Exponential Smoothing Statlet

Summary

This Statlet applies various types of exponential smoothers to a time series. It generates forecasts with associated forecast limits. Using the Statlet controls, the user may interactively change the values of the smoothing parameters to examine their effect on the forecasts.

The types of exponential smoothers included are:

1. Brown's simple exponential smoothing with smoothing parameter α.
2. Brown's linear exponential smoothing with smoothing parameter α.
3. Holt's linear exponential smoothing with smoothing parameters α and β.
4. Brown's quadratic exponential smoothing with smoothing parameter α.
5. Winters' seasonal exponential smoothing with smoothing parameters α, β, and γ.

Sample StatFolio: *expsmoothing.sgp*

Sample Data

The data file *unemployment rate.sgd* contains monthly statistics on the national unemployment rate in the United States beginning in January, 2001. For each month, both the seasonally adjusted rate and the unadjusted rate are recorded. The first several rows of the file are shown below:

Month	Unemployment rate (unadj.)	Unemployment rate (adj.)
1/2001	4.7	4.2
2/2001	4.6	4.2
3/2001	4.5	4.3
4/2001	4.2	4.4
5/2001	4.1	4.3
6/2001	4.7	4.5
7/2001	4.7	4.6
8/2001	4.9	4.9
9/2001	4.7	5.0
10/2001	5	5.3

Data Input

The data to be analyzed are specified on the following data input dialog box:
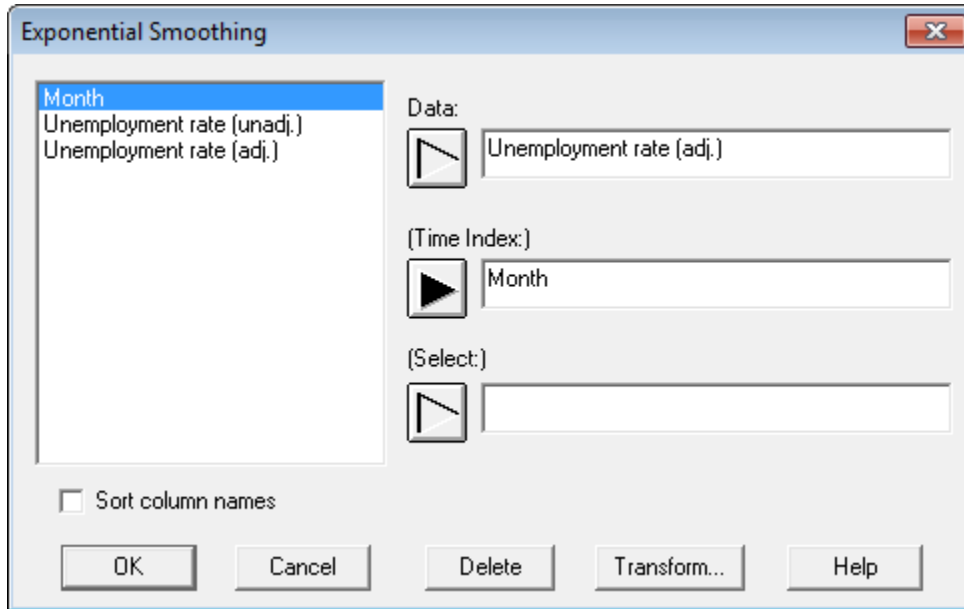
Each row of the datasheet is assumed to contain a single observation. Enter the names of the columns that contain:

- **Data:** numeric column containing the *n* time series observations, in time sequence order.

- **Time Index:** optional numeric column containing the time period corresponding to each observation. If not specified, time will be represented by the integers 1 through *n*.

- **Select:** subset selection.

Statlet

The output of this procedure is displayed in a dynamic Statlet window:
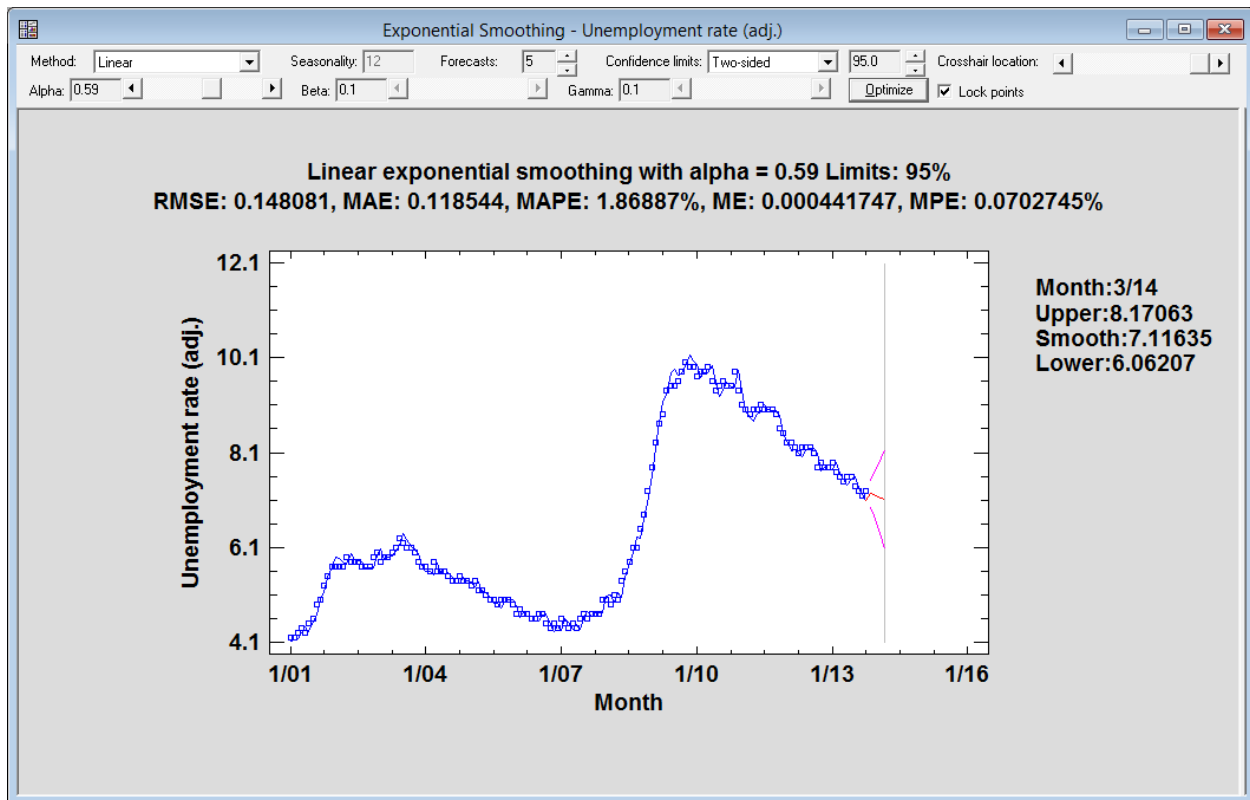
The toolbar contains a number of controls that allow the user to specify desired options:

- **Method:** the exponential smoothing method to be applied. *Simple* exponential smoothing results in a set of forecasts that are identical for all future periods. *Linear* exponential smoothing results in a set of forecasts that follow a line with constant slope. The forecast function for *quadratic* exponential smoothing is a second-order polynomial. *Seasonal* exponential smoothing generates forecasts with a seasonal pattern.

- **Seasonality**: for seasonal exponential smoothing only, the length of seasonality s. For example, monthly data has a seasonality equal to $s = 12$ months.

- **Alpha:** the first exponential smoothing parameter, which must be greater than 0 and less than 1. This parameter is used to estimate the level of the time series in all of the methods. For Brown's linear and quadratic smoothing, it is also used to estimate the slope and second-order term.

- **Beta:** the second exponential smoothing parameter, which must be greater than 0 and less than 1. This is used to estimate the slope of the forecast function when using Holt's or Winters' method.

- **Gamma:** the third exponential smoothing parameter, which must be greater than 0 and less than 1. This is used to estimate the seasonal effects when using Winters' method.

- **Forecasts:** the desired number of forecasts k.

- **Confidence limits:** the desired type of forecast limits and the percentage of confidence associated with them.

- **Optimize**: When this button is pressed, the program will automatically find the values of the smoothing parameters that minimize the root mean squared error (RMSE).

- **Crosshair cursor**: This scrollbar controls the location along the horizontal axis for which information is displayed in the right margin of the plot.

- **Lock points**: If checked, points cannot be changed. If not checked, you can drag points to new locations along the y-axis to see how that would impact the smoothed values and the forecasts.

Output

The output of the Statlet shows several important results:

1. The original data Y_t are shown as point symbols for $t = 1, 2, …, n$.

2. The smoothed values S_t are displayed as a solid line.

3. The forecasted values F_t for $t = n+1, n+2, …, n+k$ are displayed as a solid line extending from the last smoothed value.

4. The prediction limits are shown as solid lines on either side of the forecasts.

5. The values of several statistics that characterize how well the smoothed values match the original data are displayed at the top of the plot. Let $F_t(k)$ be the forecast for time $t+k$ that would have been made by smoothing the data up to and including the value at time t. The *one-ahead forecast error e_t* may then be calculated from

$$e_t = Y_t - F_{t-1}(1) \tag{1}$$

The program calculates the following statistics for these errors:

RMSE = root mean squared error

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} e_t^2}{n}} \tag{2}$$

MAPE = mean absolute percentage error

$$MAPE = 100\frac{\sum\limits_{t=1}^{n}\left|e_t / Y_t\right|}{n}\%$$ (3)

MAE = mean absolute error

$$MAE = \frac{\sum\limits_{t=1}^{n}\left|e_t\right|}{n}$$ (4)

ME = mean error

$$ME = \frac{\sum\limits_{t=1}^{n}e_t}{n}$$ (5)

MPE = mean percentage error

$$MPE = 100\frac{\sum\limits_{t=1}^{n}\dfrac{e_t}{Y_t}}{n}\%$$ (6)

It is usually desirable that the RMSE, MAPE, and MAE be as small as possible. Likewise, it is desirable that the ME and MPE be close to 0.

Calculations

Brown's Exponential Smoothing

Brown's simple, linear and quadratic exponential smoothing methods estimate forecast functions that take the form of a simple mean, a linear trend, and a quadratic trend, respectively. They do so by weighting recent observations more heavily than observations that are further in the past.

To generate the forecasts, up to three passes of an exponential smoother are made:

$$S_t' = \alpha Y_t + (1-\alpha)S_{t-1}' \tag{7}$$

$$S_t'' = \alpha S_t' + (1-\alpha)S_{t-1}'' \tag{8}$$

$$S_t''' = S_t'' + (1-\alpha)S_{t-1}''' \tag{9}$$

The initial values at time $t = 0$ are determined by backforecasting, which first smoothes the time series backwards and then uses the backforecasts to initialize the forward smoothing. The forecasts are then generated from:

Simple smoothing: $F_t(k) = S_t'$ (10)

Linear smoothing: $F_t(k) = 2S_t' - S_t'' + k\dfrac{\alpha}{1-\alpha}\left(S_t' - S_t''\right)$ (11)

Quadratic smoothing:

$$F_t(k) = \begin{aligned} &3S_t' - 3S_t'' + S_t''' + k\frac{\alpha}{2(1-\alpha)^2}\left((6-5\alpha)S_t' - (10-8\alpha)S_t'' + (4-3\alpha)S_t'''\right) \\ &+ k^2\frac{\alpha^2}{2(1-\alpha)^2}\left(S_t' - 2S_t'' + S_t'''\right) \end{aligned} \tag{12}$$

Holt's Linear Exponential Smoothing

Holt's linear exponential smoothing is similar to Brown's linear exponential smoothing in that it generates forecasts that follow a linear trend. However, Holt's procedure uses two smoothing constants, α and β, one to estimate the level of the series at time *t* and a second to estimate the slope. The procedure is as follows:

1. Smooth the data to estimate the level using

$$S_t = \alpha Y_t + (1-\alpha)\left(S_{t-1} + T_{t-1}\right) \tag{13}$$

2. Smooth the first smooth to estimate the slope using

$$T_t = \beta(S_t - S_{t-1}) + (1 - \beta)T_{t-1} \qquad (14)$$

3. Calculate the forecasts using

$$F_t(k) = S_t + kT_t \qquad (15)$$

Winters' Exponential Smoothing

Winters' exponential smoothing method extends Holt's procedure by adding an additional parameter γ to estimate seasonal effects using a third smoother. The procedure is as follows:

1. Estimate the seasonality by smoothing the ratio of the data to the estimated level at time t using:

$$I_t = \gamma \frac{Y_t}{S_t} + (1 - \gamma)I_{t-s} \qquad (16)$$

where s is the length of seasonality.

2. Estimate the level of the series by smoothing the data divided by the estimated seasonality using

$$S_t = \alpha \frac{Y_t}{I_{t-s}} + (1 - \alpha)(S_{t-1} + T_{t-1}) \qquad (17)$$

3. Estimate the slope of the series using

$$T_t = \beta(S_t - S_{t-1}) + (1 - \beta)T_{t-1} \qquad (18)$$

4. Calculate the forecasts using

$$F_t(k) = (S_t + kT_t)I_{t-s+k} \qquad (19)$$

Forecast limits

Probability limits for the forecasts are plotted at the 100(1-α)% confidence level, calculated assuming that the noise in the system follows a normal distribution. The limits are given by

$$F_n(k) \pm z_{\alpha/2} \sqrt{\hat{V}(k)} \qquad (20)$$

where $\hat{V}(k)$ equals the estimated variance of the forecast k periods past the end of the data. The variance function depends on the method being used:

Simple Exponential Smoothing

The variance function is derived from the equivalence of simple exponential smoothing to an ARIMA(0,1,1) model. The variance k periods ahead is:

$$\hat{V}(k) = \hat{\sigma}_a\left(1 + (k-1)\alpha^2\right) \tag{21}$$

Brown's Linear Exponential Smoothing

The variance function is derived from the equivalence to an ARIMA(0,2,2) model.

Brown's Quadratic Exponential Smoothing

The variance function is derived from the equivalence to an ARIMA(0,3,3) model.

Holt's Linear Exponential Smoothing

The variance function is derived from the equivalence to an ARIMA(0,2,2) model.

Winter's Exponential Smoothing

The variance function is derived from the equivalence to an ARIMA(0,1,s+1)(0,1,0)s model.