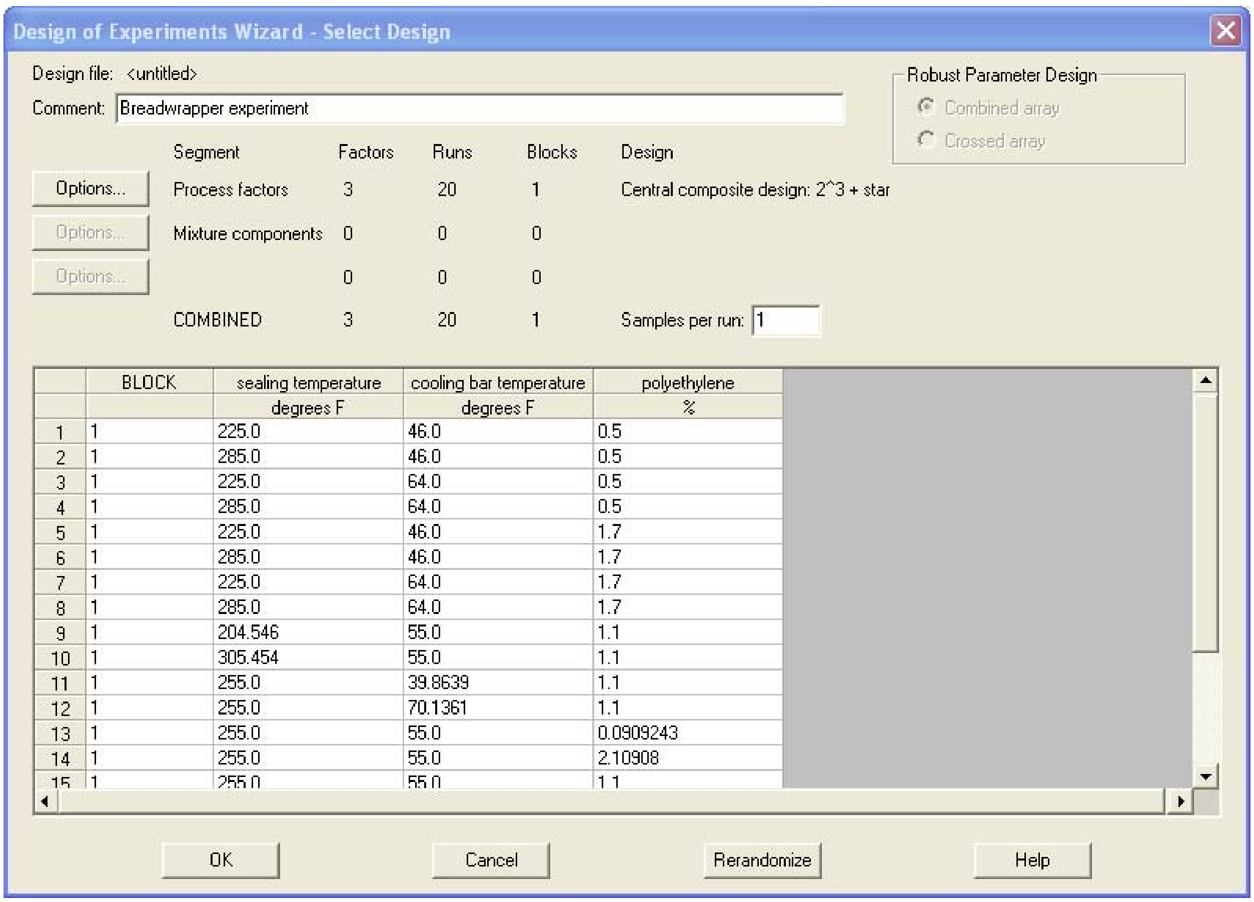
1. A section in which the *controllable non-mixture (“process”) factors* will be varied. If using the combined array approach, any noise factors will also be included in this section.

2. A section in which the *controllable mixture factors* will be varied.

3. A section in which the *noise factors* will be varied. This section is not included if using the RPD combined array approach.

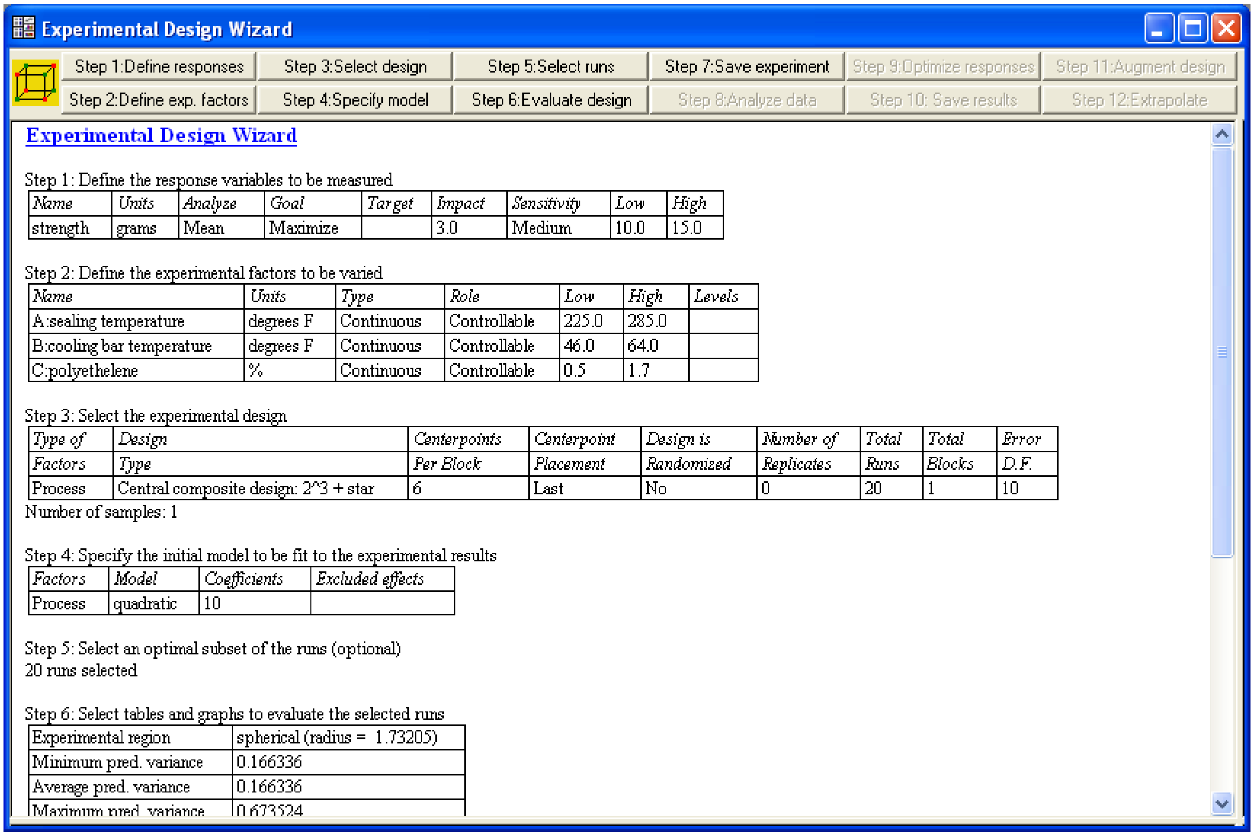
To construct the design:

1. Push the associated *Options* button for each section that applies to your factors. This will display one or more dialog boxes from which to select design options. Once a selection is made, each run in the design will be displayed in the data section in the bottom half of the dialog box.



2. Set the *Samples per run* field to a number greater than or equal to 1. This represents the number of samples that will be obtained each time an experimental run is performed. If more than one *samples per run* are specified, you will be able to analyze various statistics included the within-run variability.

3. Press *OK* to return to the main DOE Wizard window, which will reflect all of the selections:



Experimental Design Wizard

Step 1: Define the response variables to be measured

Name	*Units*	*Analyze*	*Goal*	*Target*	*Impact*	*Sensitivity*	*Low*	*High*
strength	grams	Mean	Maximize		3.0	Medium	10.0	15.0

Step 2: Define the experimental factors to be varied

Name	*Units*	*Type*	*Role*	*Low*	*High*	*Levels*
A:sealing temperature	degrees F	Continuous	Controllable	225.0	285.0	
B:cooling bar temperature	degrees F	Continuous	Controllable	46.0	64.0	
C:polyethelene	%	Continuous	Controllable	0.5	1.7	

Step 3: Select the experimental design

Type of Factors	*Design Type*	*Centerpoints Per Block*	*Centerpoint Placement*	*Design is Randomized*	*Number of Replicates*	*Total Runs*	*Total Blocks*	*Error D.F.*
Process	Central composite design: 2^3 + star	6	Last	No	0	20	1	10

Number of samples: 1

Step 4: Specify the initial model to be fit to the experimental results

Factors	*Model*	*Coefficients*	*Excluded effects*
Process	quadratic	10	

Step 5: Select an optimal subset of the runs (optional)
20 runs selected

Step 6: Select tables and graphs to evaluate the selected runs

Experimental region	spherical (radius = 1.73205)
Minimum pred. variance	0.166336
Average pred. variance	0.166336
Maximum pred. variance	0.673524

At the same time, the selections will be saved into sheet A of the main STATGRAPHICS databook:

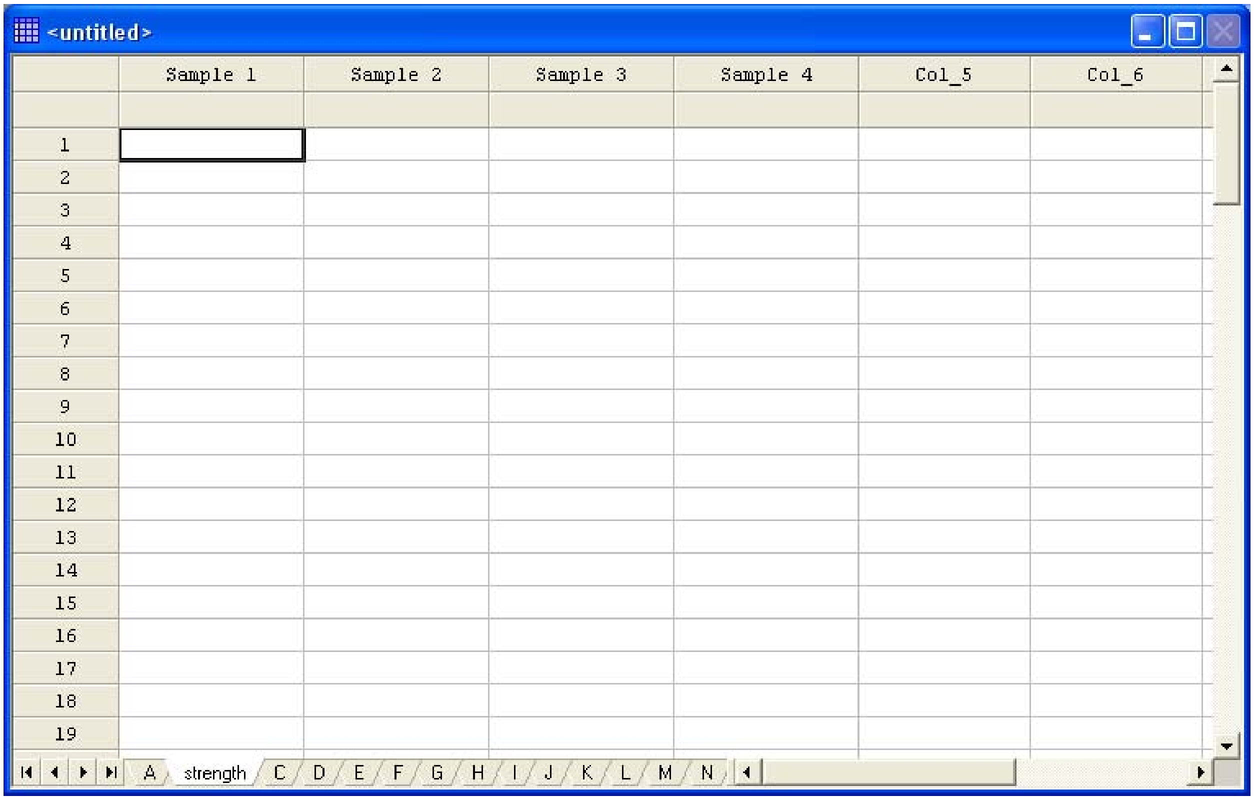
<untitled>

	BLOCK	sealing	cooling bar	polyethylene	strength
		degrees F	degrees F	%	grams per
1	1	225.0	46.0	0.5	
2	1	285.0	46.0	0.5	
3	1	225.0	64.0	0.5	
4	1	285.0	64.0	0.5	
5	1	225.0	46.0	1.7	
6	1	285.0	46.0	1.7	
7	1	225.0	64.0	1.7	
8	1	285.0	64.0	1.7	
9	1	204.546	55.0	1.1	
10	1	305.454	55.0	1.1	
11	1	255.0	39.8639	1.1	
12	1	255.0	70.1361	1.1	
13	1	255.0	55.0	0.0909243	
14	1	255.0	55.0	2.10908	
15	1	255.0	55.0	1.1	
16	1	255.0	55.0	1.1	
17	1	255.0	55.0	1.1	
18	1	255.0	55.0	1.1	
19	1	255.0	55.0	1.1	
20	1	255.0	55.0	1.1	

A / B / C / D / E / F / G / H / I / J / K

There is a separate row in the datasheet for experimental run. (Note: once the design has been created, you can edit the runs in the datasheet to round off settings if desired.)

If you had specified that multiple samples would be obtained during each run, datasheets B through Z will be used to hold the results. For example, if it had been indicated that each run would contain 4 samples, sheet B would be renamed "strength" and the first 4 columns in that sheet named as shown below:

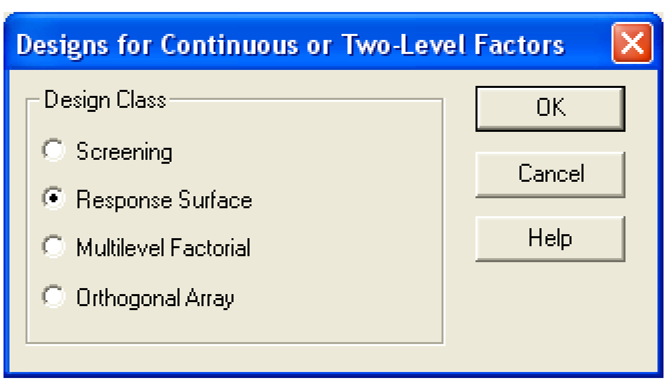


The response columns in sheet A would be linked to the other sheets and the statistics automatically calculated from the data in those sheets.

During the design selection process, other dialog boxes may be displayed. The sections below describe these dialog boxes.

Continuous Factors

If the design contains two or more process factors, all of which are either continuous or have only 2 levels, the following dialog box is displayed:



Select one of the following types of designs:

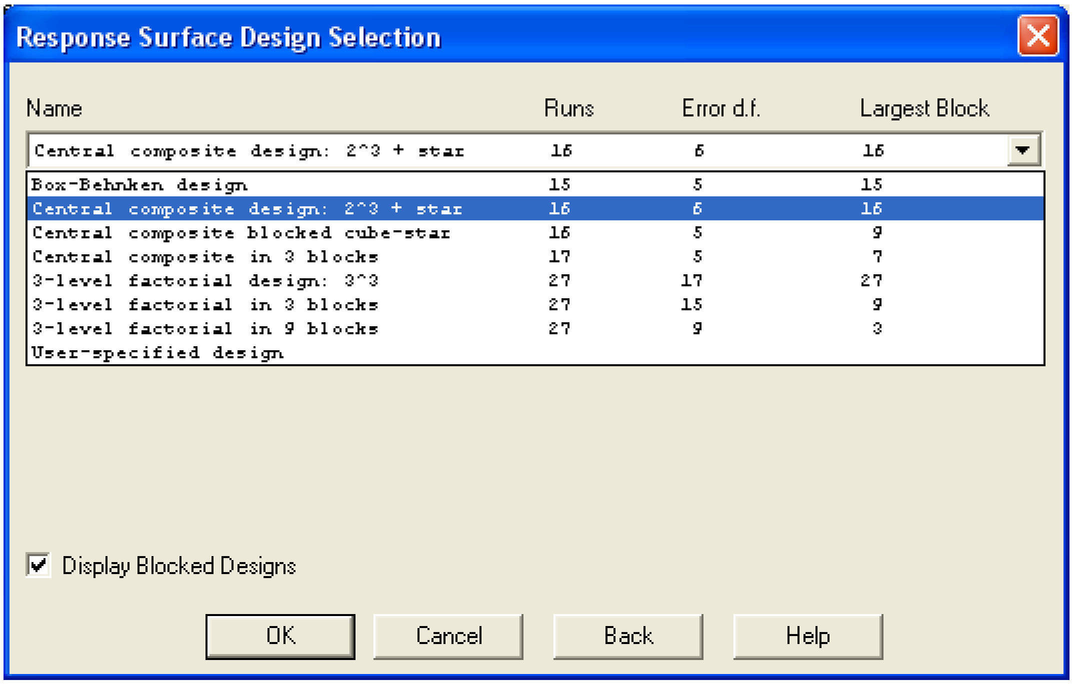
- *Screening* – These designs are intended to demonstrate which of the experimental factors have the biggest impact on the responses, but not necessarily to find the optimal levels of those factors. They consist primarily of designs in which each factor is run at only 2 levels.

- *Response Surface* – These designs are intended to find the optimal levels of the experimental factors. Each factor is run at 3 or more levels. Note: response surface designs are only available if all factors are continuous.

- *Multilevel Factorial* – These designs allow you to specify the number of levels at which each factor is to be set and consist of all combinations of the levels of those factors. It is commonly used to generate a set of candidate runs for selection by the D-optimal design creation procedure. Note: multilevel factorial designs are only available if all factors are continuous.

- *Orthogonal Array* – These designs were developed by Professor Taguchi and consist of selected combinations of the factors.

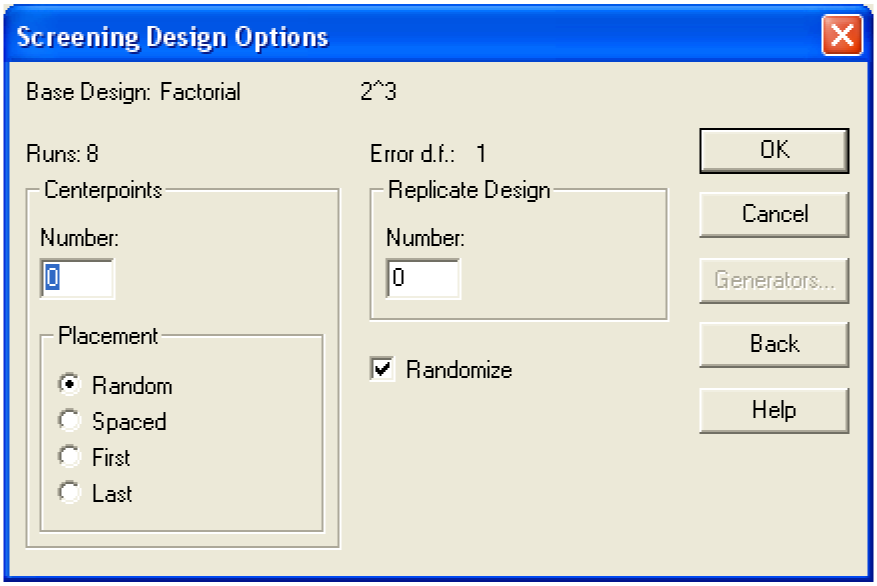
After a general class of designs is selected, additional dialog boxes will be displayed. For *screening* designs and *response surface* designs, the next dialog box displays a list of all available designs for the selected number of factors:



The dialog box shows the name of each design together with the number of runs, the resolution of the design (for screening designs only), the number of degrees of freedom available for estimating experimental error, and the size of the largest block. There is also a selection titled *User-Specified Design*, which will create an empty datasheet into which the user may enter any desired runs.

The final dialog boxes offer additional options, which vary somewhat from one type of design to another.

Screening Designs (see the document titled DOE – Screening Designs*)*



- *Centerpoints (number)* - the number of centerpoints to be added to the base design, which are additional experimental runs located at a point midway between the low and high level of all the factors. Each additional centerpoint adds one degree of freedom from which to estimate experimental error. If the design involves a single categorical factor, the centerpoints will be placed at a middle level of the quantitative factors and divided equally between the two levels of the categorical factor.

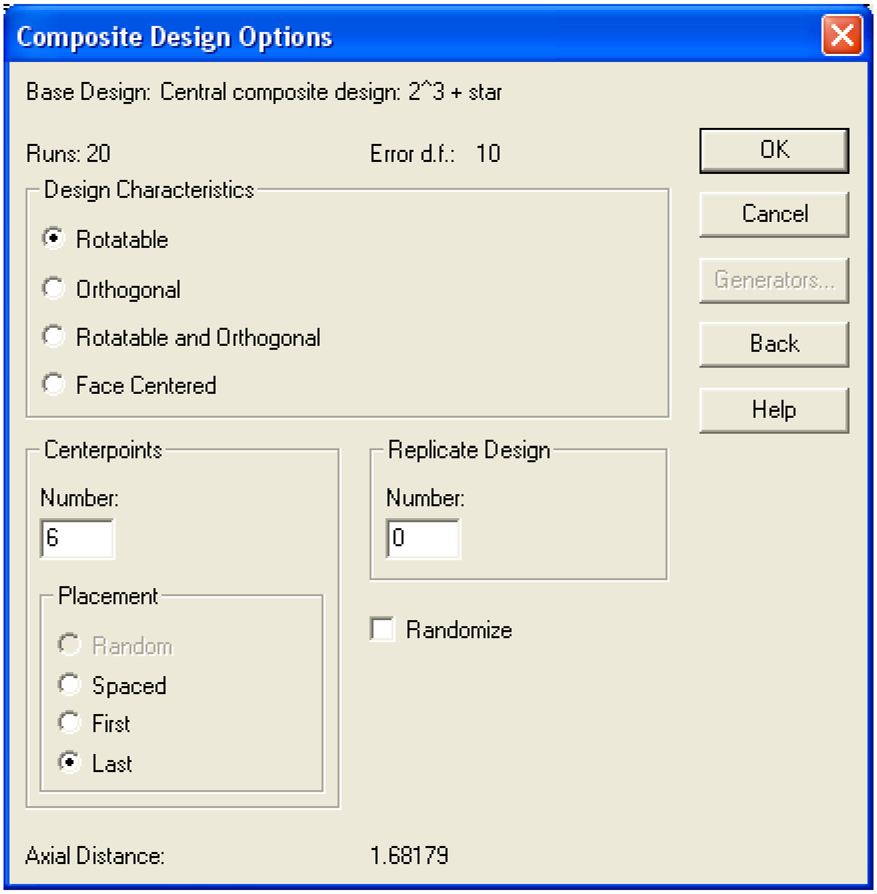
- *Centerpoints (placement)* - positioning of the centerpoints with respect to the runs in the base design. They may be *randomly* scattered throughout the other experimental runs, *spaced* evenly throughout the other runs, or placed at the *beginning* or *end* of the experiment. The first two options are usually preferable.

- *Replicate design* - if a number other than 0 is entered, the entire design will be repeated the indicated number of times.

- *Randomize* - check this box to randomly order the runs in the experiment. Randomization is generally a good idea, since it can reduce the effect of lurking variables such as trends over time.

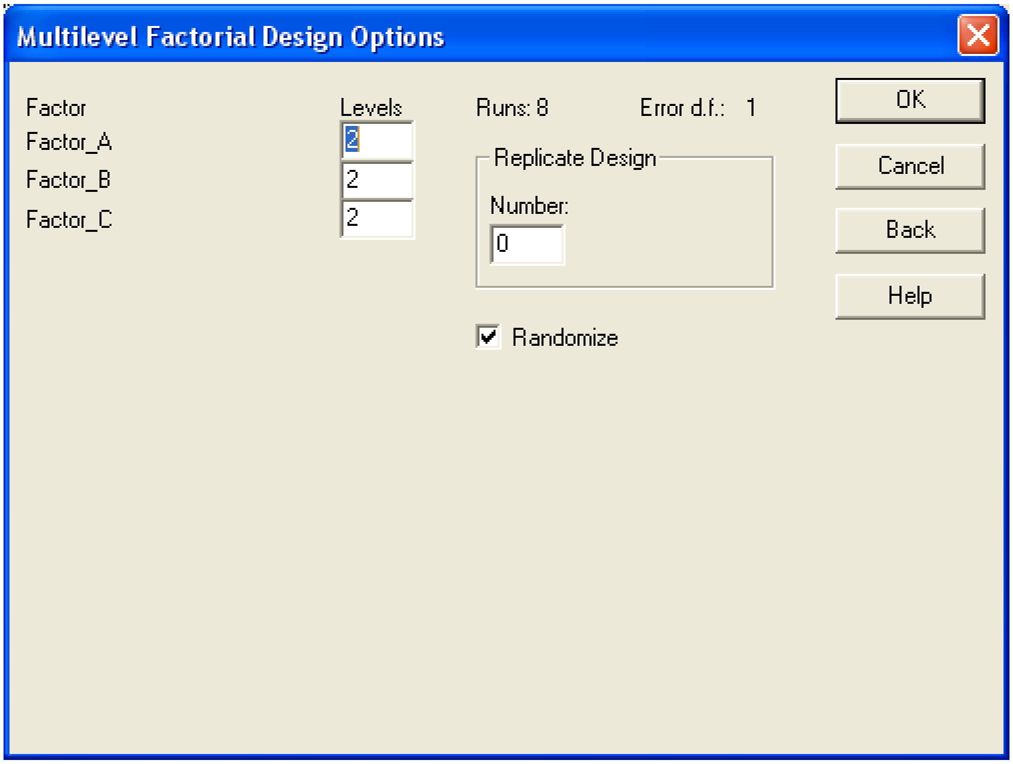
- *Generators* - this button displays a dialog box that allows experienced analysts to change the design generators for fractional factorial designs.

Response Surface Designs (see the document titled DOE – Response Surface Designs)



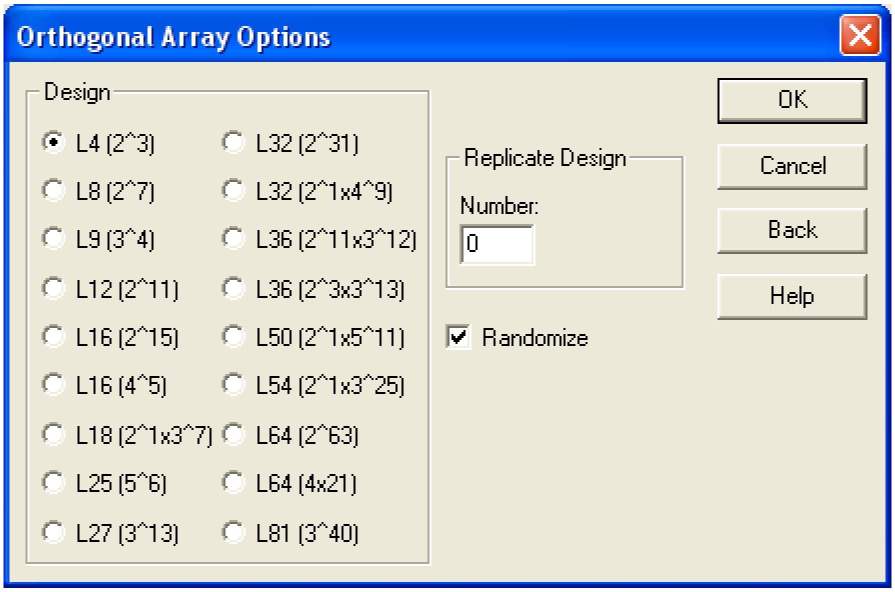
- *Design Characteristics*: specifies the properties of a central composite design, which depend on the number of centerpoints and the location of the star points.

Multilevel Factorial Designs (see the document titled DOE – Multilevel Factorial Designs)

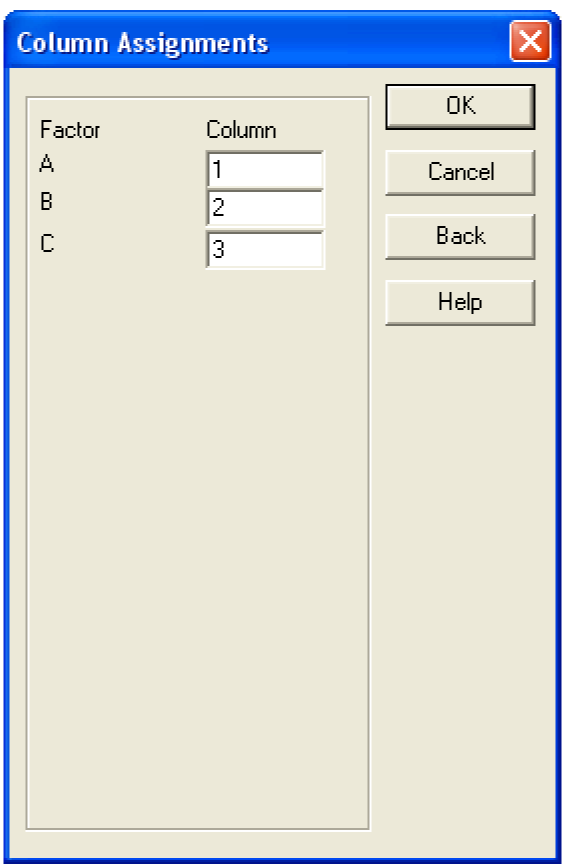


- *Levels*: the number of levels of each factor, spaced evenly between the low and high levels of the factor.

Orthogonal Arrays (see the document titled DOE – Inner/Outer Arrays)



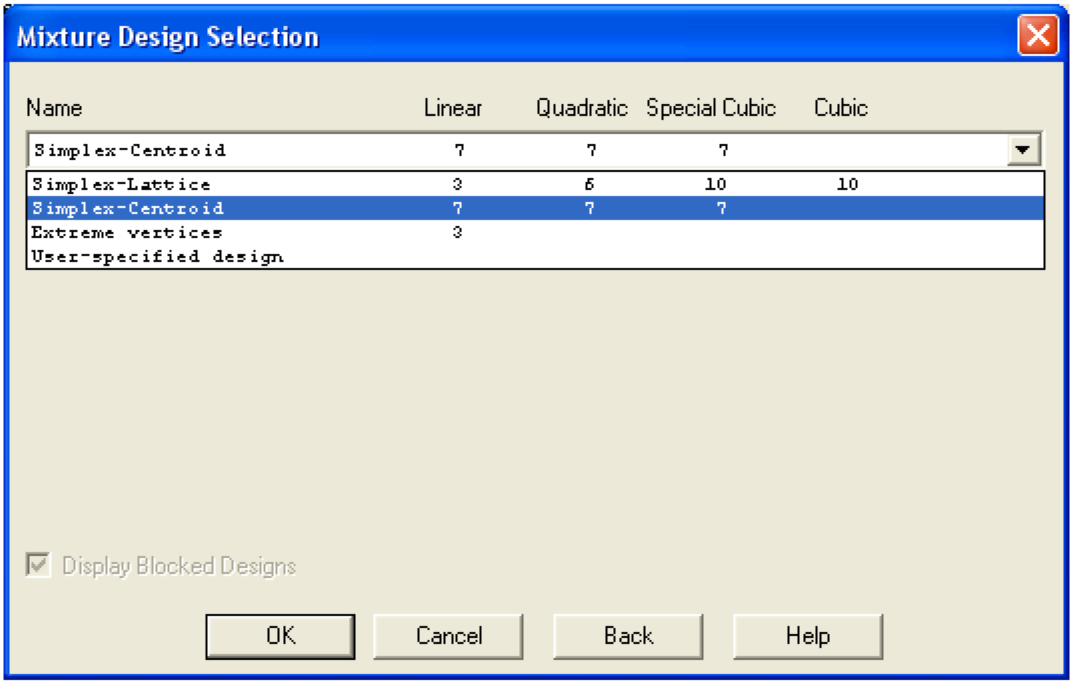
- *Design*: the type of orthogonal array, where a design such as "L18 (2^1x3^7)" indicates that the design has 18 runs consisting of 2 levels of up to 1 factor and 3 levels of up to 7 factors.



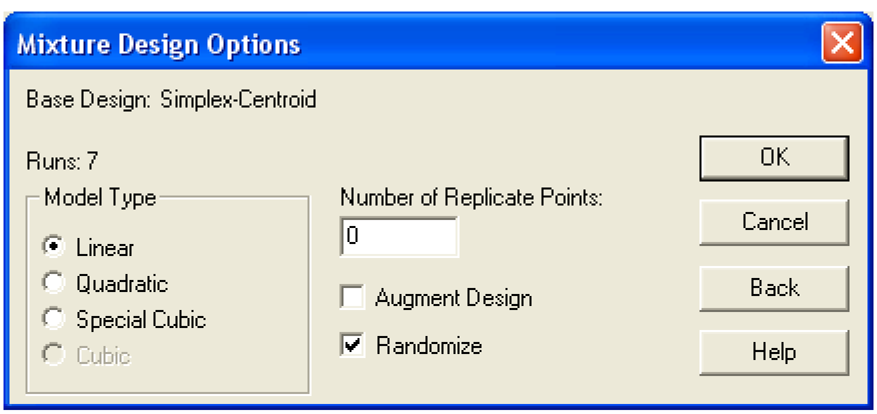
- *Column*: the column within the orthogonal array to which each factor will be assigned. If the factor is categorical with 2 levels, you must select a column of the array designed for 2-level factors. Continuous factors can be assigned to any column.

Mixture Components *(see the document titled* DOE – Mixture Designs)

When selecting the design for the mixture components, the first dialog box displayed shows a list of the available designs:



The table shows all designs that are appropriate given the specified constraints on the factors, together with the minimum number of runs that will be required to estimate each of four types of models. After selecting a specific design, a second dialog box will display any options for that design:



- *Model Type* - the most complicated model form that is to be fit. Enough runs will be added to fit the indicated model. When analyzing the results, a simpler model can be selected.

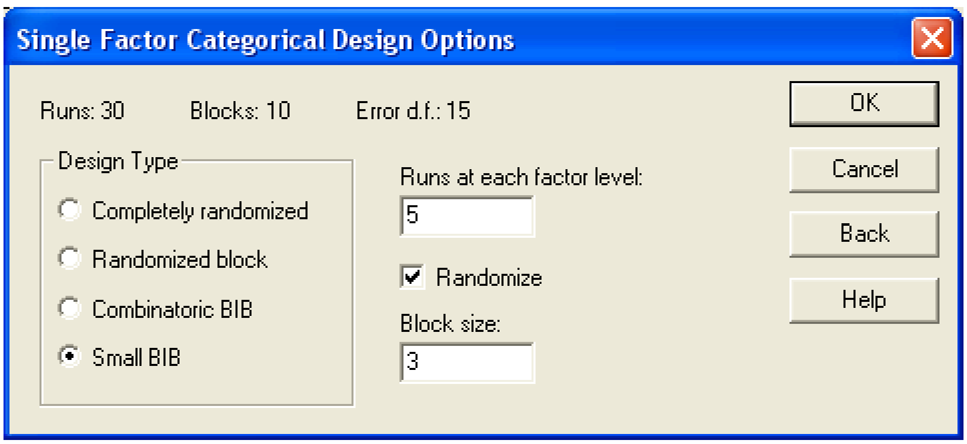
- *Number of Replicate Points* - if not zero, the specified number of points will be selected at random from the original design points and replicated. This can be used to provide additional degrees of freedom to estimate the experimental error. Alternatively, the analyst can manually choose runs to replicate rather than letting the program do so simply by adding runs to the bottom of the datasheet after the design is created.

- *Augment Design* - if selected, additional runs will be placed at special checkblends, i.e., combinations of the components that can be used to check the fit of the selected model. For the simplex-lattice and simplex-centroid designs, these runs are located along axial lines running from the pure blends through the centroid. For the extreme vertices designs, the program adds points on axial lines running from the centroid to the vertices, at the centers of the edges connecting the vertices, at the constraint plane centroids, and at the overall centroid. Note: for extreme vertices designs with simple constraints, it may be necessary to select this option in order to have enough runs to estimate all but the simplest type of model.

- *Randomize* - if selected, the runs will be arranged in random order.

One Categorical Factor *(see the document titled* DOE – Single Factor Categorical Designs)

If the section of the design to be created has only one factor and that factor is categorical, the following dialog box will be displayed:



- *Design Type*: The following types of designs may be available, depending upon the number of levels of the experimental factor:

  1. *Completely randomized design* - a design in which a random sample of measurements is taken from each of the *q* levels, with no attempt to account for the effects of any other factor.

  2. *Randomized block design* - a design in which an equal number of observations is taken from each treatment at two or more levels of a blocking or nuisance factor. Block effects are included in the model to reduce the magnitude of the experimental error.

  3. *Combinatoric BIB* - a Balanced Incomplete Block design involving a single blocking variable where the number of treatments in each block is less than *q*. If *k* treatments can

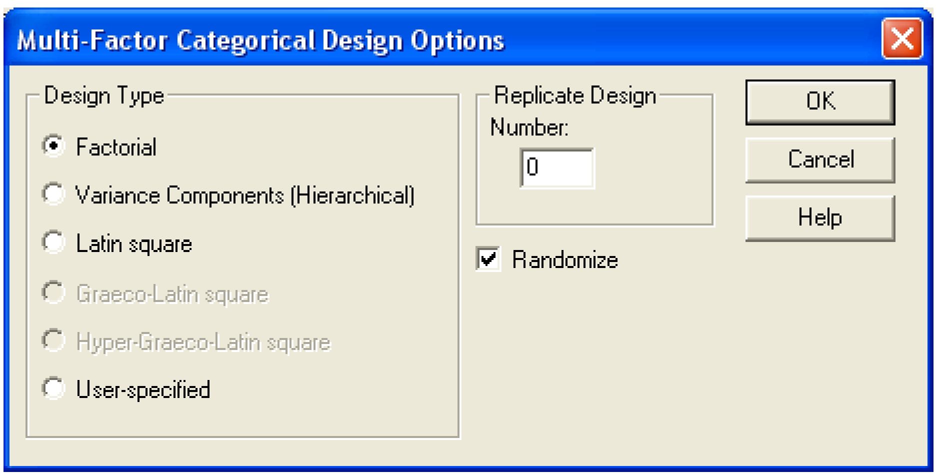
be run in each block, the design requires $\binom{q}{k}$ blocks, which represents the number of ways of choosing *k* items out of *q*.

4. *Small BIB* - a Balanced Incomplete Block design in which the number of blocks is less than that required by a full combinatoric BIB. These designs are only available for certain combinations of the number of factor levels and the block size.

- *Runs at each factor level* – the number of runs to be performed at each level of the categorical factor.
- *Randomize* - if selected, the runs will be arranged in random order.
- *Block size* – for designs with blocking, the maximum number of runs that can be performed within a single block.

Two or More Categorical Factors

If the section of the design to be created has only two or more factors and all factors are categorical, the following dialog box will be displayed:



- *Design Type*: The following types of designs may be available, depending upon the number of levels of the experimental factor:

  1. *Factorial* - a design in which data is collected at all combinations of the levels of the factors.
  2. *Variance Components (Hierarchical)* – a design in which each factor is nested in the factor above it. These designs are used to estimate variance components.
  3. *Latin square* (3 factors only) – a design in which the first factor is viewed as the response and the other 2 factors are blocking variables. The number of levels of all 3 factors must be the same.

4. *Graeco-Latin square* (4 factors only) – a design in which the first factor is viewed as the response and the other 3 factors are blocking variables. The number of levels of all 4 factors must be the same.

5. *Hyper-Graeco-Latin square* (5 factors only) – a design in which the first factor is viewed as the response and the other 4 factors are blocking variables. The number of levels of all 5 factors must be the same.

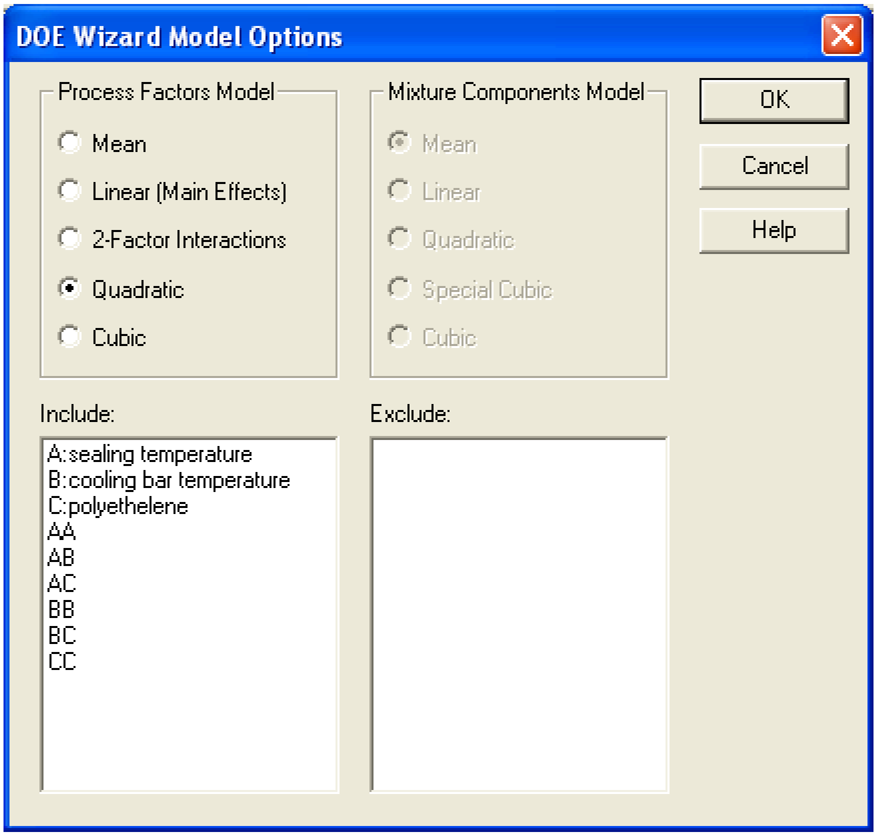
6. *User-Specified* – a design in which the user is responsible for creating the runs to be performed. For such designs, only the column headers will be created.

- *Replicate design* - if a number other than 0 is entered, the entire design will be repeated the indicated number of times.

- *Randomize* - check this box to randomly order the runs in the experiment. Randomization is generally a good idea, since it can reduce the effect of lurking variables such as trends over time.

Step 4: Specify Model

The fourth step in the *Experimental Design Wizard* is used to select the type of model that you anticipate fitting to the results of the experiment once it has been performed. The model is selected using the following dialog box:



If the experiment involves any process factors, one of the following models must be selected:

- **Mean** - This model assumes that the factors have no impact on the response. It consists of a single constant term:

$$Y = \beta_0 \tag{8}$$

- **Linear** – This model adds a coefficient that multiplies each factor:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \tag{9}$$

- **2-Factor Interactions** – This model adds cross-products for each pair of factors:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 \tag{10}$$

- **Quadratic** – This model adds a quadratic term for each factor:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 \quad (11)$$

- **Cubic –** This model adds third-order terms:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{111} x_1^3 + \beta_{222} x_2^3 + \beta_{333} x_3^3 + \beta_{112} x_1^2 x_2 + \beta_{113} x_1^2 x_3 + \beta_{122} x_1 x_2^2 + \beta_{223} x_2^2 x_3 + \beta_{133} x_1 x_3^2 + \beta_{233} x_2 x_3^2 + \beta_{123} x_1 x_2 x_3 \quad (12)$$

If the experiment involves any mixture components, one of the following models must be selected:

- **Mean** - This model assumes that the components have no impact on the response. It consists of a single constant term:

$$Y = \beta_0 \quad (13)$$

- **Linear –** This model includes a coefficient that multiplies each component:

$$Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \quad (14)$$

- **Quadratic –** This model adds cross-products for each pair of components:

$$Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 \quad (15)$$

- **Special cubic –** This model adds a special term involving the product of 3 components:

$$Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{123} x_1 x_2 x_3 \quad (16)$$

- **Cubic –** This model adds additional third-order terms:

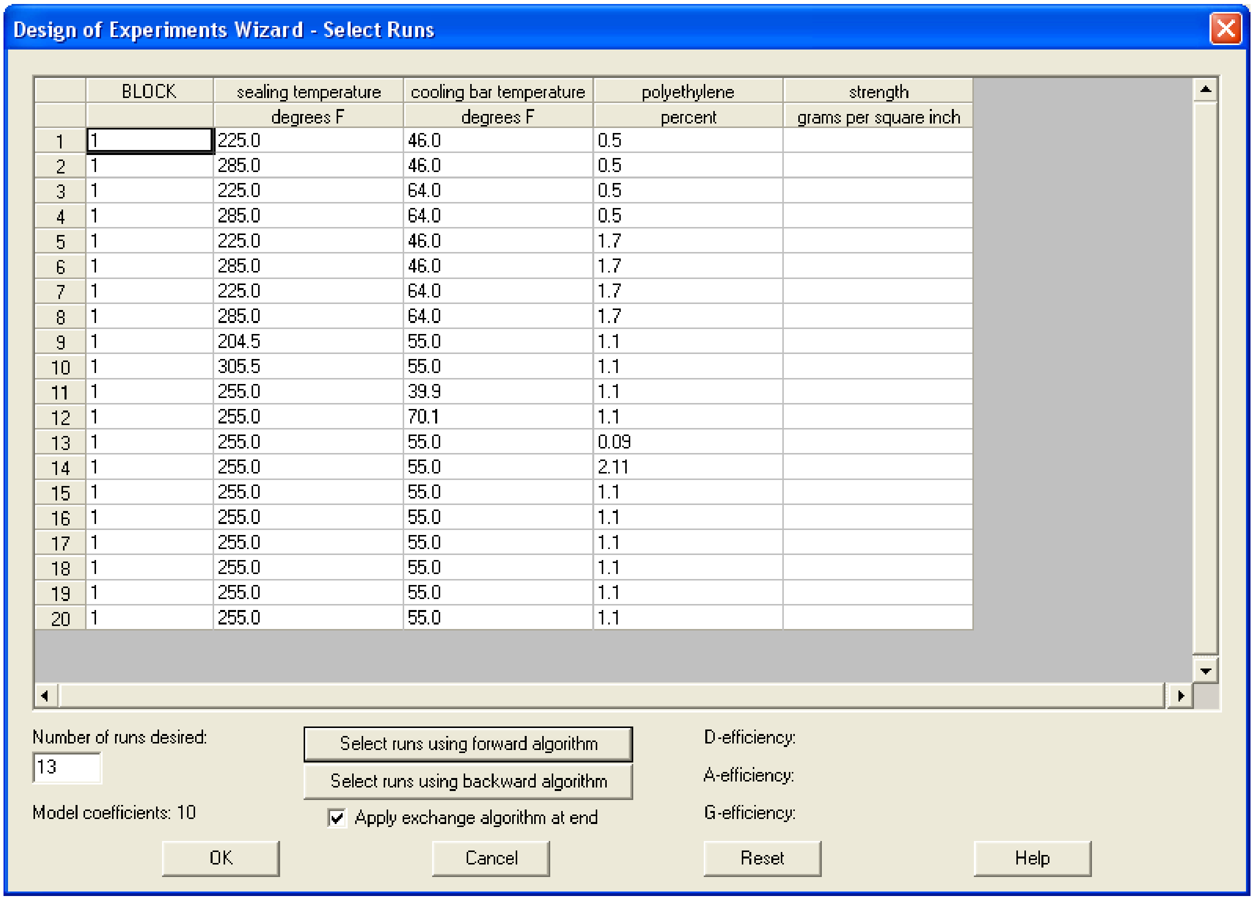
$$Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{123} x_1 x_2 x_3 + \delta_{12} x_1 x_2 (x_1 - x_2) + \delta_{13} x_1 x_3 (x_1 - x_3) + \delta_{23} x_2 x_3 (x_2 - x_3) \quad (17)$$

Specific terms may be excluded from the model by double-clicking on any term in the *Include* field, which will move that term to the *Exclude* field. Double-clicking on a term in the *Exclude* field will put it back in the model.

Step 5 – Select Runs (Optional)

For most designs, all of the runs created in Step 3 will be performed. In special cases, however, it may be desirable to select only a subset of the runs. For example, “D-optimal” designs are often constructed by creating a large number of candidate runs from which an optimal subset is then selected.

If the *Step 5: Select runs* button is pressed, the following dialog box will be displayed:



	BLOCK	sealing temperature degrees F	cooling bar temperature degrees F	polyethylene percent	strength grams per square inch
1	1	225.0	46.0	0.5	
2	1	285.0	46.0	0.5	
3	1	225.0	64.0	0.5	
4	1	285.0	64.0	0.5	
5	1	225.0	46.0	1.7	
6	1	285.0	46.0	1.7	
7	1	225.0	64.0	1.7	
8	1	285.0	64.0	1.7	
9	1	204.5	55.0	1.1	
10	1	305.5	55.0	1.1	
11	1	255.0	39.9	1.1	
12	1	255.0	70.1	1.1	
13	1	255.0	55.0	0.09	
14	1	255.0	55.0	2.11	
15	1	255.0	55.0	1.1	
16	1	255.0	55.0	1.1	
17	1	255.0	55.0	1.1	
18	1	255.0	55.0	1.1	
19	1	255.0	55.0	1.1	
20	1	255.0	55.0	1.1	

The top section of the dialog box displays each of the runs in the current experiment. The bottom of the dialog box contains the following fields:

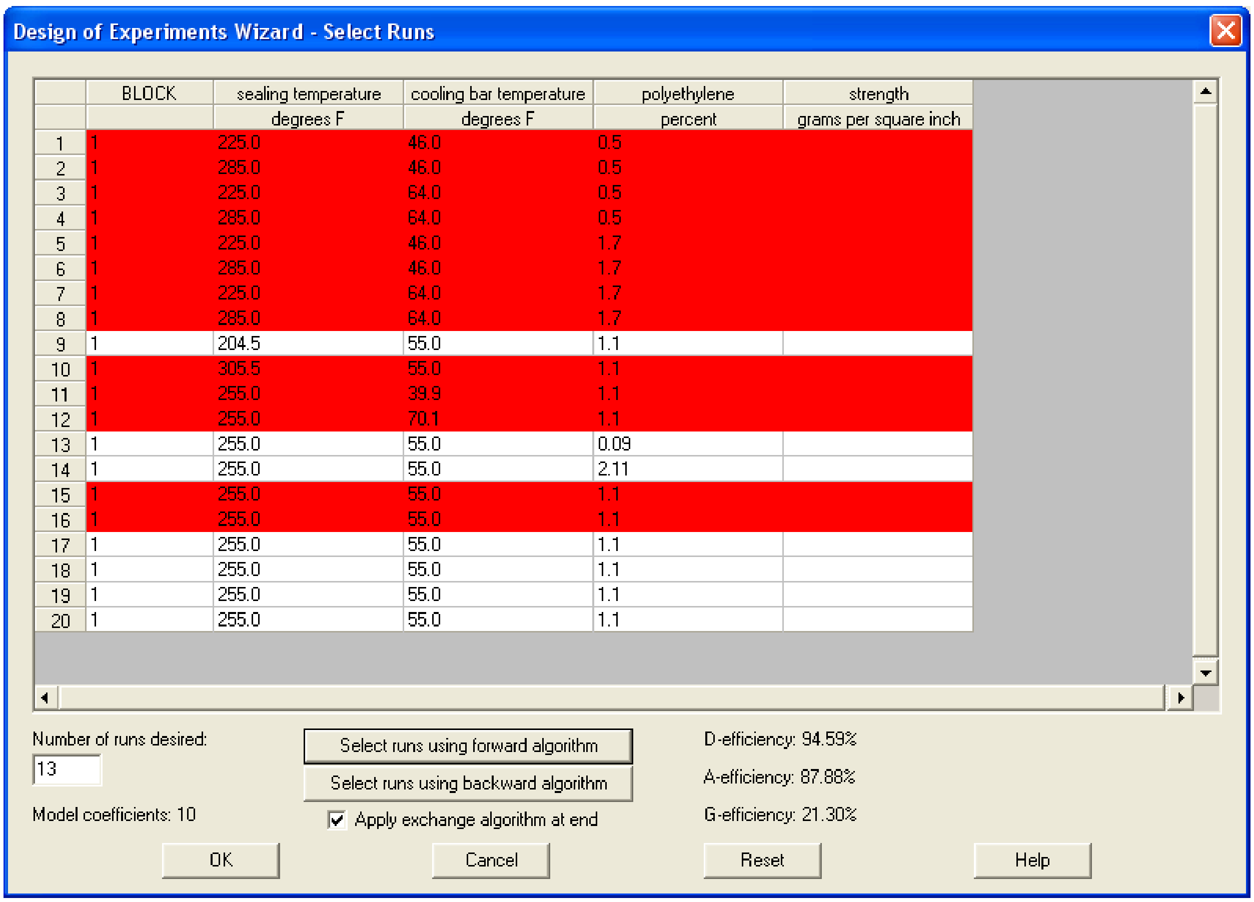
- *Number of runs desired* - Specify the number of runs to be selected, which must be greater than or equal to the number of coefficients in the model that will be fit to the data. It is usually a good idea to select at least 3 more runs than there are *model coefficients*, to provide degrees of freedom for estimating the experimental error.

- *Select runs using forward algorithm* – Press this button to select the indicated number of runs using a forward selection algorithm. The *Forward* method begins with any runs that have already been performed (if any runs contain data for the response variables) and adds runs one at a time, adding at each step the run that adds the most to the D-efficiency of the experiment.

- *Select runs using backward algorithm* – Press this button to select the indicated number of runs using a backward selection algorithm. The *Backward* method begins with all of the candidate runs and removes runs one at a time, removing at each step the run that adds the least to the D-efficiency of the experiment.

- *Apply exchange algorithm at end* – Indicate whether or not you wish to apply an exchange algorithm after the initial selection procedure is complete. This algorithm tests all pairs of runs consisting of one that has been selected and one that has not, making any exchanges that would increase the D-efficiency of the experiment. Exchanges continue until no further improvements can be made by switching one run that has been selected with one run that has not been selected.

After either of the buttons is pressed, the selection will begin. Depending on the number of candidate and the complexity of the model, selection may take several minutes. Once it is complete, the selected runs will be highlighted in red:



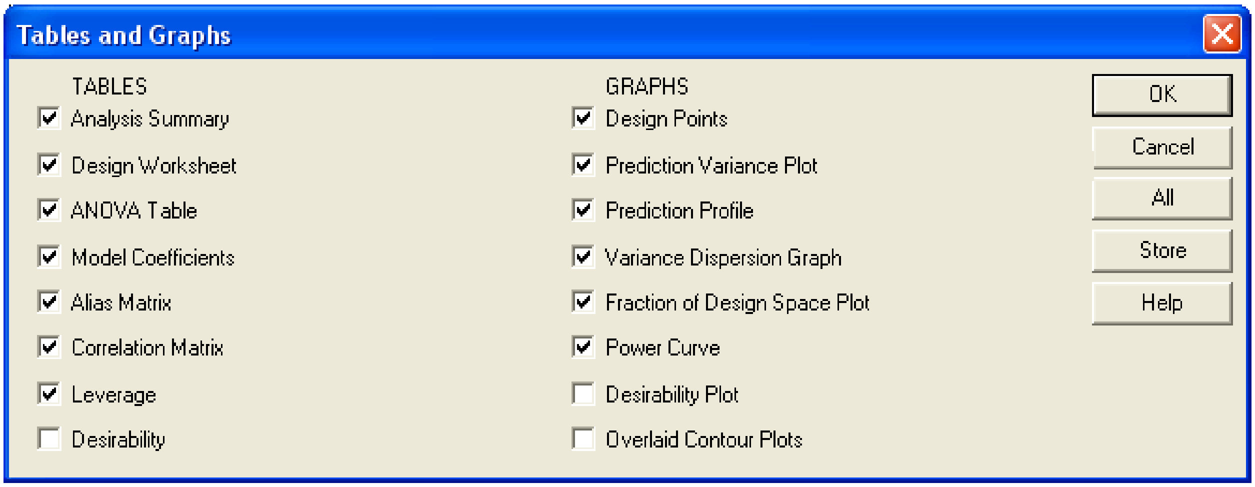
	BLOCK	sealing temperature degrees F	cooling bar temperature degrees F	polyethylene percent	strength grams per square inch
1	1	225.0	46.0	0.5	
2	1	285.0	46.0	0.5	
3	1	225.0	64.0	0.5	
4	1	285.0	64.0	0.5	
5	1	225.0	46.0	1.7	
6	1	285.0	46.0	1.7	
7	1	225.0	64.0	1.7	
8	1	285.0	64.0	1.7	
9	1	204.5	55.0	1.1	
10	1	305.5	55.0	1.1	
11	1	255.0	39.9	1.1	
12	1	255.0	70.1	1.1	
13	1	255.0	55.0	0.09	
14	1	255.0	55.0	2.11	
15	1	255.0	55.0	1.1	
16	1	255.0	55.0	1.1	
17	1	255.0	55.0	1.1	
18	1	255.0	55.0	1.1	
19	1	255.0	55.0	1.1	
20	1	255.0	55.0	1.1	

In addition, the efficiencies of the selected set of runs will be displayed.

If you press *OK*, the dialog box will be closed and the original design will be reduced to the selected subset. Press *Cancel* to abandon the selection or *Reset* to return to the original set of runs.

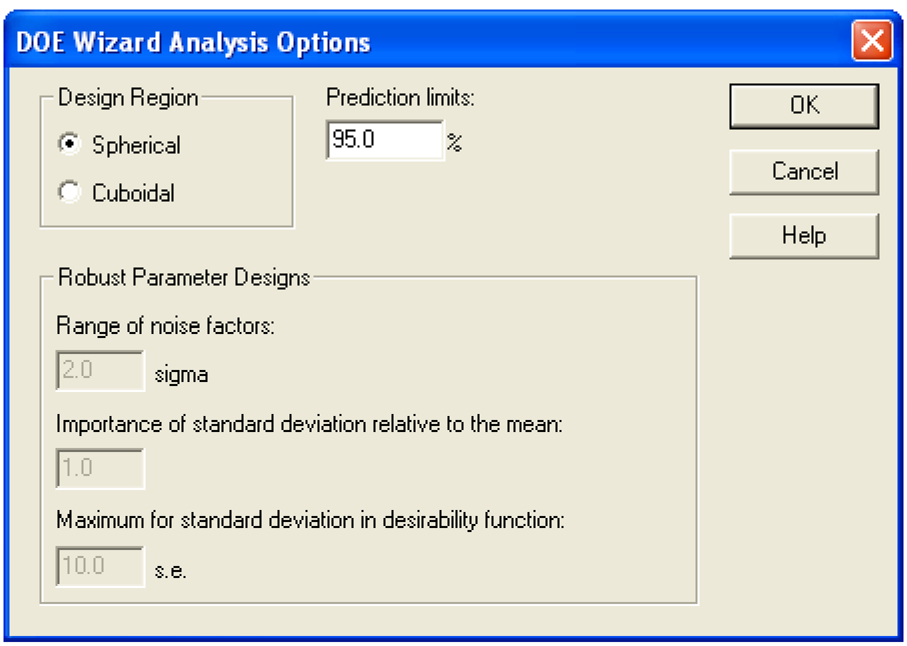
Step 6 - Evaluate Design

The DOE Wizard provides a number of tools for evaluating the design once it has been created. If you press the *Evaluate design* button, a dialog box will be displayed showing all available tables and graphs:



Each of the options checked above can be used to evaluate the design before the experiment is conducted.

If you click the right mouse button and select *Analysis Options*, the following dialog box will be displayed:



The dialog box includes:

- **Design Region** – indicates whether the shape of the experimental region should be treated as *spherical* or *cuboidal*.

- **Prediction Limits –** the probability level for the prediction limits.

Additional fields are present which apply only to robust parameter designs with crossed factors. These fields are described in the documentation for those designs.

Tabular Output

Analysis Summary

The *Analysis Summary* summarizes the experiment that has been created:

Experimental Design Wizard

Step 1: Define the response variables to be measured

Name	*Units*	*Analyze*	*Goal*	*Target*	*Impact*	*Sensitivity*	*Low*	*High*
strength	grams	Mean	Maximize		3.0	Medium	10.0	15.0

Step 2: Define the experimental factors to be varied

Name	*Units*	*Type*	*Role*	*Low*	*High*	*Levels*
A:sealing temperature	degrees F	Continuous	Controllable	225.0	285.0	
B:cooling bar temperature	degrees F	Continuous	Controllable	46.0	64.0	
C:polyethelene	%	Continuous	Controllable	0.5	1.7	

Step 3: Select the experimental design

Type of Factors	*Design Type*	*Centerpoints Per Block*	*Centerpoint Placement*	*Design is Randomized*	*Number of Replicates*	*Total Runs*	*Total Blocks*	*Error D.F.*
Process	Central composite design: 2^3 + star	6	Last	No	0	20	1	10

Number of samples: 1

Step 4: Specify the initial model to be fit to the experimental results

Factors	*Model*	*Coefficients*	*Excluded effects*
Process	quadratic	10	

Step 5: Select an optimal subset of the runs (optional)
20 runs selected

Step 6: Select tables and graphs to evaluate the selected runs

Experimental region	spherical (radius = 1.73205)
Minimum pred. variance	0.166336
Average pred. variance	0.166336
Maximum pred. variance	0.673524
D-efficiency	99.74%
A-efficiency	99.46%
G-efficiency	74.24%

As can be seen from the information above, the experimenters selected a central composite design with a total of 20 runs. The design includes 6 centerpoints and provides 10 degrees of freedom for estimating the experimental error, assuming that a full quadratic model will be fit to the resulting data. For purposes of illustration, the order of the runs has not been randomized.

The information under Step 6 is often used to compare different experimental designs. The statistics displayed involved two very important quantities:

Unscaled prediction variance – the variance of the predicted response at a location **x**, calculated from

$$Var[\hat{y}(x)] = \left(x(X'X)^{-1}x\right)\sigma^2 \tag{18}$$

where σ^2 is the variance of the experimental error. The square root of the prediction variance (called the standard error of prediction) is used to construct confidence intervals for the mean response at the location **x**.

Scaled prediction variance – the variance of the predicted response at a location **x**, calculated from

$$SPV(x) = N\left(x(X'X)^{-1}x\right)\sigma^2 \tag{19}$$

Since the experimental error variance is unknown prior to performing the experiment, the calculations displayed under Step 6 use the value $\sigma^2 = 1$. In addition, the X matrix is first converted to standardized units such that the distance from the center to the specified low and high values equals 1.

The output displays several important statistics:

- *Minimum prediction variance* – the smallest prediction variance over the experimental region, which in this case is assumed to be spherical with a radius of 1.73205 in standardized units. The selected radius is the distance from the center of the experimental region to the furthest experimental point.

- *Average prediction variance* – the average prediction variance throughout the experimental region. In general, a design with a small average prediction variance is preferred to one with a larger average prediction variance.

- *Maximum prediction variance* - the maximum prediction variance throughout the experimental region. The difference between the minimum and maximum prediction variance is a measure of the relative distribution of that variance over the experimental region.

- *D-efficiency* – This number compares the variances and covariances of the estimated model parameters to that of the best possible design (given the selected number of runs *N*). It is calculated from the determinant of the *X'X* matrix according to

$$D-efficiency = 100\left(\frac{1}{N}|X'X|^{1/p}\right)\% \tag{20}$$

  The higher the D-efficiency, the better the design is in estimating the model parameters. In this case, the D-efficiency is very high, since the design is nearly orthogonal.

- *A-efficiency* – This number is similar to D-efficiency except that it considers only the parameter variances and not their covariances. It is calculated from the trace of the *X'X* matrix according to

$$A-efficiency = 100\left(\frac{p}{trace\left(N(X'X)^{-1}\right)}\right)\% \quad (21)$$

- *G-efficiency* – This number compares the maximum scaled prediction variance *SPV(x)* over the experimental region to that of an ideal design, for which the maximum SPV equals *p*. It is calculated according to

$$G-efficiency = 100\left(\frac{p}{\max SPV(x)}\right)\% \quad (22)$$

When comparing designs with a similar number of runs, you should look for high design efficiencies.

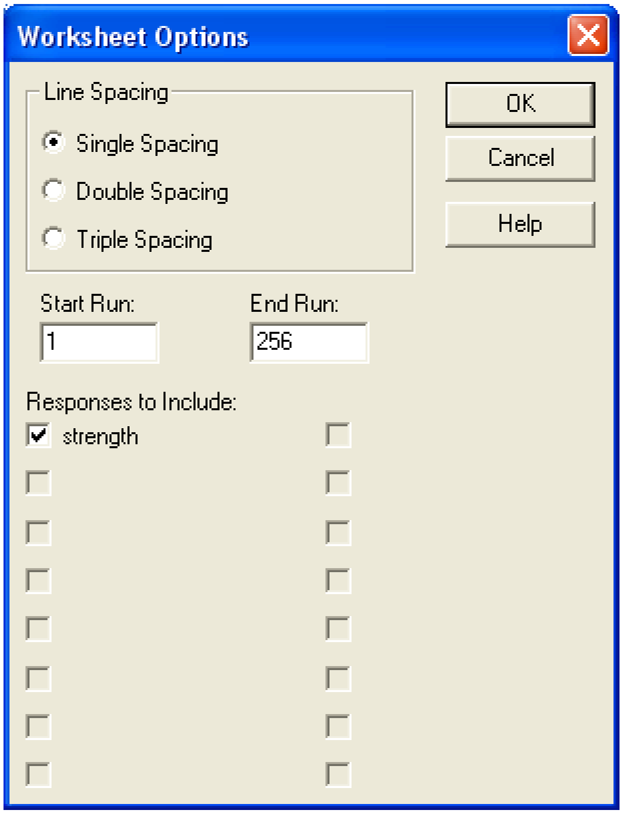
Design Worksheet

The design worksheet displays each of the experiments to be run in the desired order. It is designed to be printed and contains locations for recording each response. The worksheet for the sample data is shown below:

Worksheet for breadwrapper.sgx - Breadwrapper example from Myers and Montgomery

run	*sample*	*sealing temperature*	*cooling bar temperature*	*polyethylene*	*strength*
		Degrees F	*degrees F*	*percent*	*grams per square inch*
1	1	225.0	46.0	0.5	
2	1	285.0	46.0	0.5	
3	1	225.0	64.0	0.5	
4	1	285.0	64.0	0.5	
5	1	225.0	46.0	1.7	
6	1	285.0	46.0	1.7	
7	1	225.0	64.0	1.7	
8	1	285.0	64.0	1.7	
9	1	204.5	55.0	1.1	
10	1	305.5	55.0	1.1	
11	1	255.0	39.9	1.1	
12	1	255.0	70.1	1.1	
13	1	255.0	55.0	0.09	
14	1	255.0	55.0	2.11	
15	1	255.0	55.0	1.1	
16	1	255.0	55.0	1.1	
17	1	255.0	55.0	1.1	
18	1	255.0	55.0	1.1	
19	1	255.0	55.0	1.1	
20	1	255.0	55.0	1.1	

Pane Options



- **Line Spacing** - the spacing of runs when sent to a printer. Double or triple spacing leaves more room for writing in the results by hand.

- **Start Run** and **End Run** – the range of runs to display.

- **Responses to Include** - the responses to be included on the printed worksheet. Analysts may wish to print a separate worksheet for each response if the number of experimental factors is large.

ANOVA Table

The *Analysis of Variance Table* shows the degrees of freedom that will be available when the final data is analyzed. A typical table is shown below:

ANOVA Table

Source	D.F.
Model	9
Total Error	10
Lack-of-fit	5
Pure error	5
Total (corr.)	19

To understand this table, let:

n = total number of runs in the experiment

m = number of different combinations of the experimental factors

p = number of coefficients in the model to be estimated

For the breadwrapper example, *n* = 20, *m* = 15, and *p* = 10.

The important lines in the table are:

- *Model* – the degrees of freedom (*p* – 1) used to estimate model effects (not including the constant term).
- *Total Error* – the total degrees of freedom (*n* – *p*) available to estimate the experimental error. If this value is small, the tests of significance for the model coefficients will not have much power. These degrees of freedom are further divided into:
    - *Lack-of-fit* – the degrees of freedom (*m* – *p*) available to test whether the specified model is adequate for the data.
    - *Pure error* – the degrees of freedom (*n* – *m*) available to estimate experimental error solely from replicated observations.
- *Total (corr.)* – the degrees of freedom (*n* – 1) available once the constant term has been estimated.

In general, you should try to have at least 3 or 4 degrees of freedom available for each error term that you wish to estimate. If you wish to test the adequacy of the fitted model, there should also be a few degrees of freedom available for lack-of-fit.

Model Coefficients

The table of model coefficients lists each coefficient that will be estimated (other than the constant) when the experimental data is analyzed. For the breadwrapper experiment, it appears as shown below:

Model Coefficients

				Power at	*Power at*	*Power at*
Coefficient	*Standard Error*	*VIF*	*Ri-Squared*	*SN = 0.5*	*SN = 1.0*	*SN = 2.0*
A	0.270495	1.0	0.0	13.35%	38.67%	91.38%
B	0.270866	1.0	0.0	13.33%	38.58%	91.31%
C	0.270495	1.0	0.0	13.35%	38.67%	91.38%
AA	0.263023	1.01828	0.0179539	13.85%	40.48%	92.75%
AB	0.353553	1.0	0.0	9.83%	24.91%	72.25%
AC	0.353553	1.0	0.0	9.83%	24.91%	72.25%
BB	0.264434	1.01805	0.017731	13.75%	40.13%	92.50%
BC	0.353553	1.0	0.0	9.83%	24.91%	72.25%
CC	0.263023	1.01828	0.0179539	13.85%	40.48%	92.75%

alpha = 5.0%, sigma estimated from total error with 10 d.f.

Included in the table are:

Standard error – the estimated standard error of the standardized regression coefficient, as a multiple of σ. The standardized coefficient is the coefficient in the regression model when the X variables are scaled such that the high level of the factor equals 1 and the low level of the factor equals -1. The smaller the standard error, the more precise the estimate will be. An approximate 95% confidence interval for the coefficient is given by the estimate plus and minus 2 times the standard error.

VIF – the *Variance Inflation Factor*, which measures the extent to which any correlation amongst the estimated coefficients (multicollinearity) inflates the standard error. VIF's in excess of 10 usually indicate serious multicollinearity in the design.

R_i-squared – a measure of how correlated each coefficient is with the other coefficients in the model. Ideally, this measure should be close to 0. If it is high, it means that the corresponding factor is highly correlated with other factors in the model. Note that R_i-squared is related to the variance inflation factor according to

$$VIF = 1.0/\left(1 - R_i^2\right) \qquad (23)$$

Power at SN = 0.5, 1.0, and 2.0 – the probability that the statistical test for the coefficient will be statistically significant for different signal-to-noise ratios. The SN ratio is defined by

$$SN = \frac{2\beta}{\sigma} \qquad (24)$$

The reason for the 2 in the numerator is that the "effect" of a factor is defined as the difference between the response as the high level of the factor minus the response at the low level of the factor. For example, in the above design, there is a slightly better than 91% chance of detecting a main effect if it is twice the background sigma.

Alias Matrix

The alias matrix indicates the extent to which the effects to be estimated are confounded with effects that are not in the current model. Each row represents an effect to be estimated. Each column represents an effect that is not included in the model. A non-zero value in any cell indicates that the effect in the corresponding column, multiplied by the value in the cell, is added to the estimated effect when the model is fit. As an example, the table below shows how each effect in the second-order model is aliased with third-order effects not in the model:

Alias Matrix

Effect	AAA	AAB	AAC	ABB	ABC	ACC	BBB	BBC	BCC	CCC
constant										
A	1.7603			0.5853		0.5853				
B		0.5869					1.7497		0.5869	
C			0.5853					0.5853		1.7603
AA										
AB										
AC										
BB										
BC										
CC										

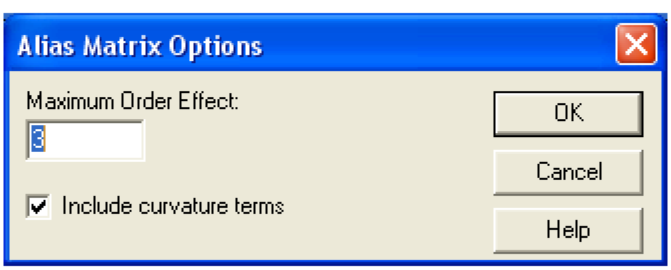
For example, the estimated main effect of factor A is actually:

$$A + 1.7603A^3 + 0.5853AB^2 + 0.5853AC^2 \qquad (25)$$

If any third-order effects exist, they consequently disturb the estimate of the main effect of factor A.

You can use *Analysis Options* to change which effects are included in the model and *Pane Options* to select the maximum order effect shown in the columns.

Pane Options



- *Maximum Order Interaction* – the highest order interaction that should be considered in the list of effects not included in the model.

- *Include curvature terms* – whether to include effects containing second-order and higher terms involving the same factor. If not checked, only simple interactions will be included.

Correlation Matrix

The correlation matrix is another method for examining the properties of a selected design. The output for the breadwrapper example is shown below:

Correlation Matrix

	A	B	C	AB	AC	BC	AA	BB	CC
A	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
B	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
C	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AB	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AC	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
BC	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
AA	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	-0.0898	-0.0910
BB	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0898	1.0000	-0.0898
CC	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0910	-0.0898	1.0000

This matrix displays the linear correlation between columns of the *X* matrix that will be used to fit a statistical model to the results of the experiment. Correlation coefficients range between -1 and +1, with 0 indicating no correlation. The non-zero correlations amongst the columns for *AA*, *BB* and *CC* indicate some correlation amongst the estimates of the quadratic terms in the model, although the correlations are not large enough to be worrisome.

Leverage

When fitting a statistical model to a set of experimental data, certain runs may have a greater influence on the estimates of the model coefficients than others. A common way of measuring the influence of a run is through a statistics called *leverage*. Leverage measures how distant a design point is from the mean of all *n* run in the space of the *independent* variables. The higher the leverage, the greater the impact of the point on the fitted values $\hat{y}$.

The leverage of the i-th run is calculated according to

$$h_i = diag\left\{X_i'\left(X'X\right)^{-1}X_i\right\} \tag{26}$$

If the model contains p coefficients and the experimental design has n runs, the average leverage is given by

$$\bar{h} = \frac{p}{n} \tag{27}$$

The table below shows the leverage of each point in the breadwrapper example:

Leverage

Run	*Leverage*	*Location*
1	0.669826	Other
2	0.669826	Other
3	0.669826	Other
4	0.669826	Other
5	0.669826	Other
6	0.669826	Other
7	0.669826	Other
8	0.669826	Other
9	0.607938	Star
10	0.607938	Star
11	0.605813	Star
12	0.605813	Star
13	0.607938	Star
14	0.607938	Star
15	0.166336	Center
16	0.166336	Center
17	0.166336	Center
18	0.166336	Center
19	0.166336	Center
20	0.166336	Center

Average leverage = 0.5

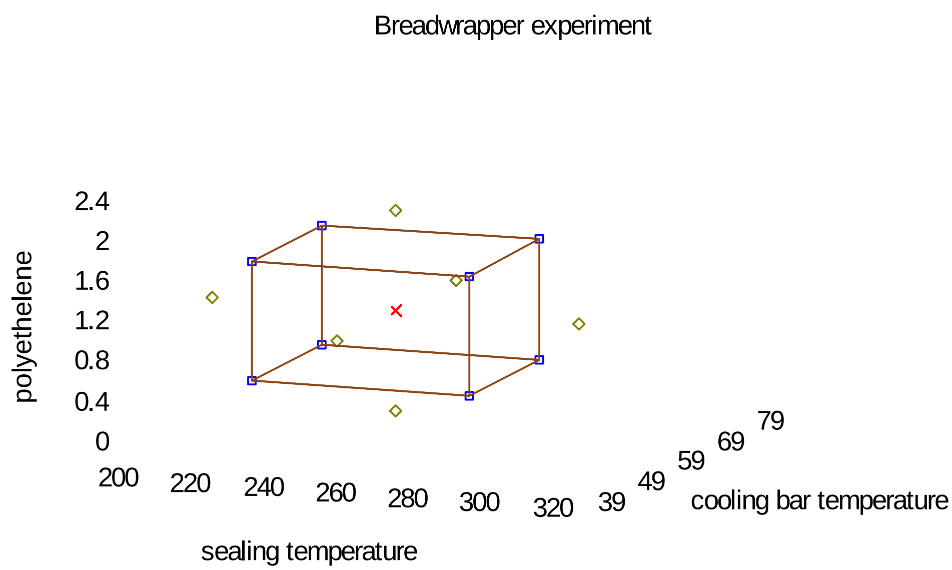
Notice that the most influential points are the factorial points, which have a leverage of approximately 0.67. The star points have slightly less leverage, while the centerpoints have the least influence. Usually, points far away from the center of the design will be most influential, which is why most runs are placed near the edges of the experimental region.

Graphical Output

A number of different graphs can also be created, displayed by pressing the *Graphs* button.

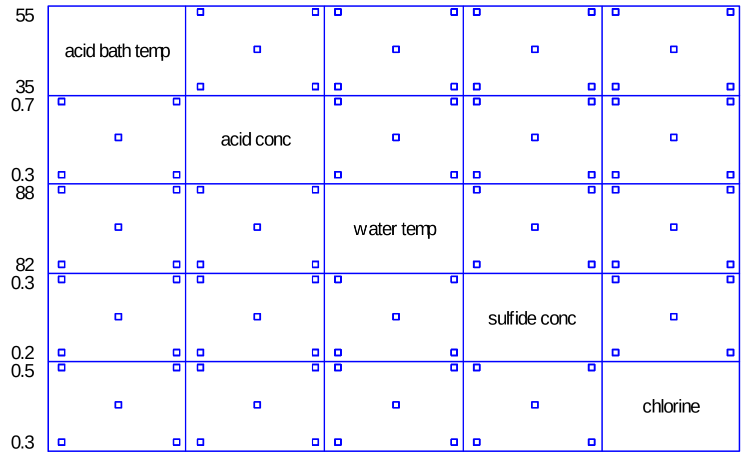
Design Points

The location of the points in the selected experiment can be displayed using this option. If the number of experimental factors equals 3, a three-dimensional scatterplot is displayed as for the breadwrapper experiment:



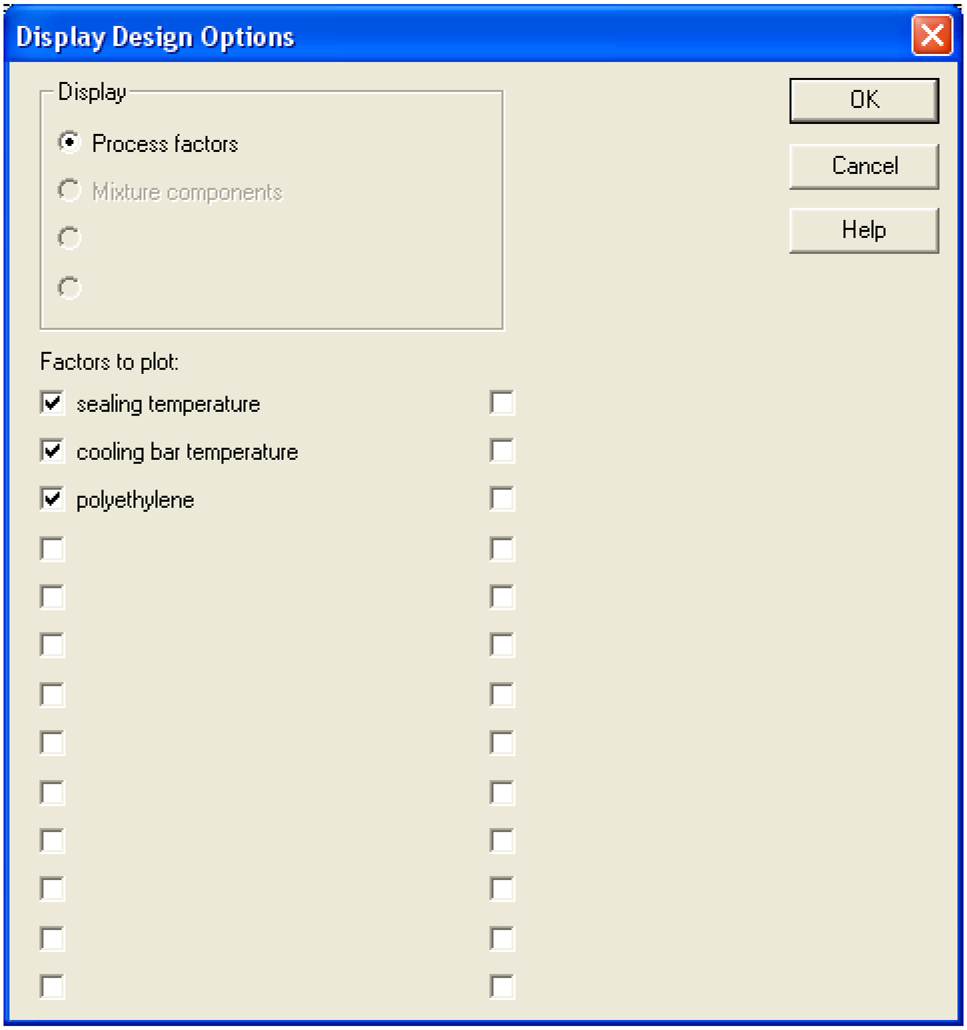
The experiment consists of 8 runs at the corners of a cube, 6 star points, and a replicated centerpoint.

If the number of experimental factors is greater than 3, a scatterplot matrix is displayed similar to that shown below:



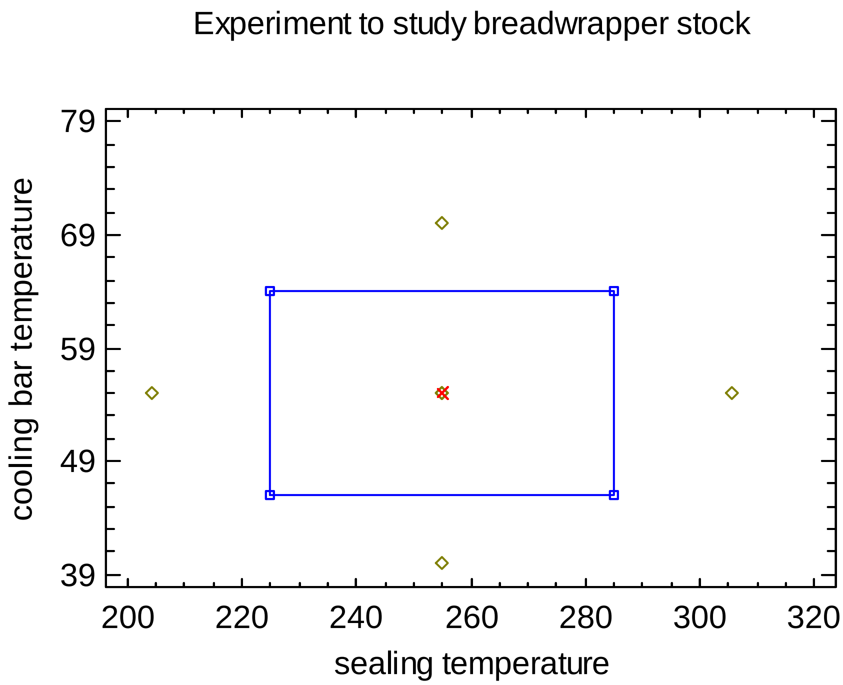
Pane Options

The *Pane Options* dialog box allows you to plot the design in lower dimensions by selecting only a subset of the factors:

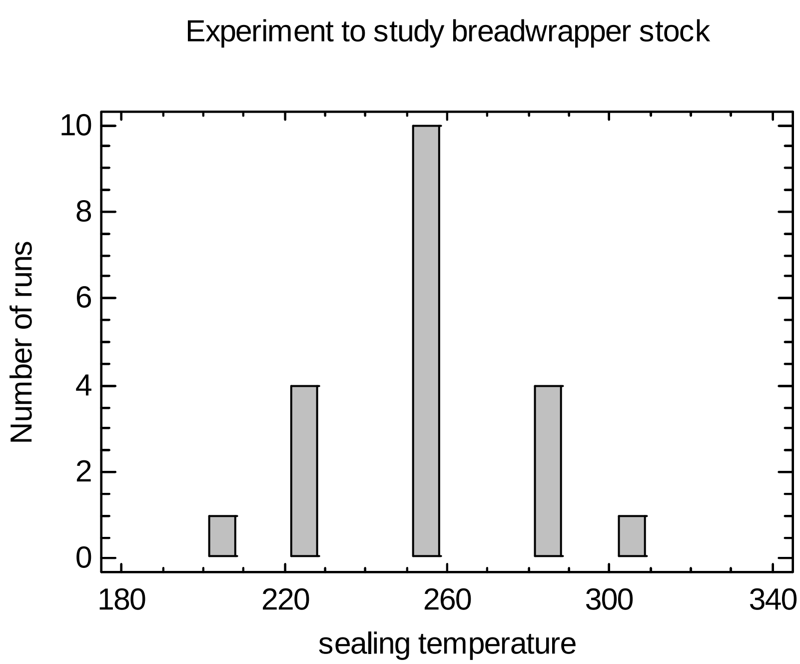


- *Display* – select the segment of the design that you wish to plot.
- *Factors to plot* – select the factors to include on the plot.

If 2 factors are selected, a 2-dimensional scatterplot is created:

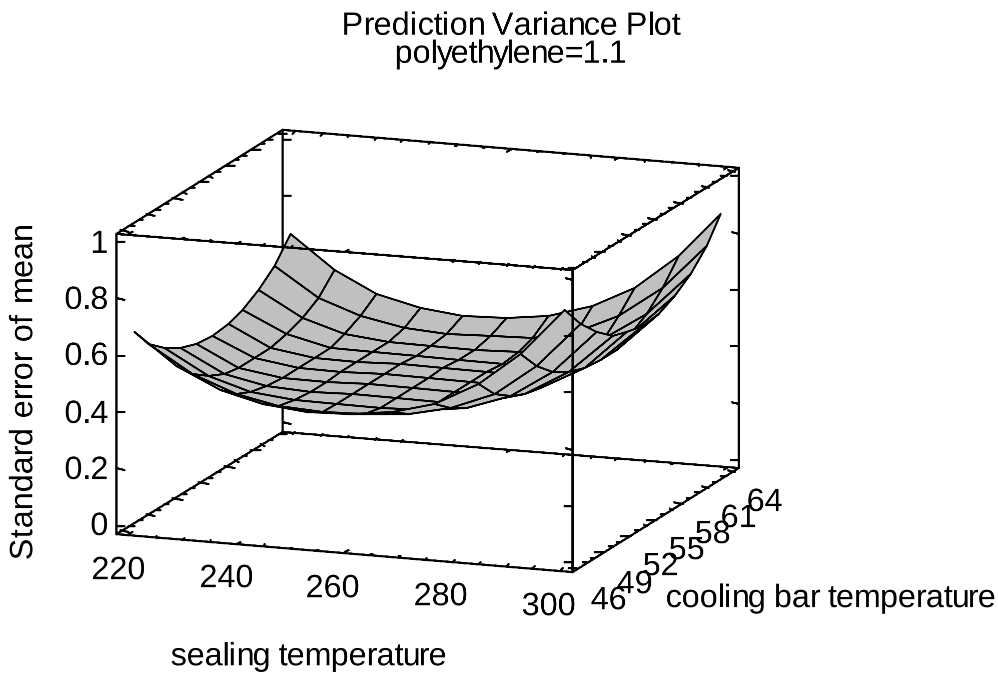


If only one factor is selected, a barchart is created displaying the number of runs at each level of the selected factor:



Prediction Variance Plot

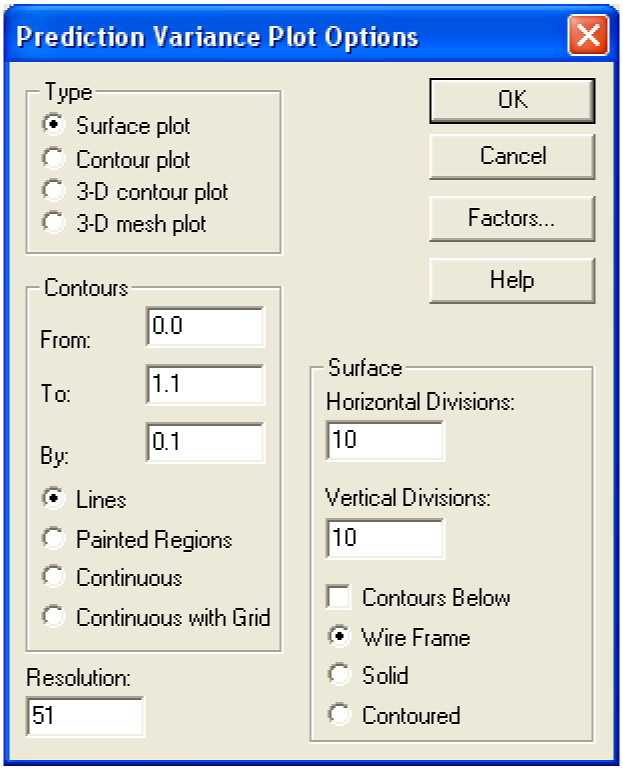
The *Prediction Variance Plot* shows how the standard error of the predicted response varies across the experimental region. The standard error displayed is the square root of the unscaled prediction variance defined in equation (11), with σ = 1. By default, a surface plot is created for the first two experimental factors, with all other factors held constant:



For the breadwrapper design, the standard error is lowest near the center of the experimental region. It increases as the location moves away from the center in any direction.

Pane Options

The *Pane Options* dialog box can be used to change the factors displayed on the plot as well as other plot features:



- **Type**: type of response plot to create. The standard error may be plotted as a surface, a two-dimensional contour plot, a three-dimensional contour plot, or a three-dimensional mesh plot.

- **Contours –** options for a contour plot.

  - **From**: location at which the first contour line is drawn, or the start of the first region.

  - **To**: location at which the last contour line is drawn, or the end of the last region.

  - **By**: spacing between contour lines or regions.

  - **Lines**: if selected, a sequence of contour lines is drawn at selected levels of the predicted response, as on a topographical map.

  - **Painted Regions**: if selected, a set of regions is drawn covering various ranges of the predicted response.

  - **Continuous**: draws contours using a continuous range of colors.

  - **Continuous with Grid**: draws contours using a continuous range of colors and adds a grid.

- **Resolution**: defines the resolution *m* of an *m*-by-*m* grid of predicted values which is used to draw the surface and contour lines. Increasing the resolution may improve the smoothness and definition of the plots, at the expense of computer time and memory.

- **Surface** – options for a surface plot.

- **Horizontal Divisions**: the number of divisions along the first experimental axis. This determines how many vertical lines will be drawn on the surface plot.

- **Vertical Divisions**: the number of divisions along the second experimental axis. This determines how many horizontal lines will be drawn on the surface plot.

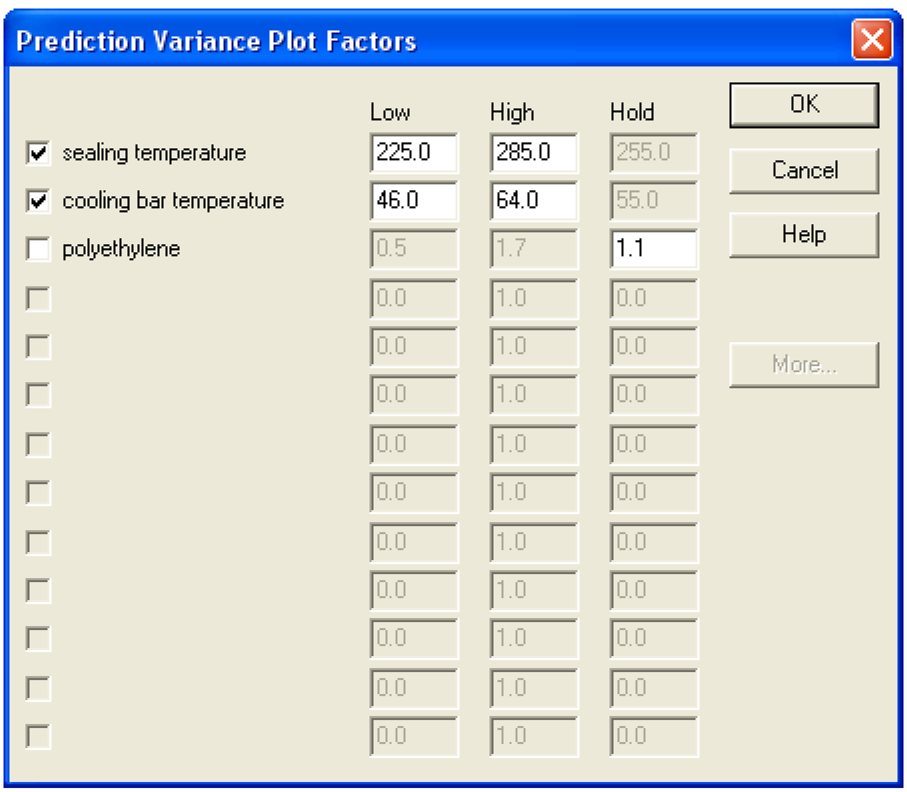
- **Contours Below**: requests that a contour plot, of type specified below, be drawn in the bottom face of the 3-D plot.

- **Wire Frame**: requests that the surface be drawn using cross-hatched lines as shown in the figure above. This is the most effective choice for black-and-white presentation.

- **Solid**: requests that the surface be drawn using a solid color.

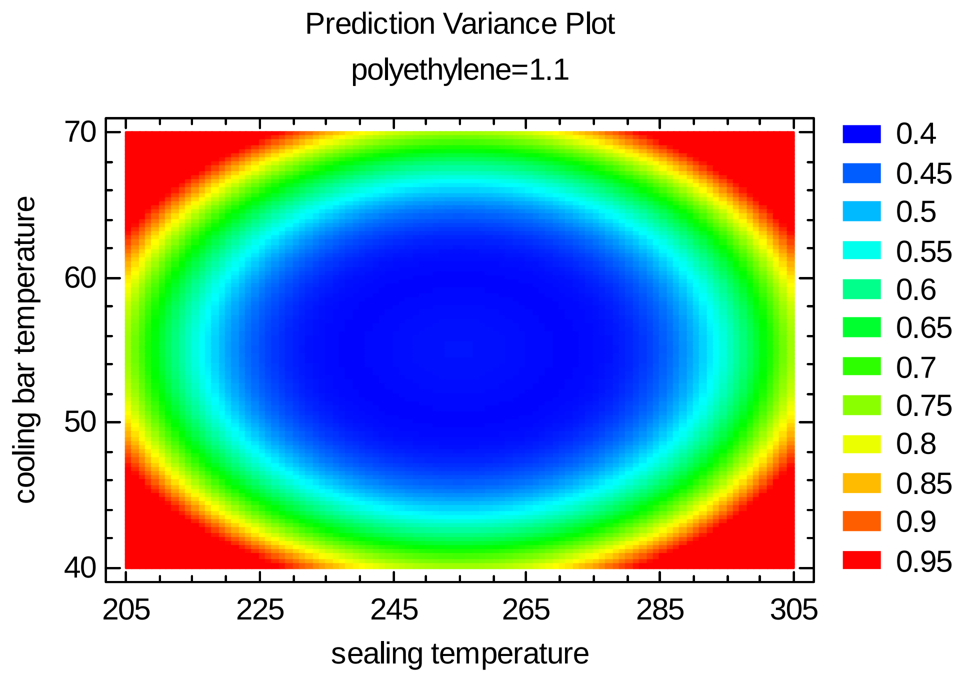
- **Contoured**: requests that the surface be drawn showing contour levels of the response.

- **Factors** button: displays a dialog box to select the factors to be plotted on each axis and the levels at which the other factors will be held:



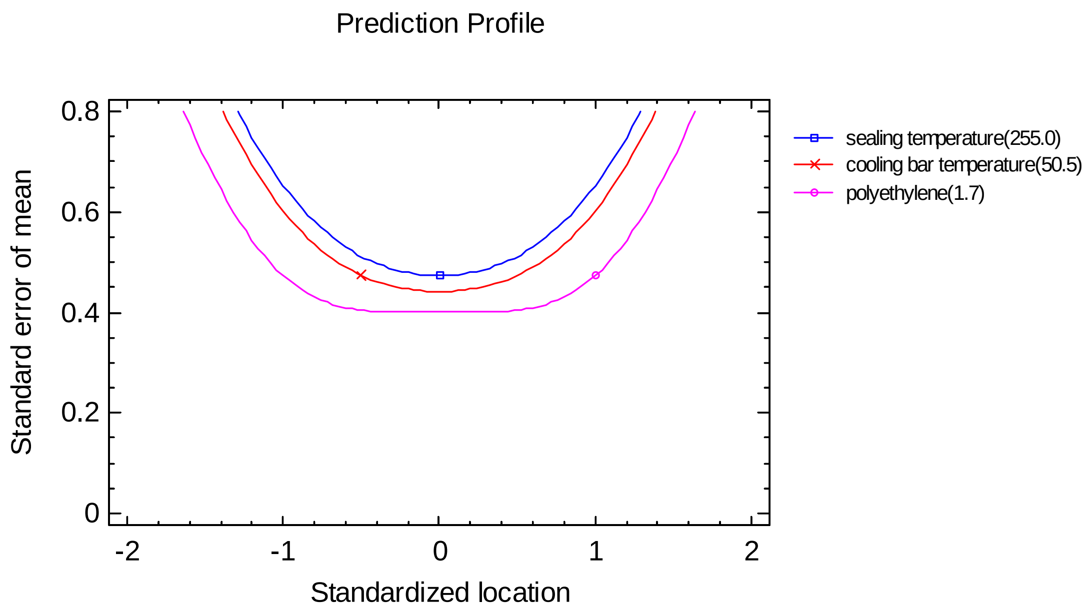
The current example plots the standard error for predicted values versus *sealing temperature* and *cooling bar temperature*, when *polyethylene = 1.1*.

The same information shown as a contour plot with continuous contours is displayed below:


Prediction Variance Plot
polyethylene=1.1
cooling bar temperature
70
60
50
40
205
225
245
265
285
305
sealing temperature
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95

Prediction Profile

The *Prediction Profile* graph displays the standard error of the predicted response as a function of each experimental factor, as the factors are moved from a specified reference point. For example, consider the location in the experimental region where *sealing temperature* = 255.0, *cooling bar temperature* = 50.5, and *polyethylene* = 1.7. At that location, the standard error of prediction equals 0.4735. The plot below shows the location each factor in standardized units:

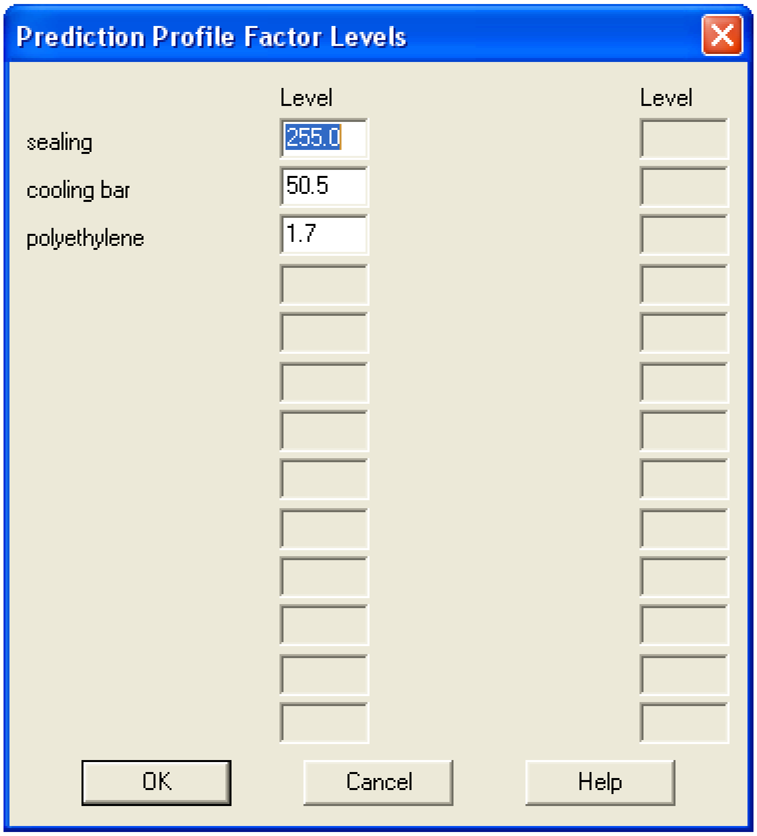


In standardized units, the specified *low* value equals -1, the center is 0, and the specified *high* value equals 1.

The lines on the plot show how the specified standard error changes as the factors are moved away from the reference location. Note that the standard errors remain small within the low to high range (-1 to 1) but start to increase rapidly outside that range.

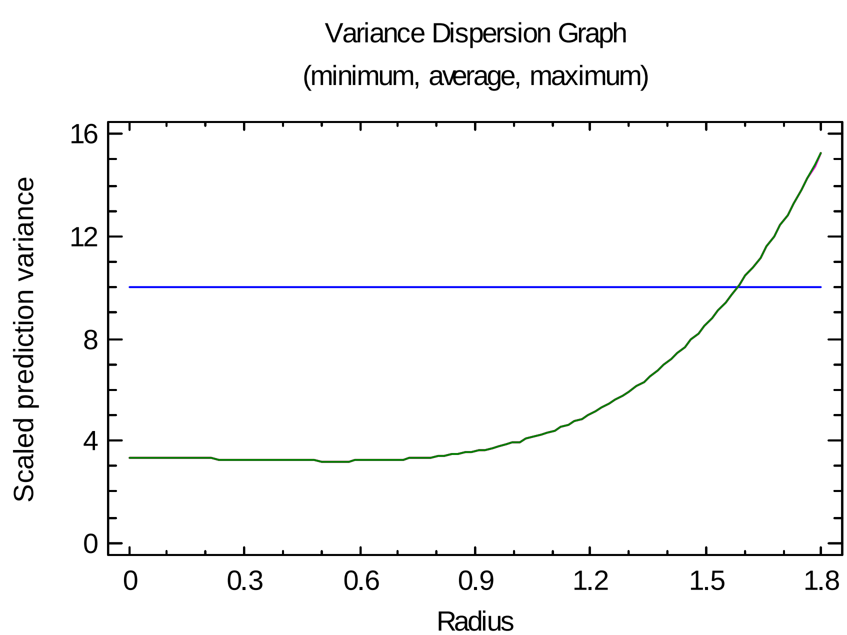
<u>Pane Options</u>

Use this dialog box to specify the reference location for the factors:



Variance Dispersion Graph

To summarize how the prediction variance in (11) changes throughout the experimental region, a *Variance Dispersion Graph* can be created:



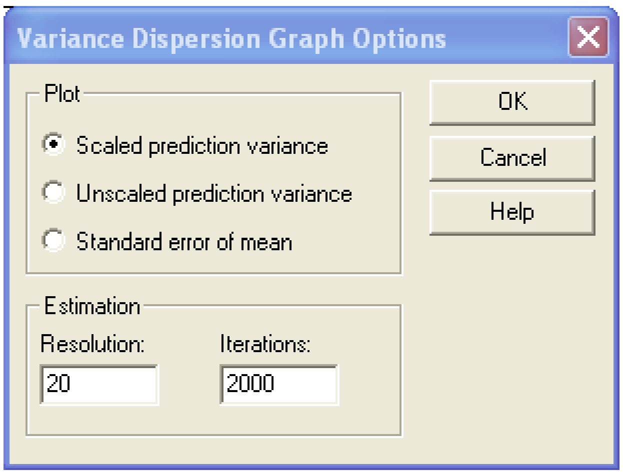
This graph shows how the variance changes with distance from the center of the experimental region. Three curves are plotted:

1. The maximum prediction variance at a specified radius (distance from the center of the experimental region in standardized units).
2. The minimum prediction variance at a specified radius.
3. The average prediction variance at a specified radius.

For the sample design, all three curves are identical, since the design is rotatable and the prediction variance changes identically with radius in all directions.

A horizontal line is also plotted at *p*, the number of model coefficients to be estimated. It can be shown that the maximum prediction variance of a design over the experimental region is at least *p*. The variance dispersion graph can be used to compare the properties of alternative designs.

Pane Options



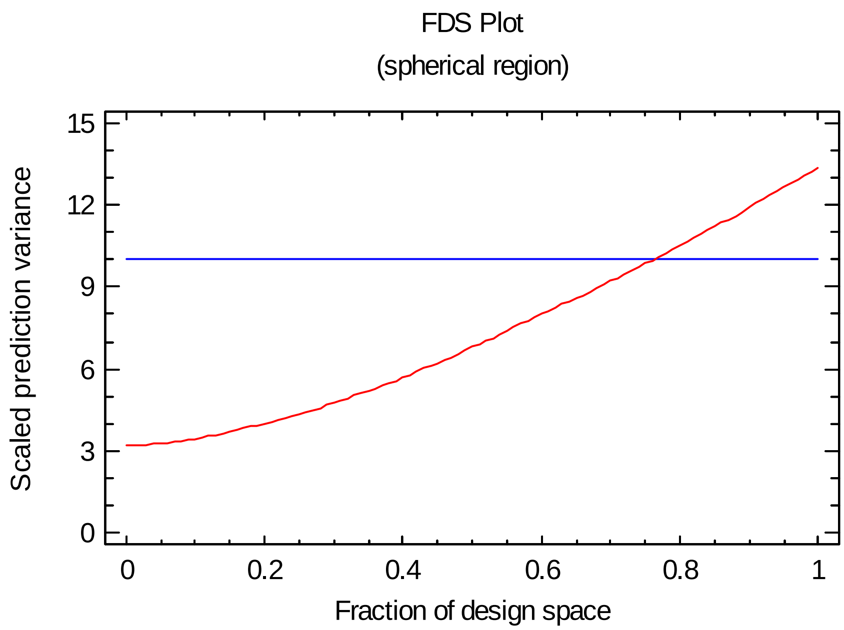
Plot – quantity to plot. Select between the scaled prediction variance defined in equation (11), the unscaled prediction variance defined in (12), or the standard error of the mean which is the square root of the unscaled prediction variance.

Resolution – number of locations along the horizontal axis at which to evaluate the function. Keep this number as small as possible, since the calculation is very time-consuming.

Iterations – the number of randomly selected points at each radius where the selected quantity is calculated. Increasing this number will cause the estimate function to be smoother but will take more time to compute.

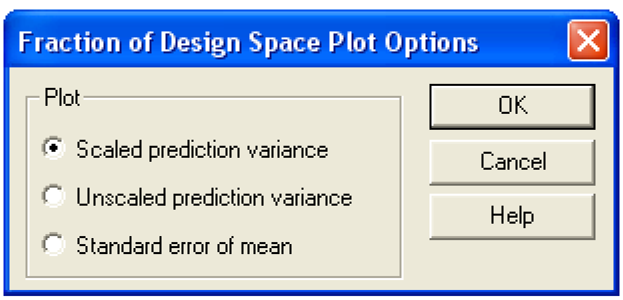
Fraction of Design Space Plot

Another way to summarize the predictive ability of a design is to calculate the fraction of the design space within which the prediction variance is less than specified values. The *Fraction of Design Space (FDS) Plot* shows this as displayed below:



For example, notice that the red line crosses the horizontal line at approximately 0.76. This implies that the SPV is less than p = 10 over approximately 76% of the design space.

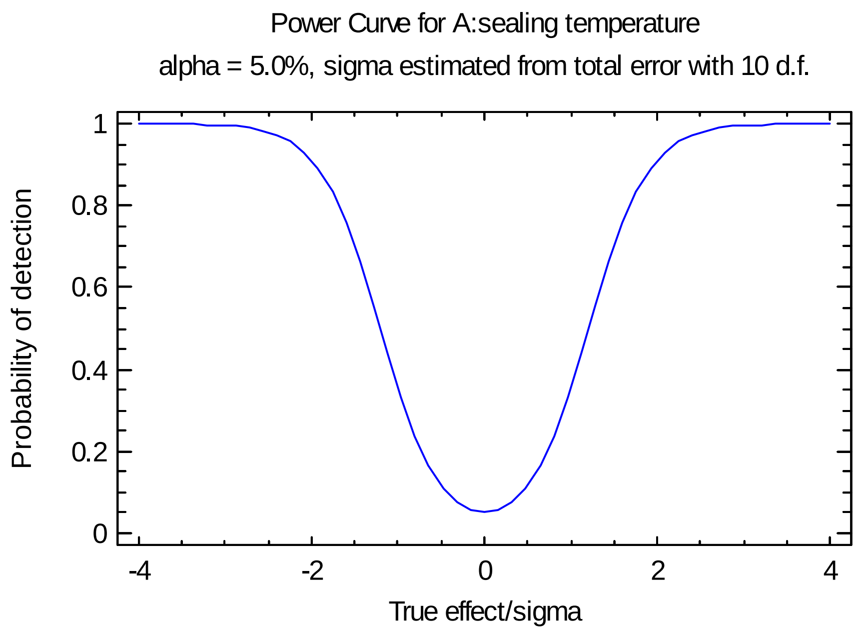
Pane Options



Plot – quantity to plot. Select between the scaled prediction variance defined in equation (19), the unscaled prediction variance defined in (20), or the standard error of the mean which is the square root of the unscaled prediction variance.

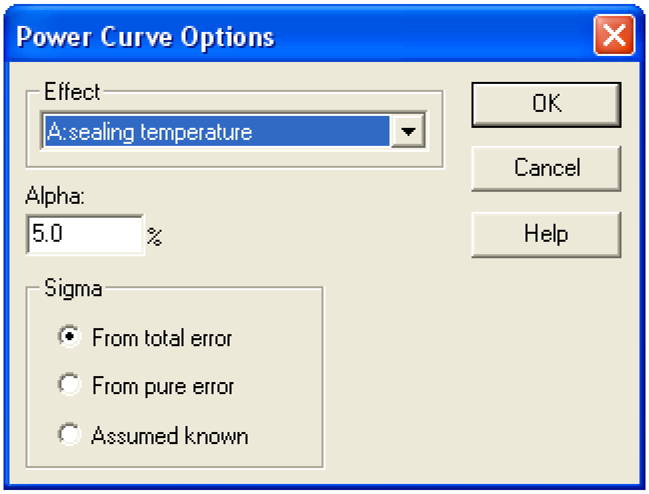
Power Curve

The *Power Curve* displays the probability that an effect will be identified as being statistically significant after the current experiment is performed and the data are analyzed. The probability is plotted ad a function of magnitude of the effect divided by the standard deviation of the experimental error. For example, the graph below plots the power for the main effect of factor A:



If the true effect is 2 or more times sigma, there is a high probability that it will be detected from this design. Anything less than 1 times the standard error is likely to be missed.

Pane Options



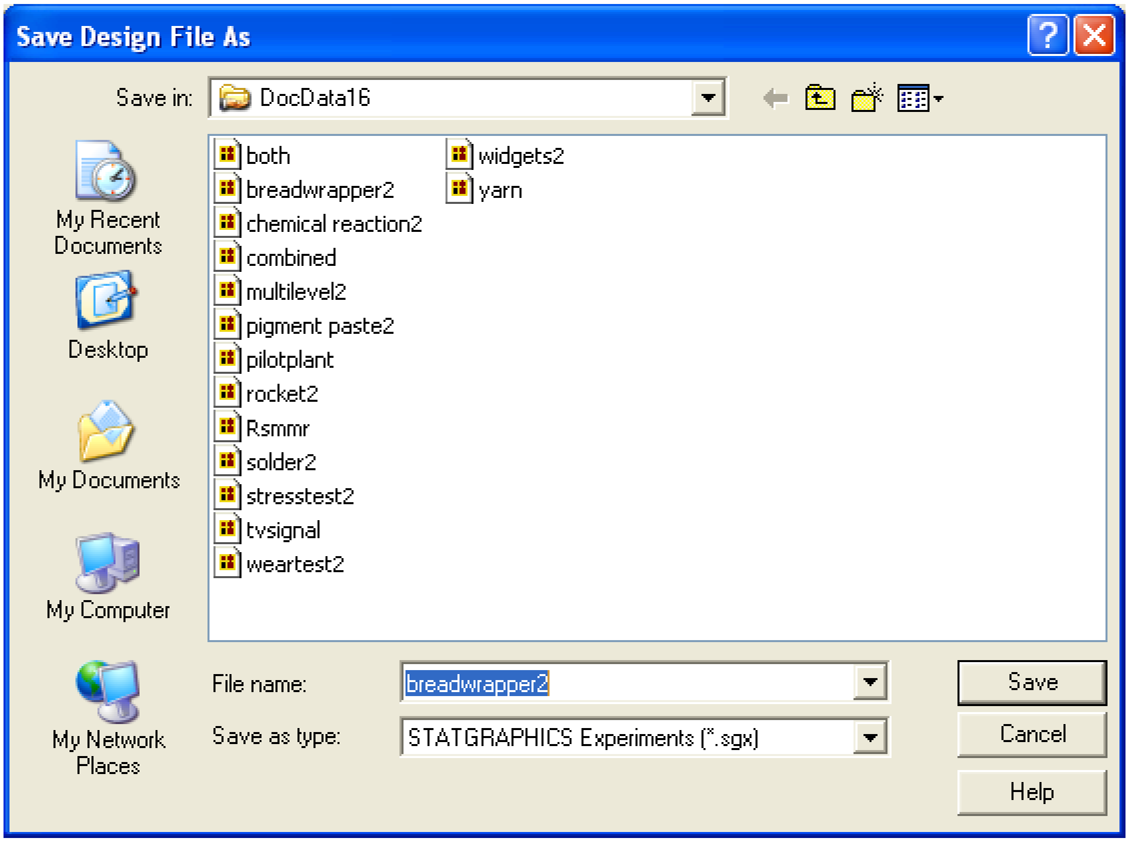
- *Effect* – the main effect or interaction for which the power will be plotted.

- *Alpha* – the alpha level at which effects are identified as being statistically significant (usually 1%, 5%, or 10%).

- *Sigma* – the method that will be used to estimate sigma when the data are analyzed.

Step 7: Save Experiment

This step saves the experiment in datasheet A as an XML file with the extension *.sgx*. A standard file save dialog box will be displayed in which to specify the desired file name:

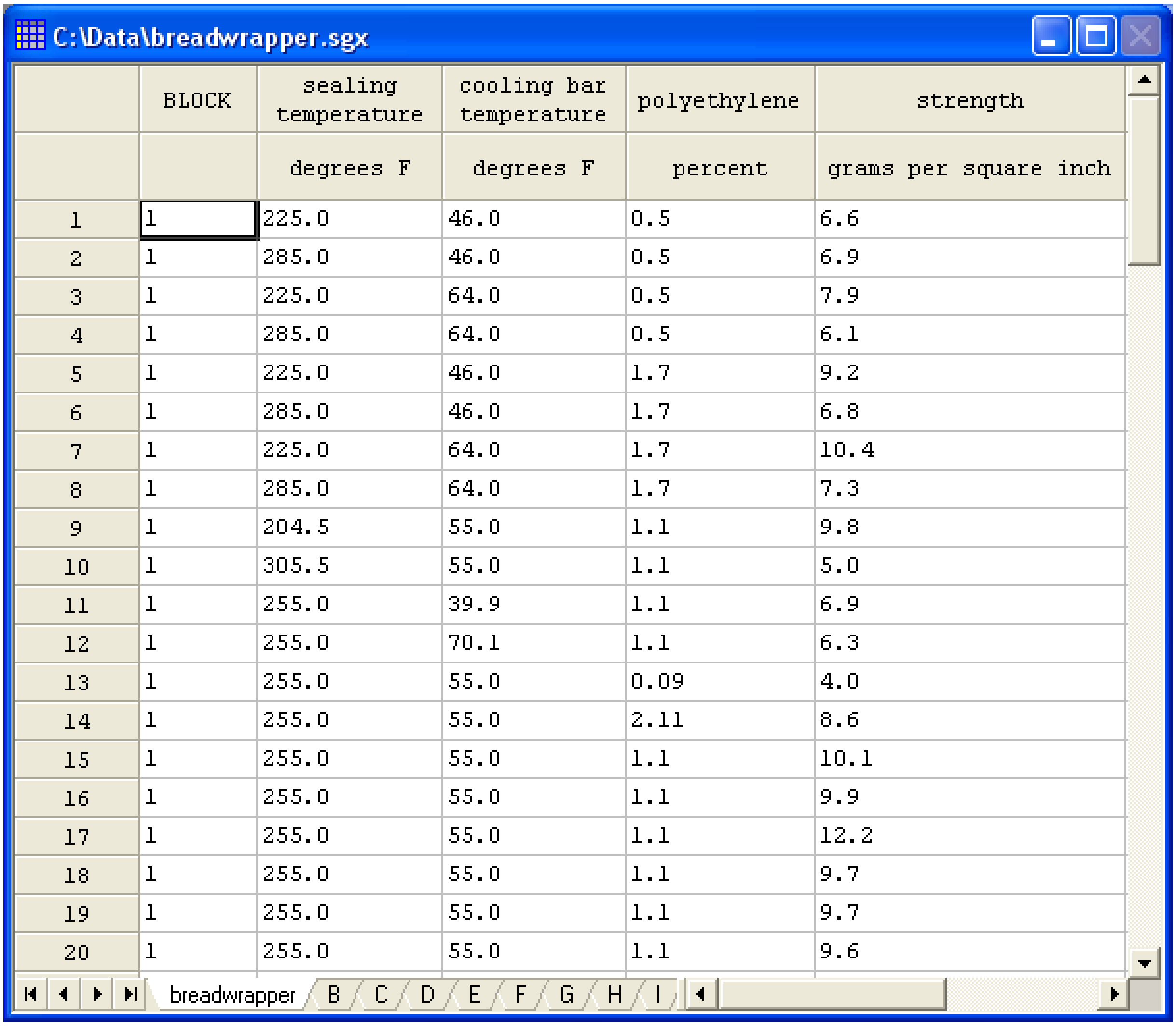


Experiment files are similar to standard STATGRAPHICS data files but contain not only the data but also additional information about the experimental structure.

Step 8: Analyze Data

After the completion of the first 7 steps, the experiment is ready to be performed. Once the experiment is completed, the analyst can return to STATGRAPHICS and reopen the saved experiment. If only one sample was taken during each experimental run, the results of the experiment will then be entered into sheet A of the databook. If multiple samples were taken during each run, the results are entered into sheets B through Z (one sheet for each response).

The results of the sample breadwrapper experiment are shown below:

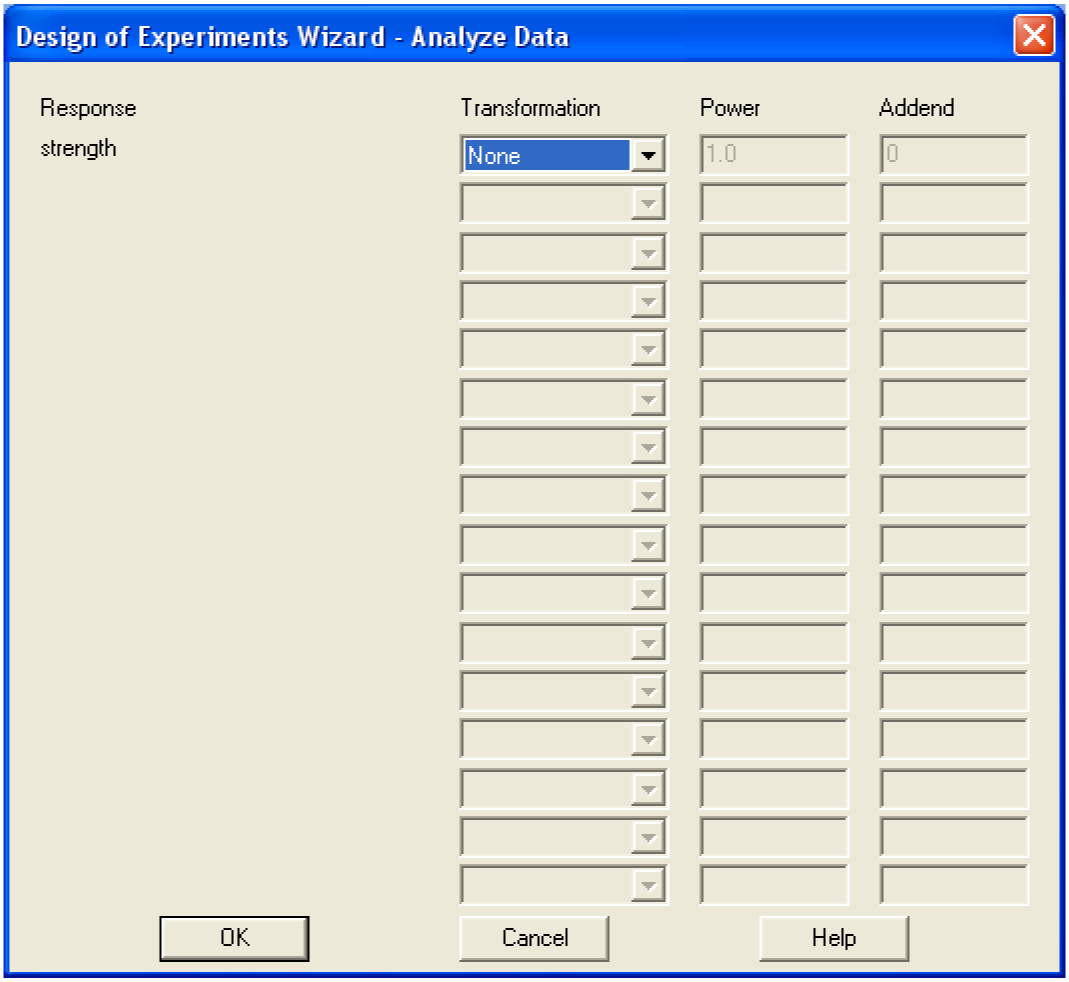


C:\Data\breadwrapper.sgx

	BLOCK	sealing temperature	cooling bar temperature	polyethylene	strength
		degrees F	degrees F	percent	grams per square inch
1	1	225.0	46.0	0.5	6.6
2	1	285.0	46.0	0.5	6.9
3	1	225.0	64.0	0.5	7.9
4	1	285.0	64.0	0.5	6.1
5	1	225.0	46.0	1.7	9.2
6	1	285.0	46.0	1.7	6.8
7	1	225.0	64.0	1.7	10.4
8	1	285.0	64.0	1.7	7.3
9	1	204.5	55.0	1.1	9.8
10	1	305.5	55.0	1.1	5.0
11	1	255.0	39.9	1.1	6.9
12	1	255.0	70.1	1.1	6.3
13	1	255.0	55.0	0.09	4.0
14	1	255.0	55.0	2.11	8.6
15	1	255.0	55.0	1.1	10.1
16	1	255.0	55.0	1.1	9.9
17	1	255.0	55.0	1.1	12.2
18	1	255.0	55.0	1.1	9.7
19	1	255.0	55.0	1.1	9.7
20	1	255.0	55.0	1.1	9.6

breadwrapper B C D E F G H I

To analyze the results, press the button labeled *Step 8: Analyze Data*. This will first display the dialog box shown below:



Each response variable is displayed so that a transformation may be selected if desired. Specify:

- **Transformation** – the type of transformation to be applied.
- **Power** – for *power* transformations, the power *P* to which the response variable will be raised.
- **Addend** – for *logarithm*, *power* and *Box-Cox* transformations, an addend Δ which will be added to the response variable before the transformation is applied. An addend is necessary for those transformations if the original response variable contains zeroes or negative values.

The selections for *transformation* are:

- o *None* – the response *Y* will be analyzed as measured without applying any transformation.
- o *Square root* – the response variable will be transformed by taking its square root.
- o *Logarithm* – the response variable will be transformed by taking its natural logarithm according to

$$Y_t = \log(Y + \Delta) \tag{28}$$

- *Reciprocal* – the response variable will be transformed by taking its reciprocal.

- *Power* – the response variable will be transformed by raising it to a power according to

$$Y_t = (Y + \Delta)^P \tag{29}$$

- *Arc sine square root* (in grads) - the response variable (all values of which must be between 0 and 1) will be transformed according to

$$Y_y = \sin^{-1}\left(\sqrt{Y}\right) \tag{30}$$

- *Box-Cox* – the Box-Cox transformation will be applied to the response variable. This transformation is defined by

$$Y_t = \begin{cases} 1 + \dfrac{(Y+\Delta)^P - 1}{Pg^{P-1}} & \text{if } P \neq 0 \\ 1 + g\ln(Y+\Delta) & \text{if } P = 0 \end{cases} \tag{31}$$

  where g is the geometric mean of the observed values of the response. The value of P is automatically determined so as to minimize the mean squared error of the fitted statistical model for the transformed values of the response variable.

The transformations defined above are commonly used in cases where the variance of the response changes as the mean changes. Note the following:

1. If the response variable is a count (which tends to follow a Poisson distribution), the *square root* transformation is appropriate.

2. If the response variable is a binomial proportion, the *arc sine square root* transformation is appropriate.

3. If the response variable is a standard deviation or a coefficient of variation, the *logarithm* is an appropriate transformation.

4. For continuous variables, power transformations (including the logarithm) are appropriate. The proper power depends on the link between the standard deviation (sigma) and the mean:

   - If sigma is proportional to the mean, use a logarithm (equivalent to $P = 0$).
   - If sigma is proportional to the square root of the mean, use a square root ($P = 0.5$).
   - If sigma is proportional to the square of the mean, use a reciprocal ($P = -1$).
   - If the link is uncertain, the *Box-Cox* transformation can be used to determine the optimal power.

After selecting the desired transformations, press *OK* to fit the selected statistical model to each of the response variables and open a new window for each response. Some important notes:

1. The contents of each response window will vary, depending on the type of experiment selected. Detailed descriptions of the analyses are contained in the following documents:

    - DOE Wizard – Inner-Outer Arrays – for robust parameter designs with crossed orthogonal arrays as proposed by Prof. Taguchi.
    - DOE Wizard – Mixture Designs – for designs involving mixture components.
    - DOE Wizard – Multi-Factor Categorical Designs – for designs involving only categorical factors.
    - DOE Wizard – Multilevel Factorial Designs – for designs with all combinations of 2 or more levels of the factors.
    - DOE Wizard – Process and Mixture Factors – for designs with both process factors and mixture components.
    - DOE Wizard – Quantitative and Categorical Factors – for designs with both quantitative and non-quantitative factors.
    - DOE Wizard – Response Surface Designs – for designs involving continuous factors at more than 2 levels.
    - DOE Wizard – Robust Parameter Designs – for robust parameter designs in which controllable and noise factors are combined in a single design.
    - DOE Wizard – Screening Designs – for designs involving factors all of which are run at 2 levels.
    - DOE Wizard – Single Factor Categorical Designs – for experiments involving a single categorical factor.
    - DOE Wizard – Variance Component Designs – for hierarchical designs in which all of the factors are nested.

2. The initial statistical model fit is based on the selection in the DOE Wizard when the experiment was first constructed. You can use the *Analysis Options* dialog box within each response window to change the model for the corresponding response.

3. To optimize each response separately, use the response windows. The optimization features within the DOE Wizard will optimize multiple responses jointly using desirability functions.

After the analysis is performed, a new section will be added to the *Analysis Summary* in the DOE Wizard window summarizing the results:

Step 8: Analyze the experimental results

Model	*strength*
Effects	9
P-value	0.0034
Error d.f.	10
Stnd. error	1.08939
R-squared	85.56
Adj. R-squared	72.56

The output includes:

- *Effects* – the number of effects in the estimated statistical model, excluding the constant term.

- *P-value* – the P-value for the F-test to determine the statistical significance of the overall model. A P-value below 0.05 indicates a statistically significant model at the 5% significance level.

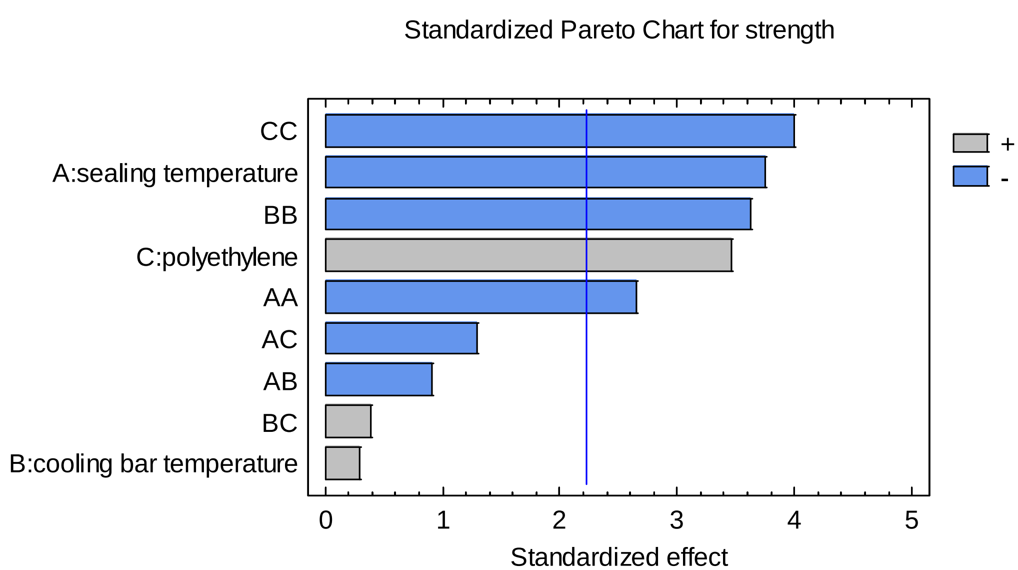
- *Error d.f.* – the number of degrees of freedom available to estimate experimental error, including any lack-of-fit.

- *Stnd. error* – the estimated standard error of the experimental noise.

- *R-squared* – the percent of the variability in the response that has been explained by the fitted model.

- *Adj. R-squared* – the adjusted R-squared value, which is often used to compare models with different numbers of coefficients.

For the breadwrapper experiment, 9 effects were estimated (3 main effects, 3 two-factor interactions, and 3 quadratic effects). The overall model is statistically significant at the 1% significance level. Overall, the statistical model explains about 85% of the variability in strength. Closer examination of the analysis window for analyzing strength shows the following Pareto chart:



All 3 factors appear to have a significant effect on *strength*, since at least one bar for each factor extends beyond the α = 5% vertical line.

Step 9: Optimize Responses

In order to find a combination of the experimental factors that provides a good result for multiple response variables, the DOE Wizard uses the concept of *desirability functions*. It will be recalled that when the response variables were first defined, several important parameters were specified for each response:

1. Whether the *goal* of the experiment was to maximize the response, minimize the response, or hit a specified *target* value.

2. The relative *impact* of the response on a scale ranging from 1 to 5.

3. A range of response values (*low* to *high*).

4. The *sensitivity* of the position within the above range.

Now let

$\hat{y}$ = predicted value of the response for a particular combination of the experimental factors **x**.

low = low end of the range for the response

high = high end of the range for the response

I = impact of the response

S = sensitivity of the response, defined as 1/5 for "very low", 1/3 for "low", 1 for "medium", 3 for "high", and 5 for "very high".

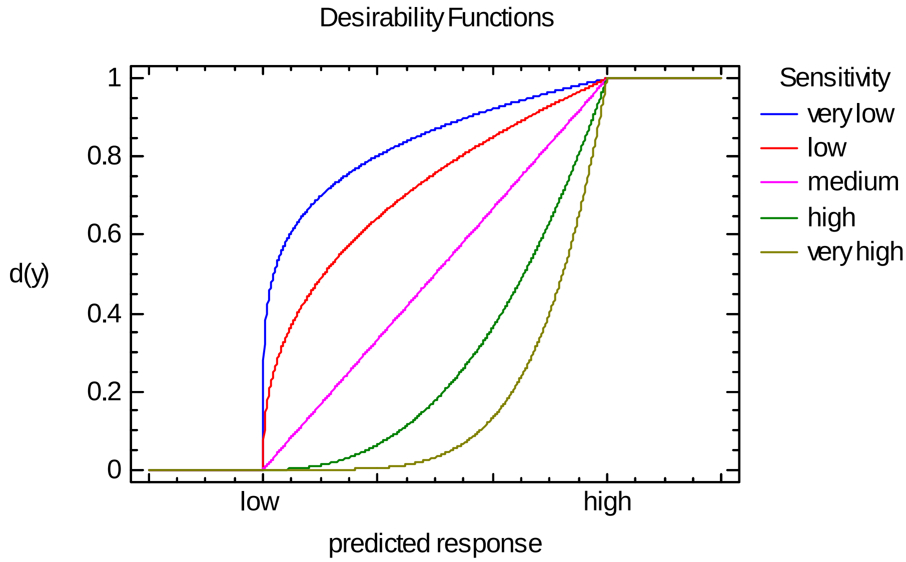
The "desirability" of the value $\hat{y}$ is defined as follows.

Response to be maximized

If a response variable is to be maximized, the desirability of the response is defined by

$$d = \begin{cases} 0 & \hat{y} < low \\ \left(\dfrac{\hat{y} - low}{high - low} \right)^S, & low \le \hat{y} \le high \\ 1 & \hat{y} > high \end{cases} \tag{20}$$

The function is plotted below:



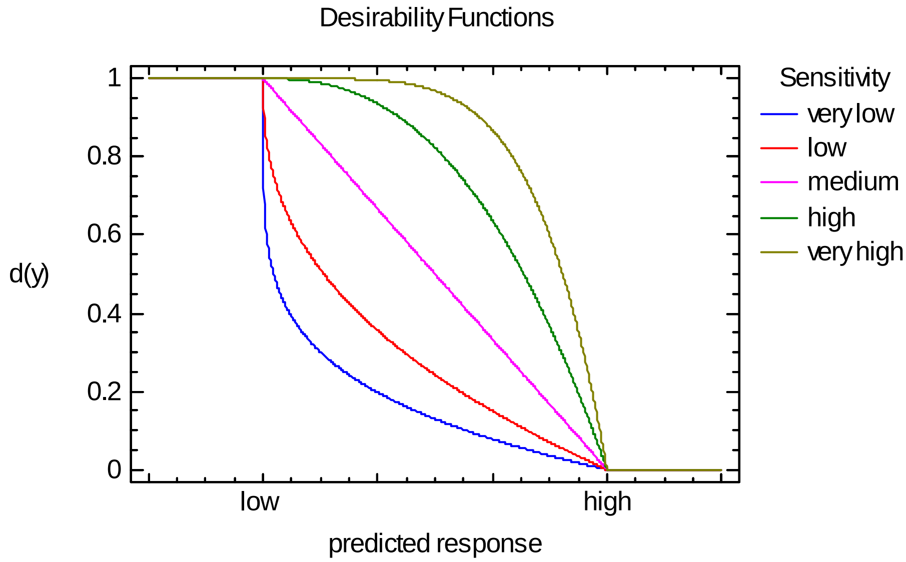
The parameter *S* defines the shape of the function. For *S = 1*, the desirability rises linearly from 0 at the *low* value to 1 at the *high* value. For *S < 1*, it increases quickly at first and then levels off. For *S > 1*, it increases slowly at first and then speeds up. The analyst can set *S* large if it is very important to be close to the maximum level or small if anywhere over the specified response range is almost equally desirable.

Response to be minimized

If a response variable is to be minimized, the desirability of the response is defined by

$$d = \begin{cases} 1 & \hat{y} < low \\ \left(\dfrac{\hat{y} - high}{low - high} \right)^{s} , & low \le \hat{y} \le high \\ 0 & \hat{y} > high \end{cases} \tag{21}$$

The function is plotted below:



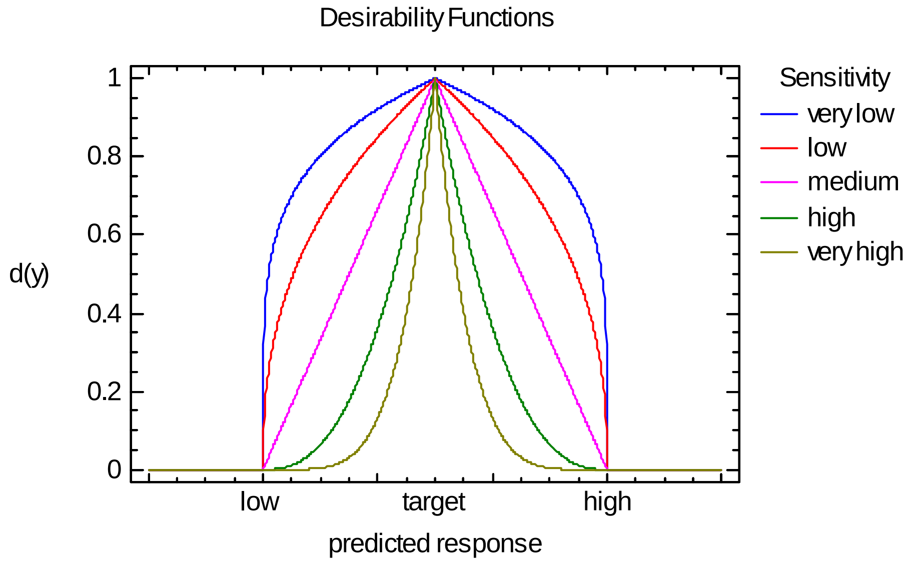
The function is the mirror image of that for maximization, starting at 1 at the *low* value and going to 0 at the *high* value.

Response to hit target

If the response variable is to be maintained at a specified value, the desirability function is defined by

$$d = \begin{cases} 0 & \hat{y} < low \\ \left(\dfrac{\hat{y} - low}{target - low} \right)^s & low \le \hat{y} \le target \\ \left(\dfrac{\hat{y} - high}{target - high} \right)^s & target \le \hat{y} \le high \\ 0 & \hat{y} > high \end{cases} , \qquad (22)$$

The function is plotted below:



Combining Responses

To combine the desirabilities of *m* responses, a single composite function *D* is created. If all of the response variables are considered to be equally important, then the composite function is the geometric mean of the separate desirabilities, given by

$$D = \{d_1 d_2 ... d_m\}^{1/m} \tag{23}$$

If some responses are more important than others, the composite function is defined as the product of the separate desirabilities after each has been raised to a power based on its *impact* coefficients:

$$D = \left\{d_1^{I_1} d_2^{I_2} ... d_m^{I_m}\right\}^{1/\left(\sum_{j=1}^{m} I_j\right)} \tag{24}$$

If you press the button labeled *Step 9: Optimize response*, the DOE Wizard will determine the combination of the experimental factors that maximizes the overall desirability *D*. It will add a table similar to that shown below to the *Analysis Summary*:

Step 9: Optimize the responses

Response Values at Optimum

Response	*Prediction*	*Lower 95.0% Limit*	*Upper 95.0% Limit*	*Desirability*
strength	11.0829	9.81658	12.3492	0.770722

Factor Settings at Optimum

Factor	*Setting*
sealing temperature	224.617
cooling bar temperature	57.3421
polyethelene	1.50968

At a *sealing temperature* of 224.6, a *cooling bar temperature* of 57.34, and a *polyethylene* level of 1.51, the predicted *strength* equals 11.08. Given the defined range for strength of (low=8, high=12), the calculated desirability is about 77%. Since there was only a single response variable, this is basically the same result that would have been obtained by maximizing *strength* directly. (Note: because of the numerical procedures used, the results may differ slightly, particularly if the response surface is relatively flat in the vicinity of the solution.)

Additional output is available by pressing the *Tables and Graphs* button while in the DOE Wizard:

Desirability

This table lists the observed and predicted desirability at each experimental point:

Desirability

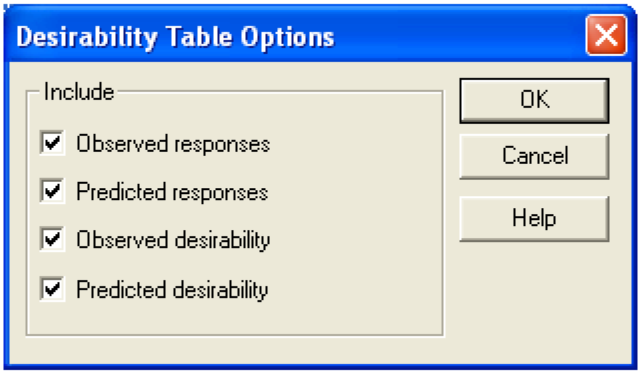
Response	*Observed Minimum*	*Observed Maximum*	*Goal*	*Impact*	*Sensitivity*	*Specified Low*	*Specified High*
strength	4.0	12.2	Maximize	3.0	Medium	8.0	12.0

Run	*Observed strength*	*Predicted strength*	*Observed Desirability*	*Predicted Desirability*
1	6.6	6.50934	0.0	0.0
2	6.9	6.0026	0.0	0.0
3	7.9	7.08444	0.0	0.0
4	6.1	5.1777	0.0	0.0
5	9.2	9.24974	0.3	0.312435
6	6.8	6.743	0.0	0.0
7	10.4	10.4248	0.6	0.606212
8	7.3	6.51811	0.0	0.0
9	9.8	9.8732	0.45	0.4683
10	5.0	6.15852	0.0	0.0
11	6.9	7.07305	0.0	0.0
12	6.3	7.36684	0.0	0.0
13	4.0	5.19852	0.0	0.0
14	8.6	8.6332	0.15	0.158301
15	10.1	10.1645	0.525	0.54112
16	9.9	10.1645	0.475	0.54112
17	12.2	10.1645	1.0	0.54112
18	9.7	10.1645	0.425	0.54112
19	9.7	10.1645	0.425	0.54112
20	9.6	10.1645	0.4	0.54112

In the breadwrapper experiment, run 7 yields the highest predicted desirability. This corresponds to a factorial point at low *sealing temperature*, high *cooling bar temperature*, and high *polyethylene*. The highest observed desirability occurred during run #17, which was one of the 6 centerpoints.

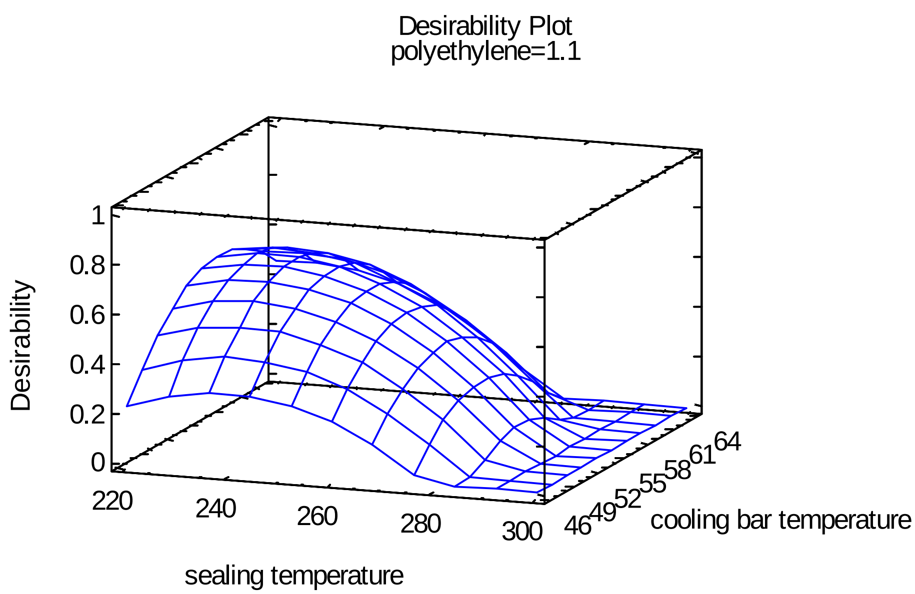
<u>Pane Options</u>

Select the items to be included in the table from the following dialog box:



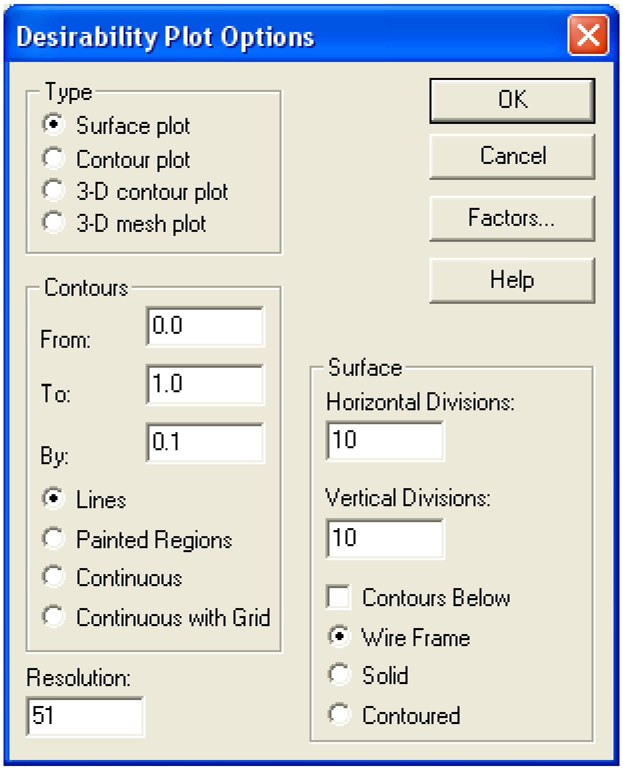
Desirability Plot

The estimated desirability can also be displayed graphically using one of three options; a surface plot, a 2-D contour plot, a 3_D contour plot, or a 3-D mesh plot. Selecting *Desirability Plot* from the *Tables and Graphs* dialog box displays the following plot by default:

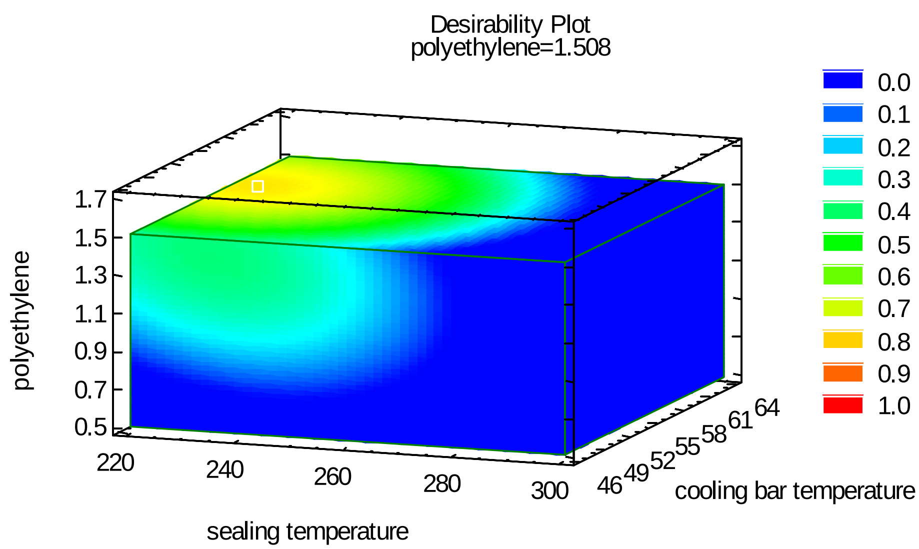


It shows the desirability function as a function of *sealing temperature* and *cooling bar temperature*, with *polyethylene* fixed at 1.1. The surface has a well-defined maximum near the center left of the plot.

<u>Plot Options</u>



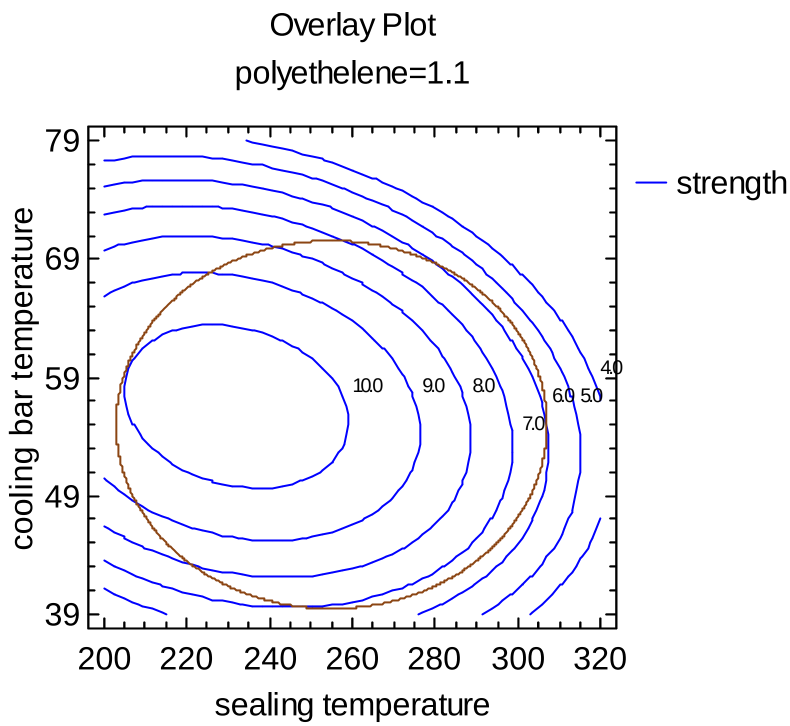
The options are similar to those described earlier for the *Variance Dispersion Plot*. Selecting the 3-D contour plot, we can display the location where *strength* is maximized:



The optimum location is clearly seen and corresponds to a small spherical region in the space of the 3 experimental factors.

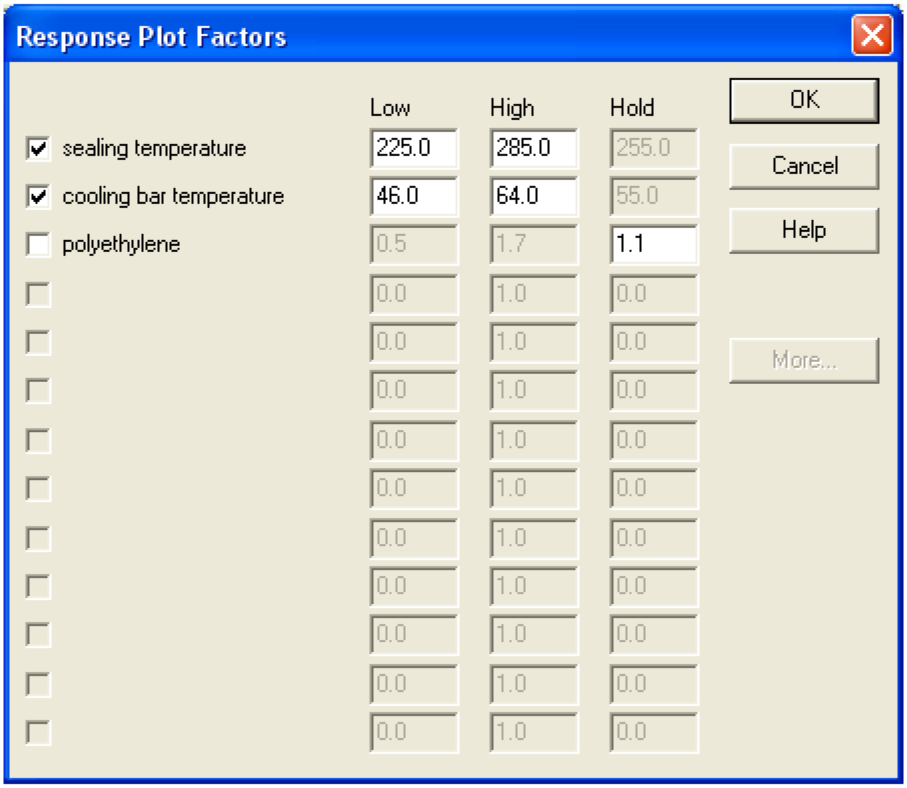
Overlaid Contour Plots

For designs with more than one response, it can be useful to overlay the contour plots of the responses. For the breadwrapper example, there is only one response:



If there had been additional responses, their contours would have been plotted on top of those for *strength*. Note: The experimental region is also drawn on the plot.

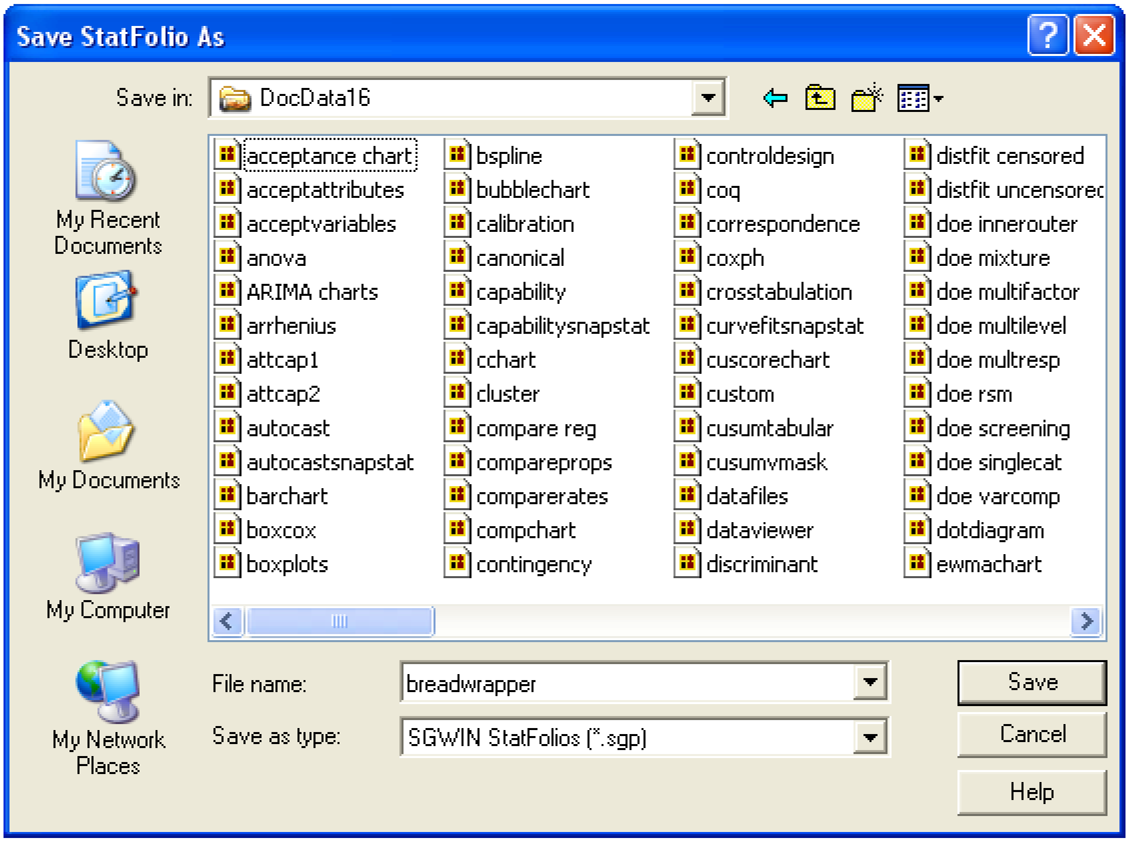
<u>Plot Options</u>



Select the factors to plot on the horizontal and vertical axes, together with their low and high limits. For other factors, select values at which they will be held constant.

Step 10: Save Results

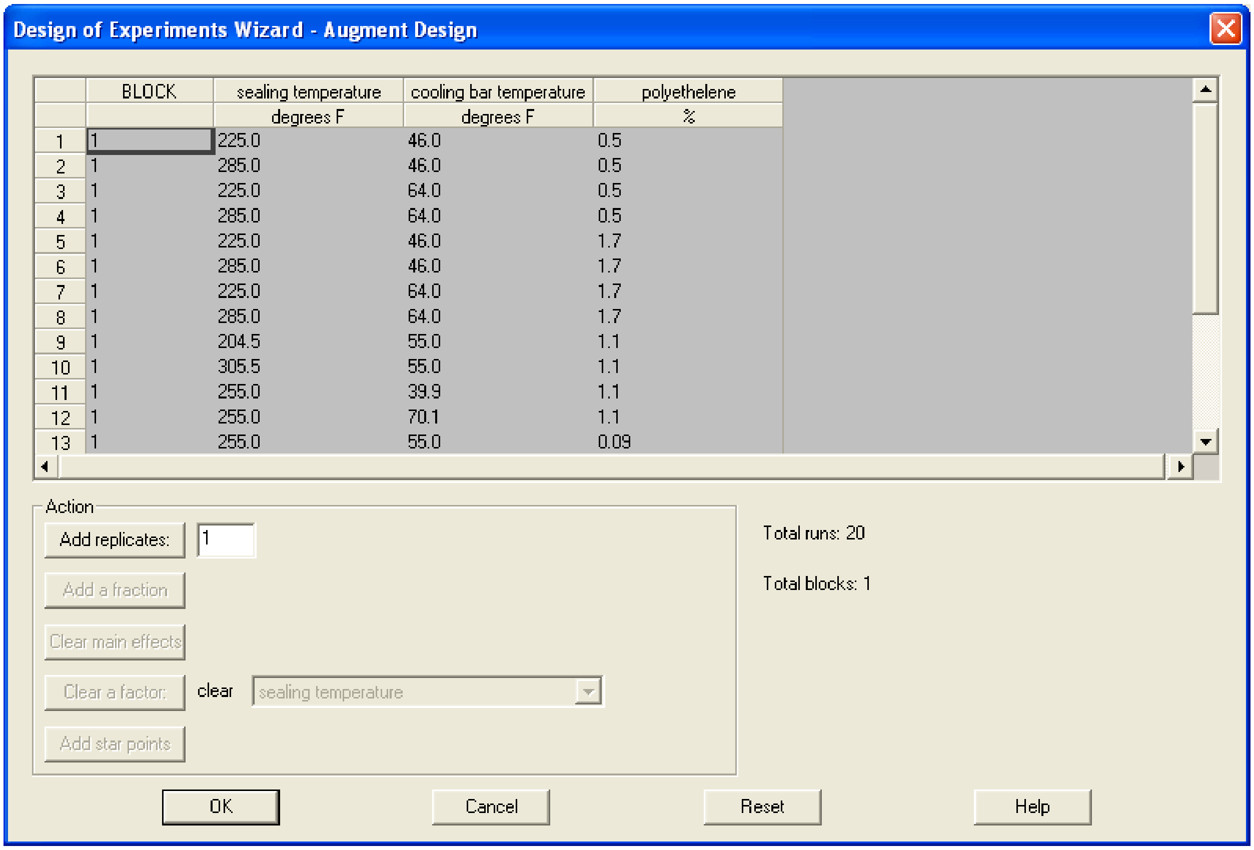
The analysis of the experiment can be saved in a StatFolio by pressing the button labeled Step 10: Save results:



Note that the analysis is saved in a file that is separate from the experiment file. Whenever the saved StatFolio is reloaded, it will automatically load the experiment file and well and place the data in Sheet A of the STATGRAPHICS DataBook.

Step 11: Augment Design

After the initial experiment is performed and the data are analyzed, it may be desirable to perform additional runs. If so, press the button labeled *Step11: Augment design*. This will display the dialog box shown below:



The top section shows the current design. The *Action* section lets you augment the design in various ways:

- *Add replicates* – Pressing this button will add additional runs containing copies of the original design. The new runs will be placed in separate blocks to account for potential changes in the mean response from the first set of runs to the second. This options is always available.

- *Add a fraction* (available only for some screening designs) – Pressing this button will add additional runs to a fractional factorial design in order to increase its resolution. This option always doubles the number of runs. In some cases, the design will be folded over, adding additional runs in which all of the low levels have been switched to high levels and vice versa.

- *Clear main effects* (available only for some Resolution III screening designs) – This option adds additional runs to a resolution III fractional factorial design in order to increase its resolution to IV. In the augmented design, each main effect is clear of (not confounded with) the two-factor interactions.

- *Clear a factor* (available only for some Resolution III screening designs) – This option adds additional runs to a resolution III fractional factorial design in order to clear the specified factor and all of its interactions. This option is sometimes used to get a detailed picture of the effects of a dominant factor.

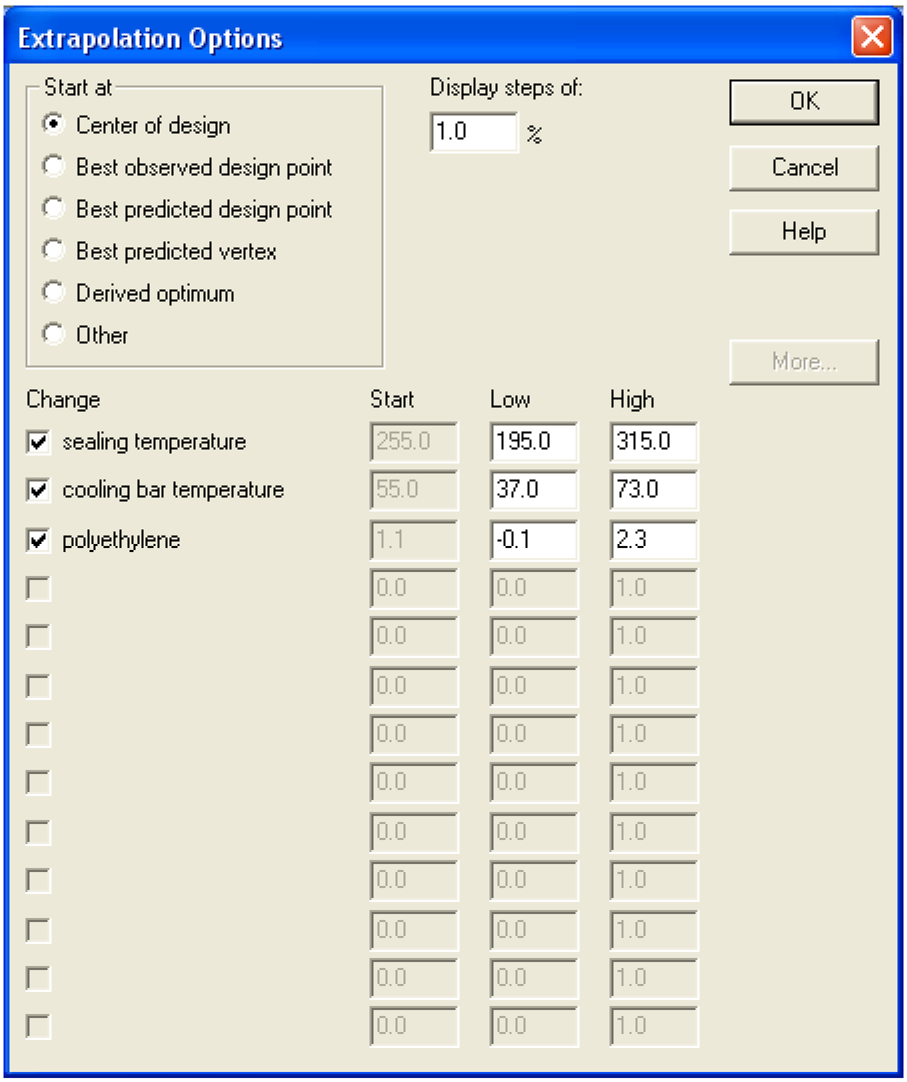
- *Add star points* (available only for some Resolution V screening designs) – This option adds star points to a resolution V factorial or fractional factorial design to create a central composite response surface design. The star points are added at the proper distance such that the design is orthogonally blocked.

When a button is pushed, the changes are made initially only within the dialog box. To accept the changes, press *OK*. To cancel the changes, press *Cancel*. You can also press *Reset* to return to the original design.

Step 12: Extrapolate

The statistical models fit to an experimental design are best used within the experimental region. Outside of the experimental region, where there is no data, anything could happen. Nonetheless, the fitted models can be used to suggest the best direction in which to look for potential improvements.

If you press the button labeled *Step 12: Extrapolate*, STATGRAPHICS will use the fitted mathematical models and the associated desirability functions to look for settings of the experimental factors which result in the highest desirability. The program first displays the following dialog box:



- *Start at* – Select location at which the search for more desirable locations should begin. The program evaluates the initial gradients at the starting location and then begins moving along the *path of steepest ascent*. As it moves, it continually reevaluates the gradients, so that it may follow a curved path.

- *Display steps of* – Indicate the magnitude of the smallest change in desirability that will be displayed.

- *Change* – Indicate which factors may be changed. If desired, you may freeze the value of one or more factors.

- *Start* – If the *Start at* location is set to “Other”, specify the location at which to start the search.

- *Low* and *High* – Specify the allowable range for the factors. Once a factor reaches either of these limits, it will not be changed anymore. By default, these values are set at twice the distance from the center of the experimental region as the original design limits.

When you press OK, two tables will be added to the *Analysis Summary* showing each step that resulted in a desirability change of at least the magnitude specified on the dialog box. The first table displays the estimated desirability and the values of the response variables:

Step 12: Extrapolate model

Extrapolated Response Values

Step	*Desirability*	*strength*
0	0.54112	10.1645
1	0.551226	10.2049
2	0.565823	10.2633
3	0.579768	10.3191
4	0.593083	10.3723
5	0.605788	10.4232
6	0.617904	10.4716
7	0.629448	10.5178
8	0.640439	10.5618
9	0.650894	10.6036
10	0.664026	10.6561
11	0.676268	10.7051
12	0.687652	10.7506
13	0.698209	10.7928
14	0.710283	10.8411
15	0.721154	10.8846
16	0.732672	10.9307
17	0.744085	10.9763
18	0.754559	11.0182
19	0.764649	11.0586
20	0.770714	11.0829

The second table displays the corresponding settings of the experimental factors:

Factor Settings for Extrapolation

Step	*sealing temperature*	*cooling bar temperature*	*polyethylene*
0	255.0	55.0	1.1
1	254.4	55.013	1.11101
2	253.5	55.0351	1.12726
3	252.6	55.0602	1.14319
4	251.7	55.0884	1.15881
5	250.8	55.1196	1.17412
6	249.9	55.1538	1.18914
7	249.0	55.191	1.20385
8	248.1	55.2312	1.21827
9	247.2	55.2743	1.23241

10	246.0	55.3364	1.25082
11	244.8	55.4038	1.26874
12	243.6	55.4763	1.28618
13	242.4	55.554	1.30315
14	240.9	55.6584	1.32372
15	239.4	55.7709	1.3436
16	237.6	55.9165	1.36658
17	235.5	56.1009	1.39221
18	233.1	56.3311	1.42005
19	229.8	56.6816	1.45599
20	224.7	57.3037	1.50705

Beginning at the center of the experimental region (Step 0), the program was able to take 20 steps to increase the desirability from approximately 54% to 77%. In this case, the most desirable location was within the experimental region. In other cases, however, the highest desirability may be obtained outside of the original region.