

Range Method (Gage Studies – Variables)



Revised: 10/10/2017



Summary	1
Data Input.....	3
Operator and Part Plot.....	3
Range Chart by Part	4
Analysis Summary	5
Analysis Options	7
Box and Whisker Plot	7
Calculations.....	9

Summary

The *Range Method* calculates an approximate estimate of the combined repeatability and reproducibility of a measurement system. It is based on a study in which 2 appraisers measure n items once each. The analysis can provide a quick estimate of the percentage of the process sigma attributable to the measurement process if an external estimate of sigma is available, or an estimate of the precision-to-tolerance ratio if specifications are available for the variable being measured.

Sample StatFolio: *gageshort.sgp*

Sample Data

The file *gageshort.sgd* contains data from a typical short study, taken from the third edition of the Automotive Industry Action Group's (AIAG) reference manual on Measurement Systems Analysis, MSA (2002). The data in that file is shown below:

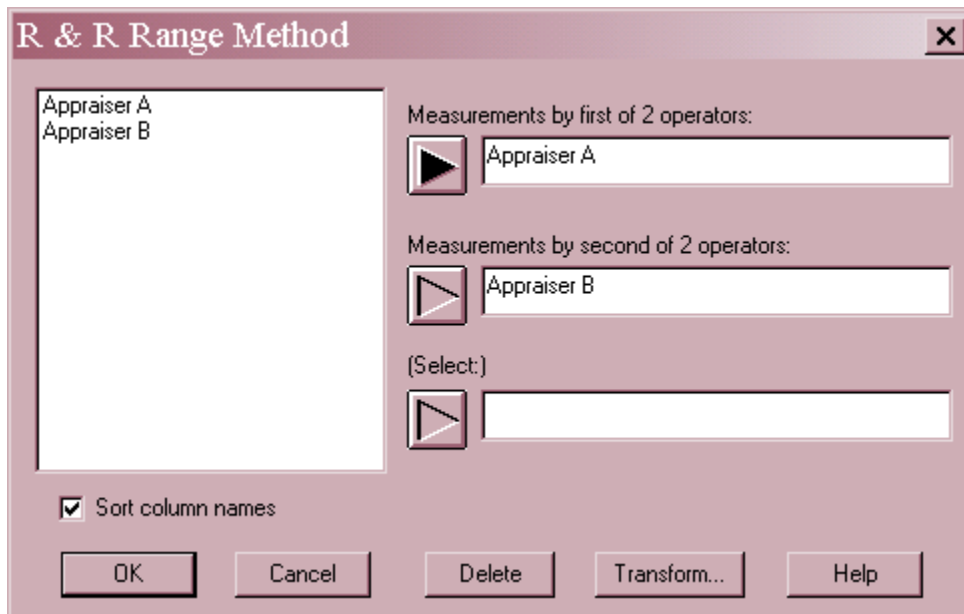
<i>Appraiser A</i>	<i>Appraiser B</i>
0.85	0.8
0.75	0.7
1	0.95
0.45	0.55
0.5	0.6

The file contains a total of $n = 5$ rows, one for each part that was measured.

Note: Data reprinted from the Measurement Systems Analysis (MSA) Manual with permission of DaimlerChrysler, Ford and GM Supplier Quality Requirements Task Force.

Data Input

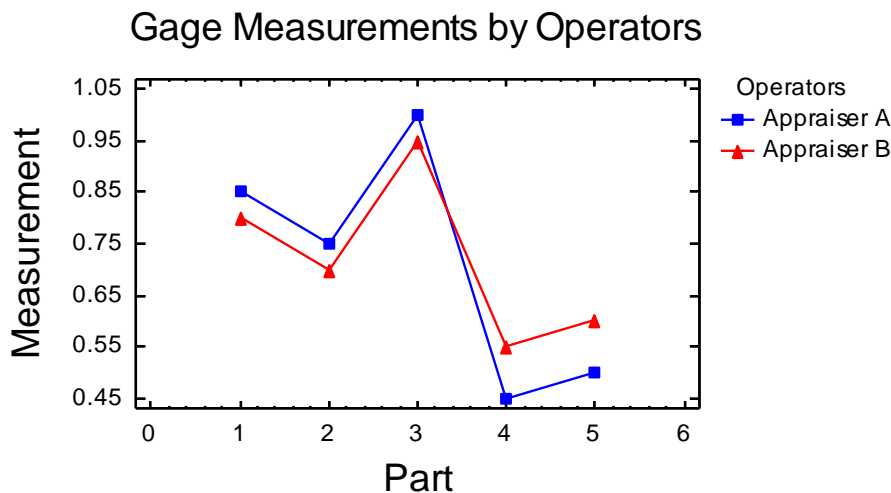
The data input dialog box requests the names of the two columns, one for each appraiser.



- **Measurements by first of 2 operators:** numeric column containing the measurements made by the first appraiser.
- **Measurements by second of 2 operators:** numeric column containing the measurements made by the second appraiser.
- **Select:** subset selection.

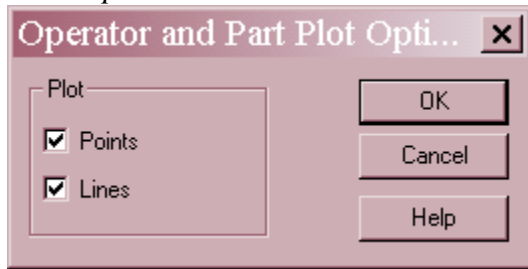
Operator and Part Plot

The *Operator and Part* plot displays the measurements made by each operator on each part:



This plot is useful for showing any consistent differences between the appraisers.

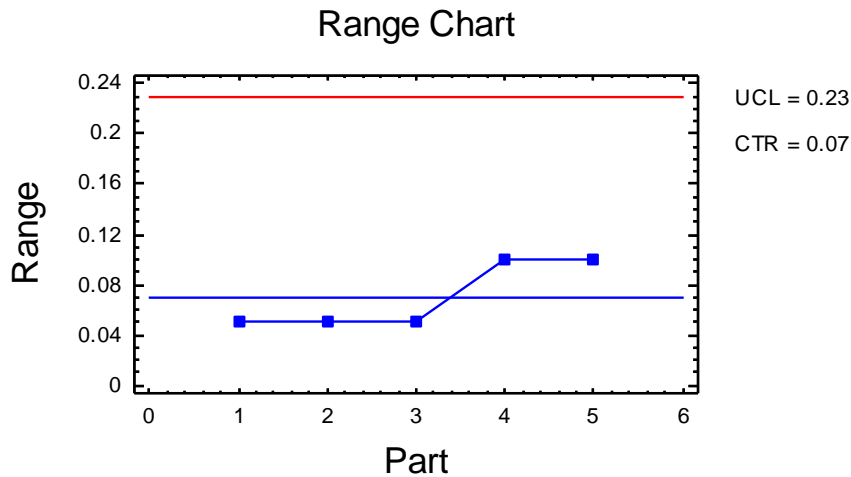
Pane Options



- **Points:** plot point symbols.
- **Lines:** connect the points with a line.

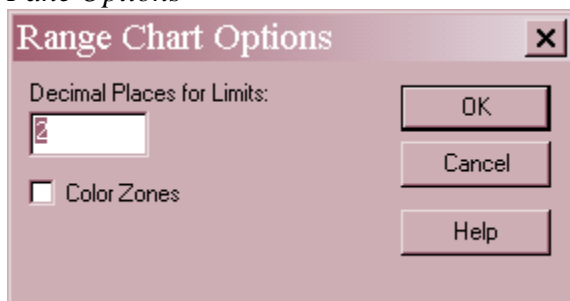
Range Chart by Part

This chart plots the range of measurements R_i made on each part.



Included on the chart is a 3-sigma upper control limit. Any ranges beyond this limit indicate an unusual difference between the two appraisers on a particular part.

Pane Options



- **Decimal Places for Limits:** number of decimal places to be displayed for the control limits.
- **Color Zones:** check this box to display green, yellow and red zones.

Analysis Summary

The *Analysis Summary* displays an estimate of the variability due to the measurement process. The estimate is of the combined repeatability and reproducibility. Since the same appraiser did not measure the same part more than once, repeatability and reproducibility can not be separated.

Gage R&R - Range Method - Appraiser A & Appraiser B

Measurements by first of 2 operators: Appraiser A
Measurements by second of 2 operators: Appraiser B

Number of parts: 5
Mean range = 0.07

Measurement Variation

	<i>Est. Std. Dev.</i>	<i>Est. Variance</i>	<i>Process Sigma</i>	<i>%GRR</i>
R&R	0.0588672	0.00346535	0.0777	75.76

This summary displays:

- **Mean range** – the average range of the measurements made on the n parts.
- **Est. Std. Dev.** – the estimated standard deviation of the measurement process, $\hat{\sigma}_{R\&R}$.
- **Est. Variance** – the estimated variance of the measurement process, $\hat{\sigma}_{R\&R}^2$.
- **Process Sigma** – if a value for the process sigma $\sigma_{process}$ is entered on the *Analysis Options* dialog box, that value is displayed here.
- **%GRR** – the estimated percentage of the process sigma due to gage repeatability and reproducibility, calculated according to:

$$\%GRR = 100 \frac{\hat{\sigma}_{R\&R}}{\sigma_{process}} \% \quad (1)$$

Alternative Analysis Based on Tolerance

Rather than entering a value for the process sigma on the *Analysis Options* dialog box, you can enter a tolerance for the variable instead, where tolerance is defined as the difference between the upper specification limit (USL) and the lower specification limit (LSL):

$$Tolerance = USL - LSL \quad (2)$$

The *Analysis Summary* table is then slightly different.

Gage R&R - Range Method - Appraiser A & Appraiser B

Measurements by first of 2 operators: Appraiser A
 Measurements by second of 2 operators: Appraiser B

Number of parts: 5
 Mean range = 0.07

Measurement Variation

	<i>Est. Smd. Dev.</i>	<i>Est. Variance</i>	<i>Tolerance</i>	<i>%GRR</i>
R&R	0.0588672	0.00346535	0.5	70.64

Based on 6 sigma limits.

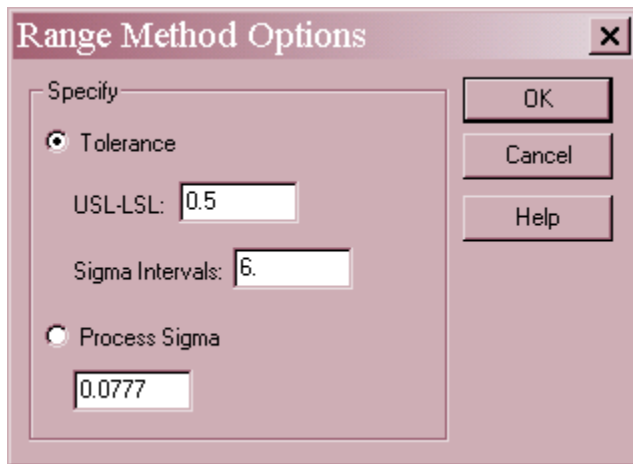
The last 2 columns of this table are:

- **Tolerance** – the specified tolerance for the variable.
- **%GRR** – the precision to tolerance ratio defined by

$$P/T = 100 \frac{K \hat{\sigma}_{R\&R}}{tolerance} \% \tag{3}$$

The multiple K is usually set to either 6.0 (for 99.73% coverage) or 5.15 (for 99% coverage). Basically, P/T is a measure of how wide the measurement error distribution is compared to the specifications for the item being measured. Values of P/T less than 10% usually imply an acceptably small measurement error, although P/T may be as high as 30% in some cases and still be acceptable.

Analysis Options

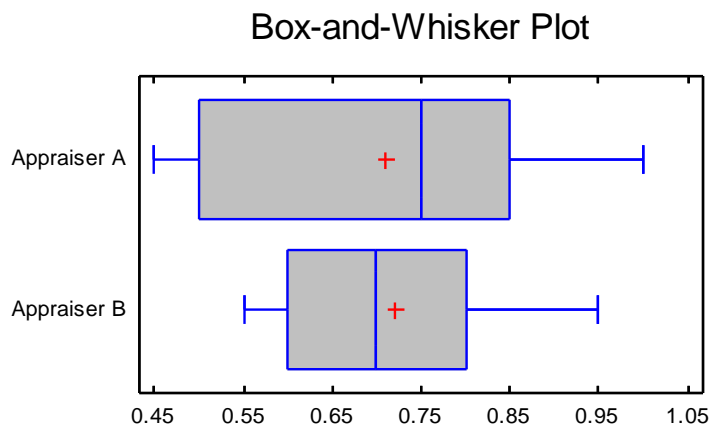


You may select to estimate %GRR in 2 different ways:

- **Tolerance:** by specifying the sigma multiple K used to compare the spread of the measurement error relative to the distance between the specifications. The default value of K is determined by the settings on the *Gage Studies* tab of the *Preferences* selection on the *Edit* menu.
- **Process Sigma:** by entering a value for the process sigma obtained by some previous study.

Box and Whisker Plot

The box-and-whisker plot provides an additional comparison between the appraisers.



For each appraiser, a box-and-whisker plot is drawn as follows:

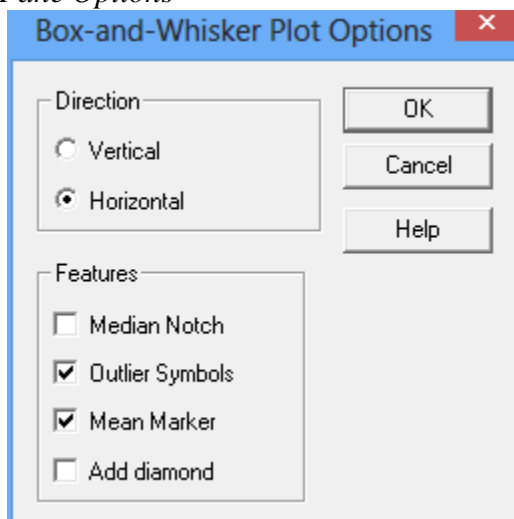
- The rectangular *box* covers the central 50% of an appraiser's measurements, ranging between the lower quartile and the upper quartile.

- A vertical line is drawn within the box at the *median* for that appraiser.
- A plus sign is drawn to indicate the *mean* measurement \bar{x}_i of each appraiser.
- *Whiskers* are drawn from each end of the box to the minimum and maximum value for each operator, unless outside points are detected, in which case the whiskers are drawn to the most extreme data values that are not outside points.
- Any *outside points* are indicated using point symbols such as a small square, or a square with a plus sign through it if the points are *far outside*.

For more details on outside points and other features of box-and-whisker plots, refer to the documentation for the standalone *Box-and-Whisker Plot* procedure.

In the above plot, no single measurements appear to be outliers since there are not any outside points.

Pane Options



- **Direction:** the orientation of the plot, corresponding to the direction of the whiskers.
- **Median Notch:** if selected, a notch will be added to the plot showing an approximate $100(1-\alpha)\%$ confidence interval for the median at the default system confidence level (set on the *General* tab of the *Preferences* dialog box on the *Edit* menu).
- **Outlier Symbols:** if selected, indicates the location of outside points.
- **Mean Marker:** if selected, shows the location of the sample mean as well as the median.
- **Add diamond:** if selected, a diamond will be added to the plot showing a $100(1-\alpha)\%$ confidence interval for the mean at the default system confidence level.

Calculations

- Average range

$$\bar{R} = \frac{\sum_{i=1}^m R_i}{m} \quad (4)$$

- Measurement Error (Repeatability and Reproducibility)

$$\hat{\sigma}_{R\&R} = \frac{\bar{R}}{d_2^*(2, n)} \quad (5)$$

where

$$d_2^*(s, t) = \left(1 + \frac{1}{4v}\right) d_2(s) \quad (6)$$

and

$$v = \frac{1}{-2 + 2\sqrt{1 + \frac{2[d_3(s)/d_2(s)]^2}{t}}} \quad (7)$$

Values for $d_2(s)$ and $d_3(s)$ are tabulated in textbooks such as Introduction to Statistical Quality Control, sixth edition, by Douglas Montgomery (Wiley, 2009).