Analytic Method (Gage Studies – Attributes)

Summary

The *Analytic Method* estimates the repeatability and bias in a measurement system where the observations consist of an attribute rather than a variable. In this type of study, *n* items are evaluated *m* times, and the total number of acceptances *a* for each item is recorded.

Unlike the *Risk Analysis Method*, this method requires that there be a quantitative variable whose value is known for each item in the study, even though the routine appraisals of the item are based on an attribute rather than measurements of that variable. The item with the smallest reference value must be rejected every time, while the item with the largest reference value must be accepted every time. At least 6 other items must be obtained between the smallest and the largest and have some acceptances and some rejections.

Sample StatFolio: *gageanalytic.sgp*

Sample Data:

The file *gageanalytic.sgd* contains data from a typical attributes gage study, taken from the third edition of the Automotive Industry Action Group's (AIAG) reference manual on <u>Measurement Systems Analysis, MSA</u> (2002). The data in that file are shown below:

Xt	a	m
-0.016	0	20
-0.015	1	20
-0.014	3	20
-0.013	5	20
-0.012	8	20
-0.011	16	20
-0.0105	18	20
-0.01	20	20
-0.008	20	20

The file contains a total of n = 9 rows, one for each part. The first column shows the reference value for the part, the second column shows the number of acceptances, and the third column shows the number of trials.

Note: Data reprinted from the Measurement Systems Analysis (MSA) Manual with permission of DaimlerChrysler, Ford and GM Supplier Quality Requirements Task Force.

Data Input

The data input dialog box displayed for this procedure is shown below.

- **Reference Values:** the value of the quantitative reference value for each item, covering a range wide enough that at least one part is rejected on all trials and at least one part accepted on all trials.

- **Number of Acceptances:** number of acceptances a for each part.

- **Number of Trials**: number of trials m for each part. If $m = 20$ for all items, then an adjusted repeatability value will be calculated and a t-test will be performed to test the statistical significance of the estimated bias.

- **Upper Specification Limit:** the upper specification limit for the reference variable.

- **Lower Specification Limit:** the lower specification limit for the reference variable.

- **Study Header:** optional header to be printed at the top of each output page.

- **Select:** subset selection.

Probability Plot

The first step in the analysis of the study data is to calculate the probability of acceptance corresponding to each part and to plot those probabilities on a normal probability plot.
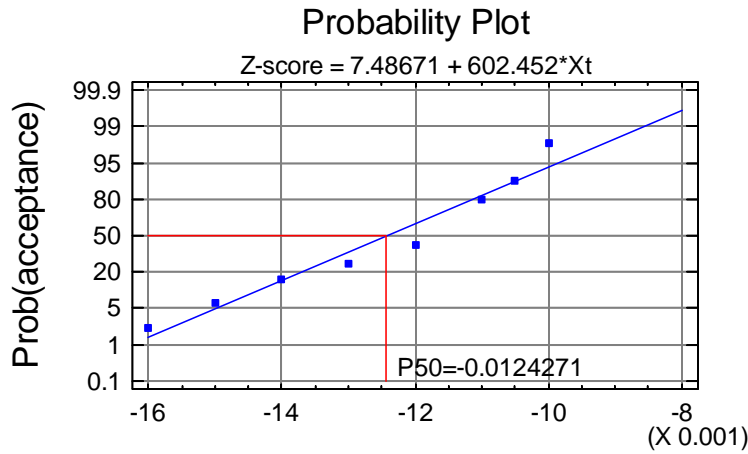
If the measurement errors follow a normal distribution, then the calculated probabilities should fall along a straight line. A regression line is fit to the probabilities, the equation of which is shown along the top of the plot. This equation is then used to calculate percentiles, such as the P_{50} value, which is the value of *Xt* at which the probability of acceptance equals 50%.

Analysis Summary

The *Analysis Summary* tabulates the results illustrated above. The upper section of that output is shown below:

Gage Study for Attributes - Analytical Method - Xt
AIAG Example p. 137

Reference values: Xt
Number of acceptances: a
Number of trials: m

Upper spec. limit: 0.01
Lower spec. limit: -0.01

Regression model
Innermost reference value with no rejections: -0.01
Innermost reference value with no acceptances: -0.016
Z-score = 7.48671 + 602.452*Xt
R-squared = 95.508%

Bias Analysis
P(acceptance) = 0.5 at -0.0124271
Comparison spec. limit = -0.01
Estimated bias = 0.00242706

There two important sections:

- **Regression Model**

This section shows the innermost reference values for which $a = 0$ and $a = m$. Also displayed is the equation of the fitted line from the normal probability plot, together with an R-Squared statistic. R-squared values in excess of 90% imply a very good fit.

- **Bias Analysis**

This section shows the estimated P_{50} value from the fitted model, together with an indication of the specification limit around which the study has been performed. Note: the study may be structured such that the reference values spans either the lower specification limit or the upper specification limit. Also displayed is the estimated bias. Assuming that the lower specification limit has been spanned, the bias is estimated from:

$$bias = lower\ specification\ limit - P_{50} \qquad (1)$$

In this case, there is a slight positive bias, since the probability of acceptance equals 50% at a reference value slightly below the lower spec limit. If the upper specification limit had been spanned, the bias would be estimated from

$$bias = upper\ specification\ limit - P_{50} \qquad (2)$$

The lower section of the *Analysis Summary* displays the following:

Repeatability Analysis
P(acceptance) = 0.005 at -0.0167026
P(acceptance) = 0.995 at -0.00815147
Estimated repeatability (unadjusted) = 0.00855116
Estimated repeatability (adjusted) = 0.00791774
AIAG t-statistic = 9.59451
P-value = 1.0209E-8

- **Repeatability Analysis**

This section estimates the repeatability of the measurement process by first computing $P_{99.5}$ and $P_{0.5}$, the estimated reference values for which the probability of acceptance equals 99.5% and 0.5%, respectively. Repeatability is then estimated from:

$$repeatability = P_{99.5} - P_{0.5} \qquad (3)$$

The calculated value is an estimate of the range over which the probability of acceptance changes by 99%.

If $m = 20$ for all items, the adjusted repeatability is also calculated according to as recommended in the AIAG manual (2002):

$$adjusted\ repeatability = \frac{P_{99.5} - P_{0.5}}{1.08} \qquad (4)$$

In such cases, a t-test for the measurement bias is also performed and its *P-Value* displayed. Small P-Values (less than 0.05 at the 5% significance level) indicate that the bias is significantly different from zero.

Gage Performance Plot

This plot summarizes the results of the analysis.

Gage Performance Plot
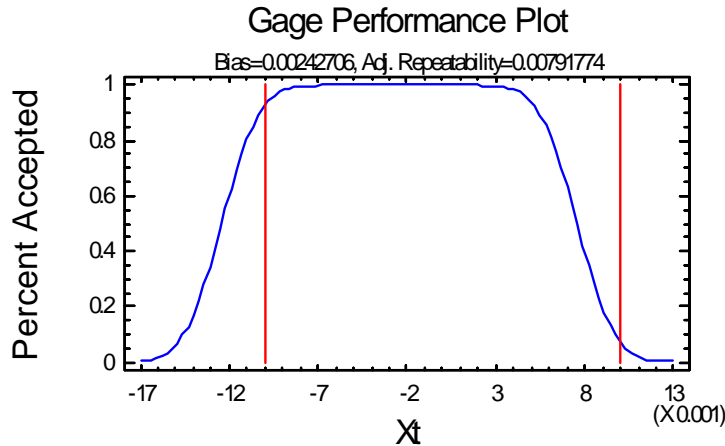
Important features of the plot include:

- **Solid blue line**: estimated probability of acceptance as a function of the reference value for the item. This is based upon the fitted regression equation and the assumption of a normal distribution. In drawing this curve, it is assumed that the distribution of measurement errors is the same around both specification limits, except that the mean is shifted by the amount of the estimated bias.

- **Vertical lines**: drawn at the upper and lower specification limits.

- The estimated bias and the repeatability (or adjusted repeatability if available).

In the example, you will note that there is a fairly high probability of accepting an item whose true value is less than the lower specification limit, although the same is not true above the upper limit. If the lower specification limit is critical, then some improvement is necessary.

Calculations

- **Probability of Acceptance**

    $P'_a = 0$ if $a = 0$ except for the largest reference among such cases.   (5)

    $P'_a = \dfrac{1}{2m}$ for the largest reference with $a = 0$.   (6)

$P'_a = 1$ if $a = m$ except for the smallest reference among such cases. (7)

$$P'_a = 1 - \frac{1}{2m} \text{ for the smallest reference with } a = m. \tag{8}$$

$$P'_a = \frac{a + 0.5}{m} \text{ if } \frac{a}{m} < 0.5 \text{ and } a \neq 0 \tag{9}$$

$$P'_a = \frac{a - 0.5}{m} \text{ if } \frac{a}{m} > 0.5 \text{ and } a \neq m \tag{10}$$

$$P'_a = 0.5 \text{ if } \frac{a}{m} = 0.5 \tag{11}$$

- **P_{50}, $P_{99.5}$, and $P_{0.5}$**

  Calculated from the fitted regression line by determining the appropriate Z-value and solving for *Xt*.

- **Bias t-Test**

$$t = \frac{31.3|bias|}{repeatability} \tag{12}$$

  A two-sided P-value is calculated based on Student's t distribution with 19 degrees of freedom.