EWMA Charts

Summary

The **EWMA (Exponentially Weighted Moving Average) Charts** procedure creates control charts for a single numeric variable where the data have been collected either individually or in subgroups. In contrast to traditional X-Bar or X charts, a point plotted on the EWMA chart represents not just the last observation or subgroup but a weighted moving average of current and past data. Such charts are very useful in Phase 2 monitoring of processes against established control limits, since they have a shorter average run length in detecting small process shifts.

The procedure creates both an EWMA chart and an R chart, S chart, or MR(2) chart. The charts may be constructed in either *Initial Study* (Phase 1) mode, where the current data determine the control limits, or in *Control to Standard* (Phase 2) mode, where the limits come from either a known standard or from prior data.

Sample StatFolio: *ewmachart.sgp*

Sample Data:

The file *process shift.sgd* contains a sample of random numbers described by Montgomery (2005). The data consist of $m = 30$ observations, to be treated as individuals. A partial list of the data in that file is shown below:

Sample	X
1	9.45
2	7.99
3	9.29
4	11.66
5	12.16
6	10.18
7	8.04
8	11.46
9	9.20
10	10.34

The first 20 observations were randomly generated from a normal distribution with $\mu = 10$ and $\sigma = 1$. The last 10 observations were randomly generated from a normal distribution with $\mu = 11$ and $\sigma = 1$, representing a 1-sigma shift in the process mean. Montgomery uses this example to illustrate the properties of various types of time-weighted charts.

Data Input

There are two menu selections that create EWMA charts, one for individuals data and one for grouped data. In the case of grouped data, the original observations may be entered, or subgroup statistics may be entered instead.

Case #1: Individuals
The data to be analyzed consist of a single numeric column containing *n* observations. The data are assumed to have been taken one at a time.

- **Observations:** numeric column containing the data to be analyzed.

- **Date/Time/Labels:** optional labels for each observation.

- **LSL, Nominal, USL**: optional lower specification limit, nominal (target) value, and upper specification limit.

- **Select:** subset selection.

Case #2: Grouped Data – Original Observations
The data to be analyzed consist of one or more numeric columns. The data are assumed to have been taken in groups, in sequential order by rows.

- **Observations:** one or more numeric columns. If more than one column is entered, each row of the file is assumed to represent a subgroup with subgroup size *m* equal to the number of columns entered. If only one column is entered, then the *Date/Time/Labels or Size* field is used to form the groups.

- **Date/Time/Labels or Size**: If each set of *m* rows represents a group, enter the single value *m*. For example, entering a 5 as in the example above implies that the data in rows 1-5 form the first group, rows 6-10 form the second group, and so on. If the subgroup sizes are not equal, enter the name of an additional numeric or non-numeric column containing group identifiers. The program will scan this column and place sequential rows with identical codes into the same group.

- **LSL, Nominal, USL**: optional lower specification limit, nominal (target) value, and upper specification limit.

- **Select:** subset selection.

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Case #3: Grouped Data – Subgroup Statistics
In this case, the statistics for each subgroup have been computed elsewhere and entered into the
datasheet, as in the table below:

Sample	Means	Ranges	Sizes
1	10.11	4.17	5
2	9.844	3.42	5
3	10.098	2.44	5
4	9.932	2.32	5
5	10.924	2.96	5

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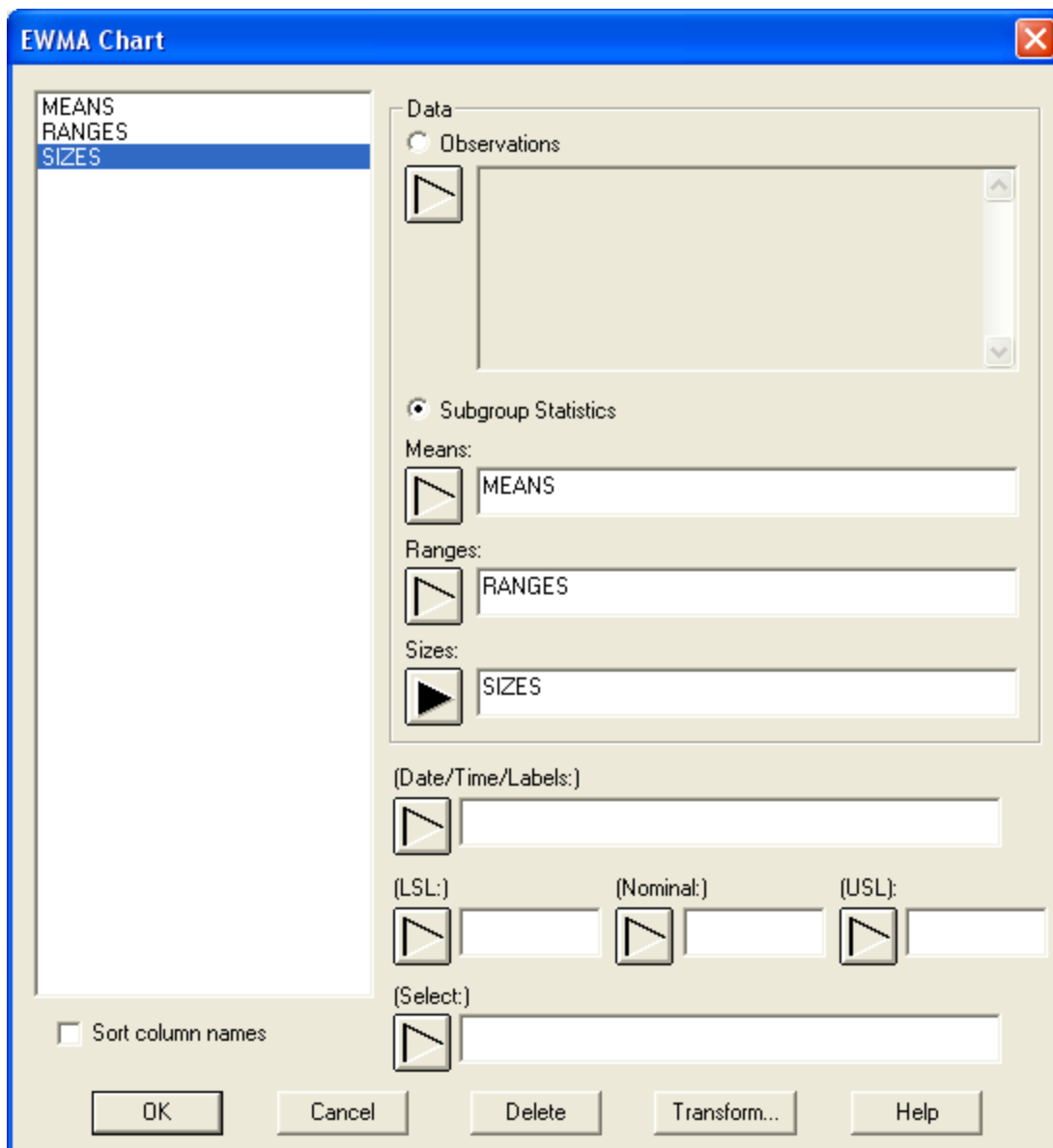
- **Subgroup Statistics**: the names of the column containing the subgroup means, subgroup ranges, and subgroup sizes.

- **Date/Time/Labels:** optional labels for each subgroup.

- **LSL, Nominal, USL**: optional lower specification limit, nominal (target) value, and upper specification limit.

- **Select:** subset selection.

EWMA Chart

This chart plots an exponentially weighted moving average of the current observation and all previous observations.

EWMA Chart for X
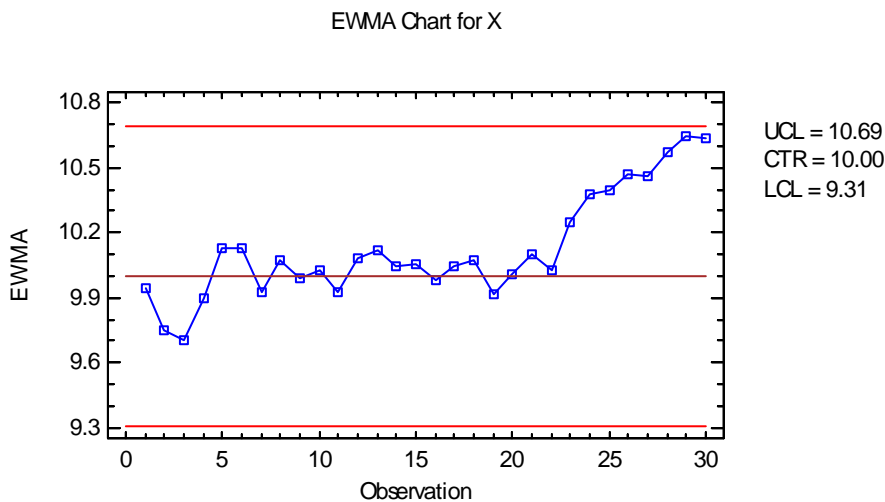
The exponentially weighted moving average at location *j* is defined by the recursive equation:

$$EWMA_j = \lambda x_j + (1-\lambda)EWMA_{j-1} \qquad (1)$$
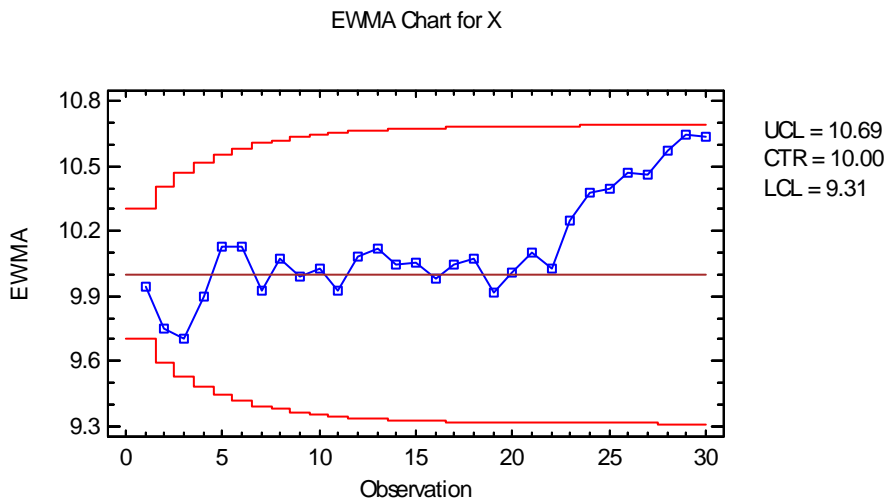
for individuals data or by

$$EWMA_j = \lambda \bar{x}_j + (1-\lambda)EWMA_{j-1} \qquad (2)$$

for data collected in subgroups, where $0 < \lambda < 1$ is called the *smoothing parameter* of the chart. To evaluate the EMWA at $j = 1$, an assumption must be made about the EWMA at time $j = 0$. The *Control Charts* tab of the *Preferences* dialog box, accessible from the *Edit* menu, provides 3 choices:
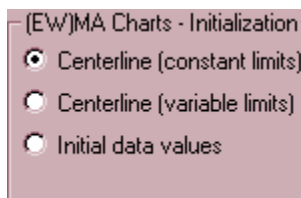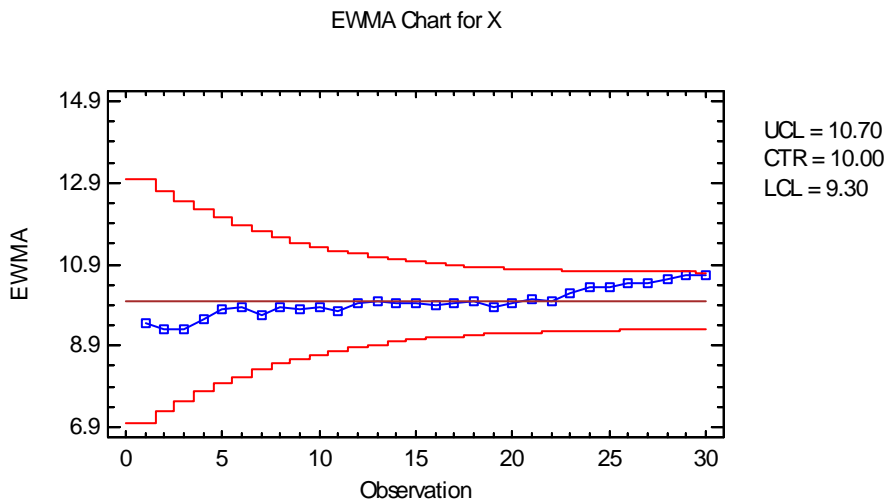
(1) **Centerline (constant limits):** $EWMA_0$ is set equal to the centerline. Although the initial values of the EWMA then vary less about the centerline than for large *j,* this decreased variability is ignored when plotting the control limits so that they are horizontal lines. This is the format of the chart shown above.

(2) **Centerline (variable limits):** $EWMA_0$ is set equal to the centerline. The control limits are drawn as step functions, reflecting the decreased variability of the EWMA for small *j*.

(3) **Initial data values:** $EWMA_0$ is set equal to x_1 or $\overline{x}_1$. The control limits are drawn as step functions, reflecting the increased variability of the EWMA for small *j*.

Example: Initializing at Centerline with Variable Limits

EWMA Chart for X

UCL = 10.69
CTR = 10.00
LCL = 9.31

Example: Initializing using First Data Value

EWMA Chart for X

UCL = 10.70
CTR = 10.00
LCL = 9.30

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In Phase 1 (*Initial Studies*) mode, the centerline and control limits are determined from the data. The centerline is located at the average of the subgroup means or observations:

$$\overline{\overline{x}} = \frac{\sum_{j=1}^{m} \overline{x}_j}{m} \tag{3}$$

The control limits are placed above and below the centerline at:

$$\overline{\overline{x}} \pm k \frac{\hat{\sigma}}{\sqrt{n}} \sqrt{\frac{\lambda}{2 - \lambda}} \tag{4}$$

where *k* is the sigma multiple specified on the *Control Charts* tab of the *Preferences* dialog box, *n* is the subgroup size, and $\hat{\sigma}$ is the estimate of the process sigma.

The method for estimating the process sigma also depends on the settings on the *Control Charts* tab of the *Preferences* dialog box, as discussed in the *Analysis Summary* section below.

Any points beyond the control limits will be flagged using a special point symbol. Any point excluded from the analysis, usually by clicking on a point on the chart and pressing the *Exclude/Include* button, will be indicated by an X. In the current chart, no out-of-control signals are indicated.

Pane Options
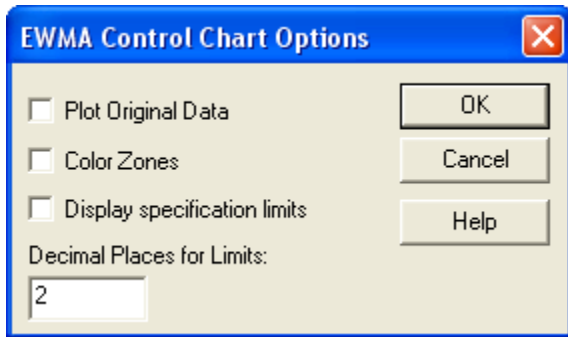
- **Plot Original Data**: check this box to plot the original observations or subgroups in addition to the moving averages.

- **Decimal Places for Limits:** the number of decimal places used to display the control limits.

- **Color Zones:** check this box to display green, yellow and red zones.

- **Display Specification Limits**: whether to add horizontal lines to the chart displaying the location of the specification limits (if any).
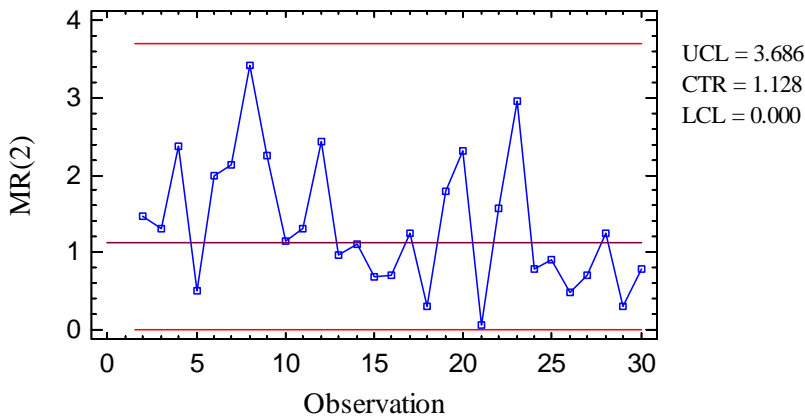
Example: Plotting Original Data

EWMA Chart for X

UCL = 10.69
CTR = 10.00
LCL = 9.31
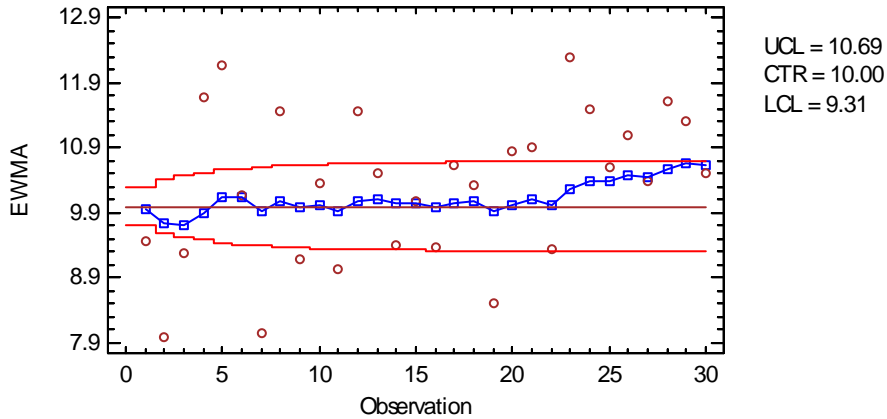
MR(2)/R/S Chart

A second chart is also included to monitor the process variability.

MR(2) Chart for X

UCL = 3.686
CTR = 1.128
LCL = 0.000

For individuals data, the chart displayed is an MR(2) chart, described in the *Individuals Control Charts* documentation. For grouped data, either an R chart or an S chart is plotted, depending on the setting on the *Control Charts* tab of the *Preferences* dialog box:

Preferred Dispersion Chart
⦿ Range
○ Sigma
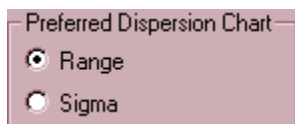
These charts are described in the *X-Bar and R Charts* and the *X-Bar and S Charts* documents.

EWMA Chart Report

This pane tabulates the values plotted on the control charts:

EWMA Individuals Chart Report		
Observations Beyond Limits		
X = Excluded * = Beyond Limits		
Observation	*EWMA*	*MR(2)*

Out-of-control points are indicated by an asterisk. Points excluded from the calculations are indicated by an X.

Pane Options
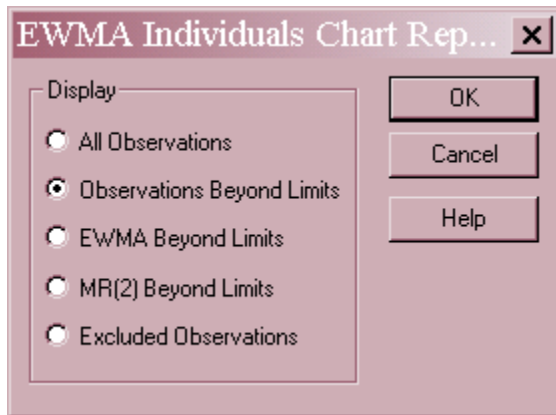
- **Display:** specify the observations or subgroups to display in the report.

Analysis Summary

The *Analysis Summary* summarizes the data and the control charts.

EWMA Individuals Chart - X
Number of observations = 30
0 observations excluded

Distribution: Normal
Transformation: none

EWMA Chart - Lambda = 0.1

Period	#1-30
UCL: +3.0 sigma	10.6876
Centerline	10.0
LCL: -3.0 sigma	9.31237

0 beyond limits

MR(2) Chart

Period	#1-30
UCL: +3.0 sigma	3.6855
Centerline	1.128
LCL: -3.0 sigma	0.0

0 beyond limits

Estimates

Period	#1-30	Standard
Process mean	10.315	10.0
Process sigma	1.19987	1.0
Mean MR(2)	1.35345	

Sigma estimated from average moving range

Included in the table are:

- **Subgroup Information**: the number of observations or subgroups *m* and the subgroup size *n* (if not individuals). If any observations or subgroups have been excluded from the calculations, that number is also displayed.

- **Distribution:** the assumed distribution for the data. By default, the data are assumed to follow a normal distribution. However, one of 26 other distributions may be selected using *Analysis Options*.

- **Transformation:** any transformation that has been applied to the data. Using *Analysis Options*, you may elect to transform the data using either a common transformation such as a square root or optimize the transformation using the Box-Cox method.

- **EWMA Chart:** a summary of the centerline and control limits for the EWMA chart.

- **MR(2)/R/S Chart:** a summary of the centerline and control limits for the dispersion chart.

- **Estimates:** estimates of the process mean μ and the process standard deviation σ. The methods for estimating the process sigma depending upon the settings on the *Control*

Charts tab of the *Preferences* dialog box, accessible through the *Edit* menu. The options are described in the *Individuals Control Charts* and *X-bar and R Charts* documentation.

- **Mean MR(2), Mean Range, or Mean S:** the average of the values plotted on the dispersion chart.

Analysis Options

- **Type of Study:** determines how the control limits are set. For an *Initial Study* (Phase 1) chart, the limits are estimated from the current data. For a *Control to Standard* (Phase 2) chart, the control limits are determined from the information in the *Control to Standard* section of the dialog box.

- **Lambda:** the smoothing parameter of the moving average λ where $0 < \lambda < 1$. The smaller lambda is, the more influence past data has on the plotted values. The value of λ and the sigma multiple k are often selected so as to achieve a desired sensitivity or average run length in the presence of a small process shift.

- **ARL:** the approximate average run length for an EWMA chart with the selected λ and k. If you enter new values, the ARL will be updated instantaneously. Note that the sigma multiple for the EWMA chart has been lowered to $k = 2.7$ rather than $k = 3.0$ in order to achieve the same false alarm ARL as a normal 3-sigma Shewhart chart.

- **Initialization**: method for initializing the EWMA. If *Centerline (constant limits)*, then *EWMA₀* is set equal to the centerline, but the decreased variability is not reflected in the control limits. If *Centerline (variable limits)*, then *EWMA₀* is set equal to the centerline and the control limits are tightened to reflect the decreased variability. If *Initial data values*, then *EWMA₀* is set equal to the value of the first observation or first subgroup mean and the control limits are loosened to reflect the increased variability.

- **EWMA Control Limits:** specify the multiple *k* to use in determining the upper and lower control limits on the EWMA chart. To suppress a limit completely, enter 0.

- **MR(2) Control Limits:** specify the multiple *k* to use in determining the upper and lower control limits on the MR(2), R, or S chart. To suppress a limit completely, enter 0.

- **Control to Standard:** to perform a Phase 2 analysis, select *Control to Standard* for the *Type of Study* and then enter the established standard process mean and sigma (or other parameters if not assuming a normal distribution).

- **Exclude button:** Use this button to exclude specific subgroups from the calculations.

- **Transform Button**: Use this button to specify a transformation or non-normal distribution.

For a discussion of the *Exclude* and *Transform* features, see the documentation for *Individuals Control Charts*.
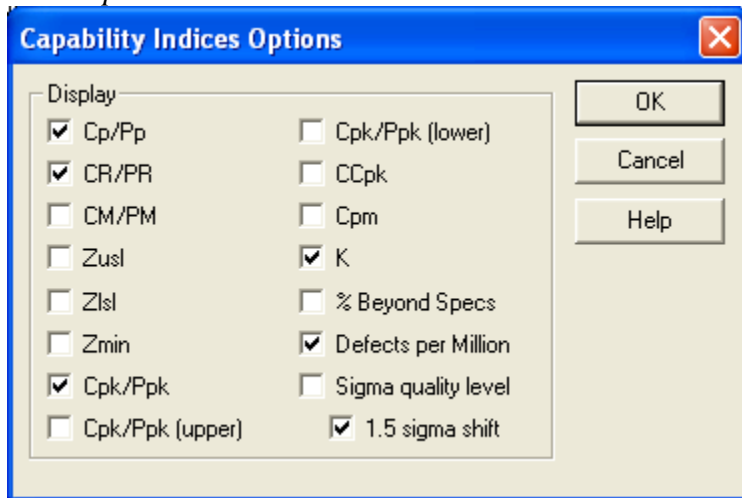
Capability Indices

The *Capability Indices* pane displays the values of selected indices that measure how well the data conform to the specification limits.

Capability Indices for X
Specifications
   USL = 15.0
   Nom = 10.0
   LSL = 5.0

	Short-Term Capability	Long-Term Performance
Sigma	1.19987	1.15354
Cp/Pp	1.38904	1.44483
CR/PR	71.9919	69.2122
Cpk/Ppk	1.38904	1.44483
K		0.0
DPM	30.8637	0.574209

Based on 6.0 sigma limits. Short-term sigma estimated from average moving range.

The indices displayed by default depend on the settings of the *Capability* tab on the *Preferences* dialog box. A detailed discussion of these indices may be found in the documentation for *Process Capability (Variables)*.
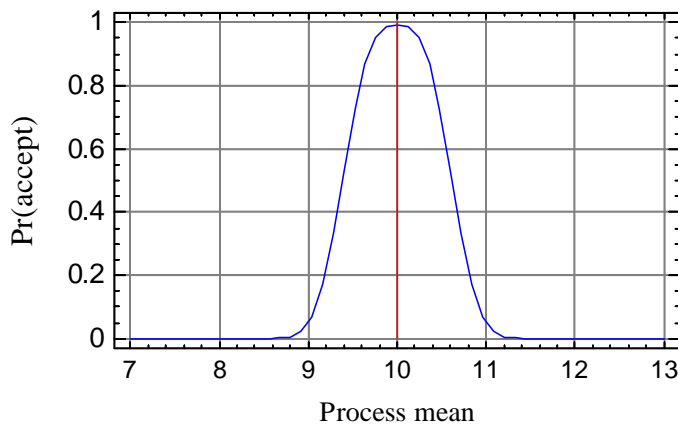
Pane Options

- **Indices**: select the indices to be displayed.

Note: Capability indices will only be calculated if specification limits are entered on the initial data input dialog box.

OC Curve

The *OC (Operating Characteristic) Curve* is designed to illustrate the properties of a Phase 2 control chart.


OC Curve for EWMA

The chart displays the probability that a plotted value will be within the control limits on the EWMA chart, as a function of the true process mean. For example, if the process mean were to shift to 11, the probability that the next EWMA will be within the control limits is approximately 5%.

ARL Curve

The *ARL Curve* is another way to view the performance of a Phase 2 chart.

ARL Curve for EWMA
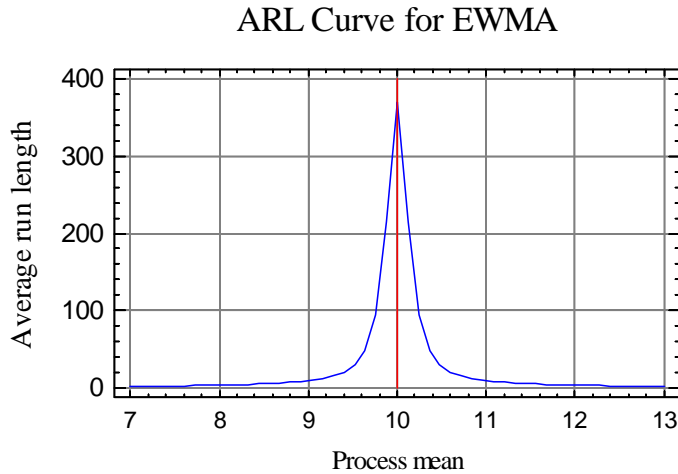
The ARL curve plots the average run length (average number of values plotted up to and including the first point beyond the control limits) as a function of the true process mean. Assuming that the process mean suddenly shifts to a new value, the chart shows how long it takes on average until an out-of-control signal is generated. For very small shifts, it can take in excess of 350 points on average to detect the shift. At a shift to $\mu = 11$, the ARL is approximately 10.

Save Results

The following results can be saved to the datasheet, depending on whether the data are individuals or grouped:

1. *EWMA* – the values plotted on the EWMA chart.
2. *Ranges, sigmas, or moving ranges* – the values plotted on the dispersion chart.
3. *Sizes* – the subgroup sizes.
4. *Labels* – the subgroup labels.
5. *Process Mean* – the estimated process mean.
6. *Process Sigma* – the estimated process standard deviation.
7. *Included Observations* – a column of 0's and 1's for excluded and included observations, respectively. This column can then be used in the *Select* field on other data input dialog boxes.

<u>Calculations</u>

ARL

Assuming a shift in the mean of magnitude $\Delta\sigma$:

$$ARL(\Delta) = 1 + (1/\lambda)\int_{-3}^{+3} ARL(x) f\left\{\left[x - (1-\lambda)\Delta\right]/\lambda\right\}dx \qquad (5)$$

The integral is evaluated using 32-point Gaussian quadrature following the method described by Crowder (1987a).