

How To: Perform an Optimization Experiment Using STATGRAPHICS Centurion

by

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Introduction

STATGRAPHICS Centurion contains an extensive set of procedures for creation and analysis of designed experiments. The intent of this guide is to explore a typical use of those procedures, beginning with the creation of a screening experiment to determine the most important factors affecting a process and ending with the determination of optimal operating conditions. Since most experimentation is sequential in nature, the narrative is structured in a step-by-step format.

Scenario

The scenario we will consider is that of an injection molding process. Following the example in Statistics for Experimenters, 2nd edition by Box, Hunter and Hunter (Wiley, 2005), we consider the case of researchers interested in studying the effects of various controllable factors on several response variables, including *shrinkage* and *warpage*. After much discussion, they narrowed the list of experimental factors to the following eight:

- Factor A: *mold temperature*
- Factor B: *moisture content*
- Factor C: *holding pressure*
- Factor D: *cavity thickness*
- Factor E: *booster pressure*
- Factor F: *cycle time*
- Factor G: *gate size*
- Factor H: *screw speed*

Their eventual goal was to find levels of each factor that would minimize both shrinkage and warpage.

Step 1: Design a Screening Experiment

It has long been recognized that optimization of most processes requires a multivariate approach. Except in situations where the experimental factors act in a purely additive manner and never interact, one cannot expect to find the optimal operating conditions by optimizing the process with respect to only one factor at a time. On the other hand, attempting to optimize more than 3 or 4 factors at one time can be both expensive and confusing. In most cases, a handful of factors will be responsible for most of the variation seen in a product. Identifying that set of factors from a large list of possible candidates is usually a necessary prerequisite to the actual optimization.

Procedure: Screening Design Selection

The experimental design section of STATGRAPHICS Centurion contains a procedure designed to help create a screening experiment. To use this procedure:

- If using the Classic menu, select: *DOE – Design Creation – Screening Design Selection*.
- If using the Six Sigma menu, select: *Improve – Experimental Design Creation – Screening Design Selection*.

The procedure begins by displaying the dialog box shown below:

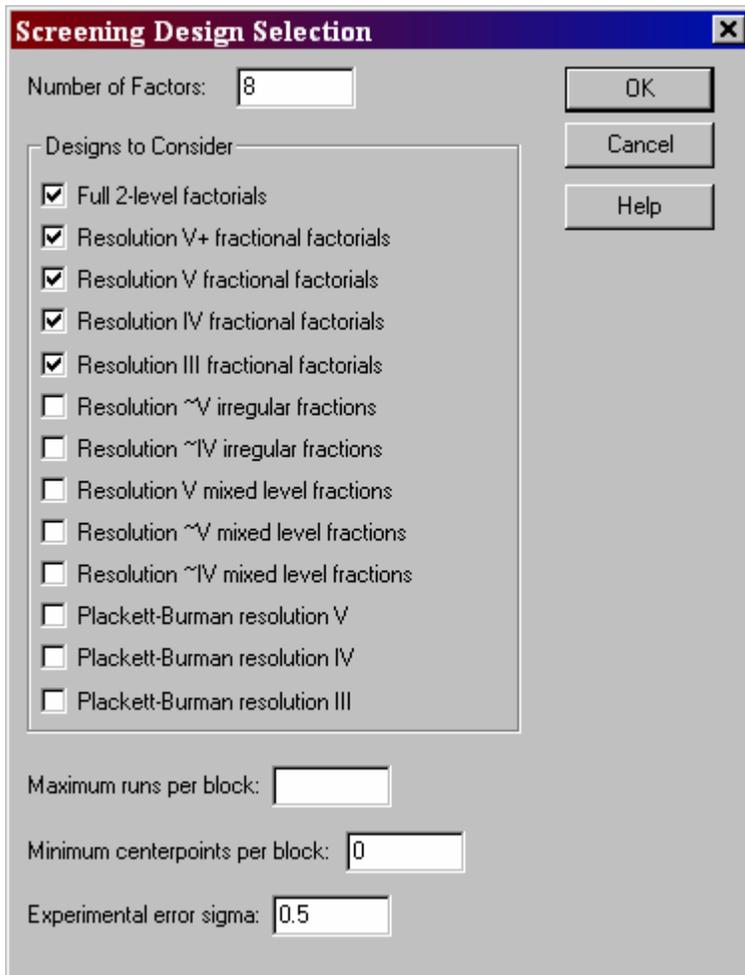


Figure 1: First Screening Design Selection Dialog Box

On this dialog box, you indicate the type of designs you wish to consider and other important information. In this case, we have requested that the program look at only pure two-level factorials and fractional factorial designs, since we expect that we may need to augment the design after it is run. These designs consist of a subset of runs at two levels of each experimental factor, with the total number of runs being a power of 2.

A very important entry on the dialog box is the *Experimental error sigma*. This refers to the repeatability of any experimental run. It encompasses the noise in setting up the run, variations in external factors, measurement error, and everything else that causes the results to differ when we try to repeat the same experiment more than once under similar conditions. From other experiments, it is supposed that we believe that σ for shrinkage (considered to be the more

important response variable of the two) will be approximately equal to 0.5%. We could also add restrictions about block size or minimum centerpoints if we wanted, but have elected not to.

The procedure then displays a second dialog box:

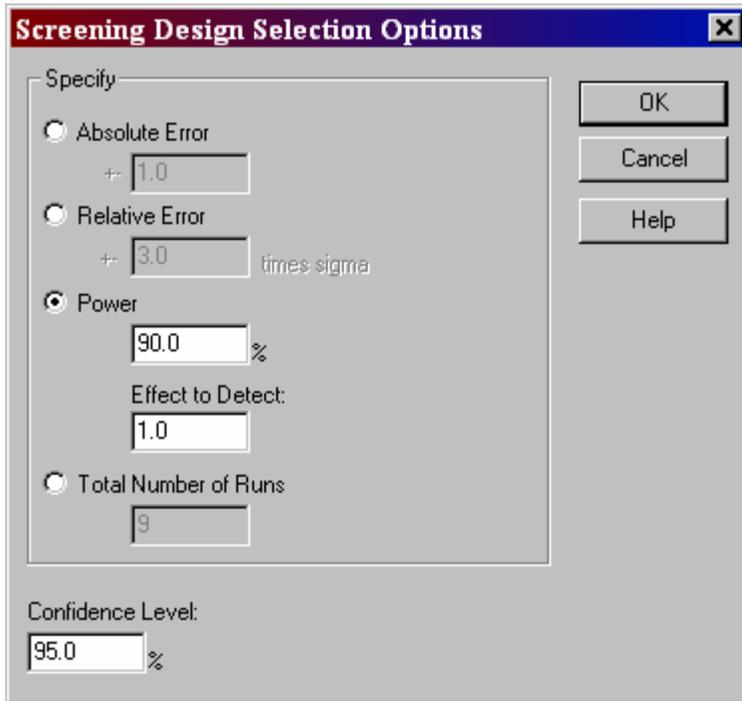


Figure 2: Second Screening Design Selection Dialog Box

On this dialog box, we indicate the precision that we desire for this experiment. By selecting the *Power* option and specifying the *Effect to Detect*, we have requested designs that have a 90% chance of detecting (declaring as statistically significant) any factors or interactions that make a difference of 1.0% or more in shrinkage.

The procedure then opens an analysis window and shows us the smallest designs of each type that meet the desired criteria:

<u>Screening Design Selection</u>									
Input									
<i>Number of</i>	<i>Min. Centerpoints</i>	<i>Max. Runs</i>	<i>Exp. Error</i>	<i>Desired</i>	<i>Target</i>	<i>Confidence</i>			
<i>Factors</i>	<i>per Block</i>	<i>per Block</i>	<i>Sigma</i>	<i>Power</i>	<i>Effect</i>	<i>Level</i>			
8	0		0.5	90.0%	1.0	95.0%			
Selected Designs									
<i>Design</i>	<i>Runs</i>	<i>Resol.</i>	<i>Corner</i>	<i>Center</i>	<i>Error</i>	<i>D.F.</i>	<i>Reps.</i>	<i>Blocks</i>	<i>Power (%)</i>
Half fraction 2 ⁸⁻¹	128	V+	128	0	91	1	1	1	100.0
Quarter fraction 2 ⁸⁻²	64	V	64	0	27	1	1	1	99.9999
Sixteenth fraction 2 ⁸⁻⁴	22	IV	16	6	6	1	1	1	91.4633
The StatAdvisor									
The table shows 3 experimental designs which have at least a 90% chance of detecting an effect of magnitude 2.									

Figure 3: Suggested Screening Designs

There are 3 designs, ranging from 22 runs up to 128 runs. The smallest resolution V design, which is a design that can estimate the main effects of the 8 factors and all 2-factor interactions,

requires that we perform 64 runs. If we can live with a resolution IV design, which estimates main effects clearly but confounds 2-factor interactions, we can get away with only 22 runs.

The 22-run design consists of 16 different combinations of high and low levels of the 8 experimental factors (corner points), plus 6 replicates at the center of the design region. The 6 replicates are needed to get a good estimate of the experimental error σ . If we don't want to run quite so many centerpoints, we can press the *Analysis Options* button  on the analysis toolbar and change the settings to reduce the experiment to 19 runs:

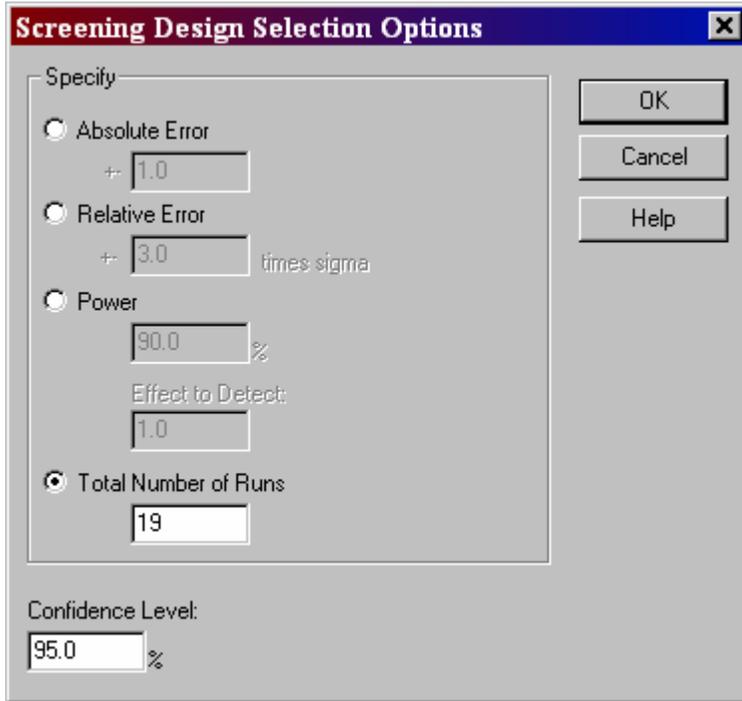


Figure 4: Restricting the Number of Runs

The *Analysis Summary* then displays just one selection:

<u>Screening Design Selection</u>								
Input								
<i>Number of Factors</i>	<i>Min. Centerpoints per Block</i>	<i>Max. Runs per Block</i>	<i>Exp. Error Sigma</i>	<i>Total # of Runs</i>	<i>Confidence Level</i>			
8	0		0.5	19.0	95.0%			
Selected Designs								
<i>Design</i>	<i>Runs</i>	<i>Resol.</i>	<i>Corner Points</i>	<i>Center Points</i>	<i>Error D.F.</i>	<i>Reps.</i>	<i>Blocks</i>	<i>Tolerance</i>
Sixteenth fraction 2 ⁸⁻⁴	19	IV	16	3	3	1	1	0.795612
The StatAdvisor								
The table shows 1 experimental designs which have no more than 19 runs.								

Figure 5: Restricted Selection of Screening Designs

With only 19 runs, the resolution IV design shows a *Tolerance* equal to 0.8. The *Tolerance* is defined as the margin of error associated with estimating the effect of each factor, based on a 95% confidence interval of the form

$$\text{estimated effect} \pm \text{tolerance}$$

For the 19-run design, we can expect to be able to estimate any effect to within about $\pm 0.8\%$ with 95% confidence.

The *Screening Design Selection* procedure also displays a power curve for the experiment:

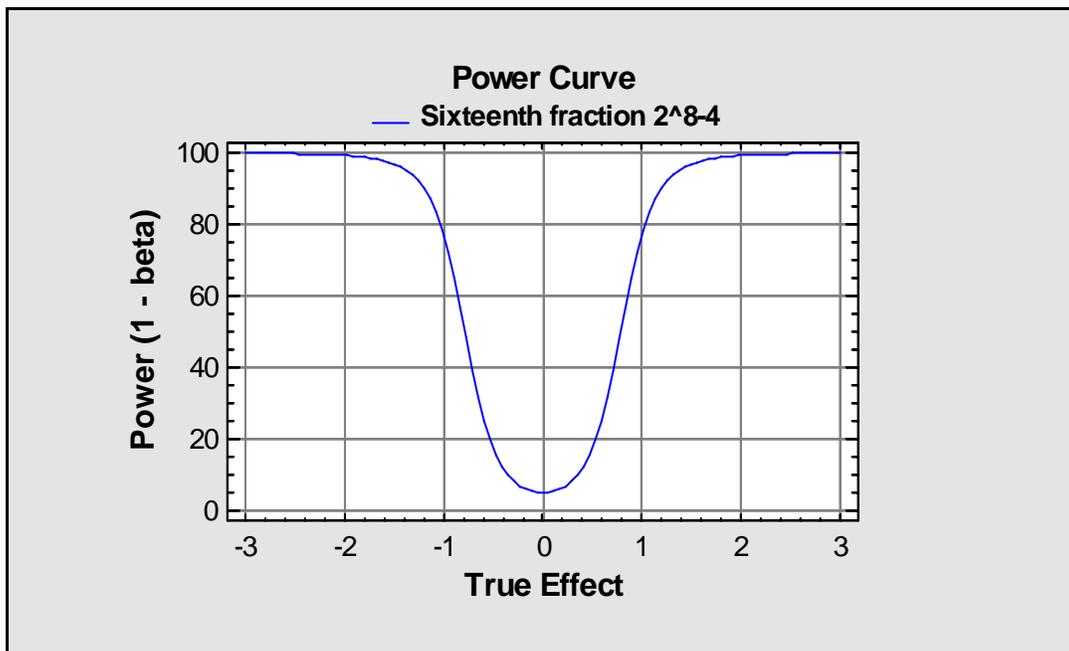


Figure 6: Power Curve for the 19-Run Design

The *Power Curve* plots the probability of obtaining a statistically significant result if an experimental factor is changed from one level to the other, as a function of its true magnitude or effect. Note that the curve is approximately 80% at a true effect of plus or minus 1. Although not the 90% chance that we originally hoped for, there is still a 4 out of 5 chance that we will declare as significant any factor that makes a 1% difference in shrinkage.

Step 2: Construct the Screening Design

Once we've decided on the design to run, we can now ask STATGRAPHICS Centurion to construct the design and place it in a datasheet.

Procedure: Create New Design

To create a new design:

- If using the Classic menu, select: *DOE – Design Creation – Create New Design*.
- If using the Six Sigma menu, select: *Improve – Experimental Design Creation – Create New Design*.

A sequence of dialog boxes will then be presented on which to specify the desired design attributes. The first dialog box specifies the type of design, the number of responses, and the number of experimental factors:

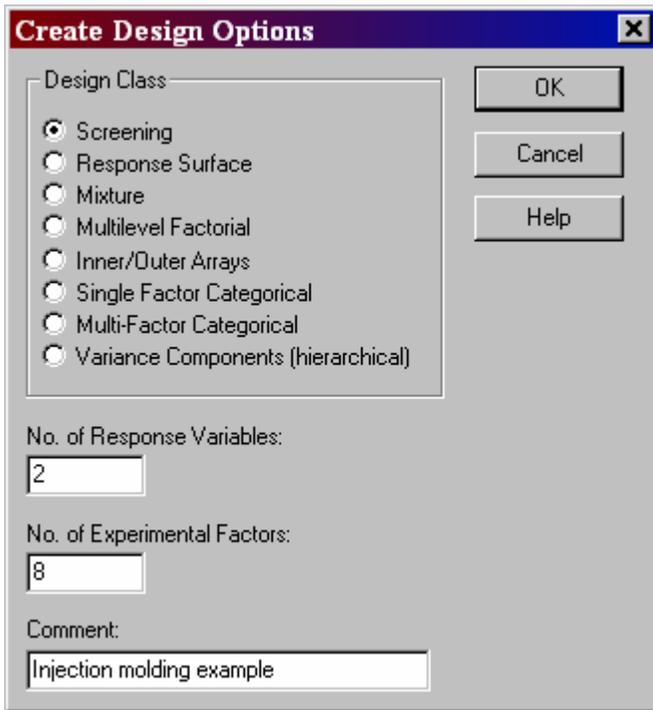


Figure 7: First Design Creation Dialog Box

The second dialog box requests names for each factor and the limits over which that factor will be varied:

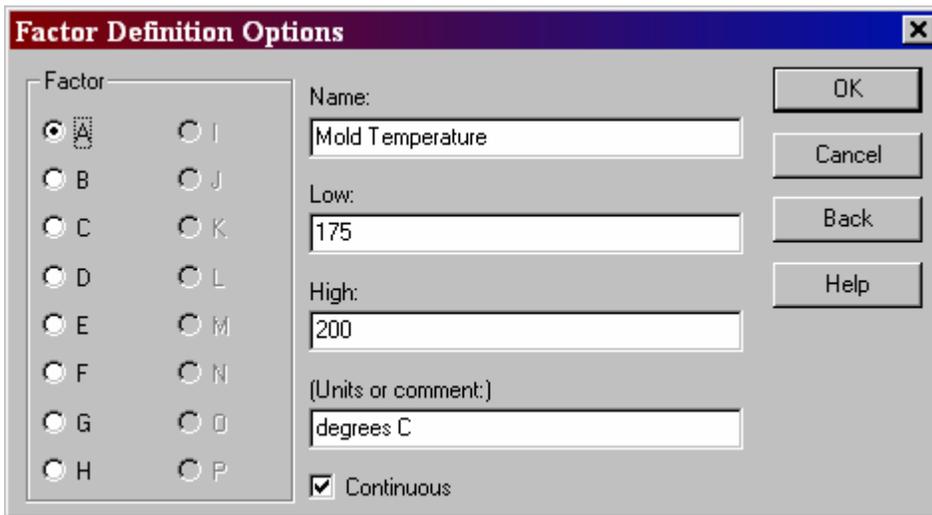


Figure 8: Second Design Creation Dialog Box

Since all factors in this experiment can be varied continuously between their low and high levels, the *Continuous* box should be checked for each.

The third dialog box specifies the names and units of each response variable:

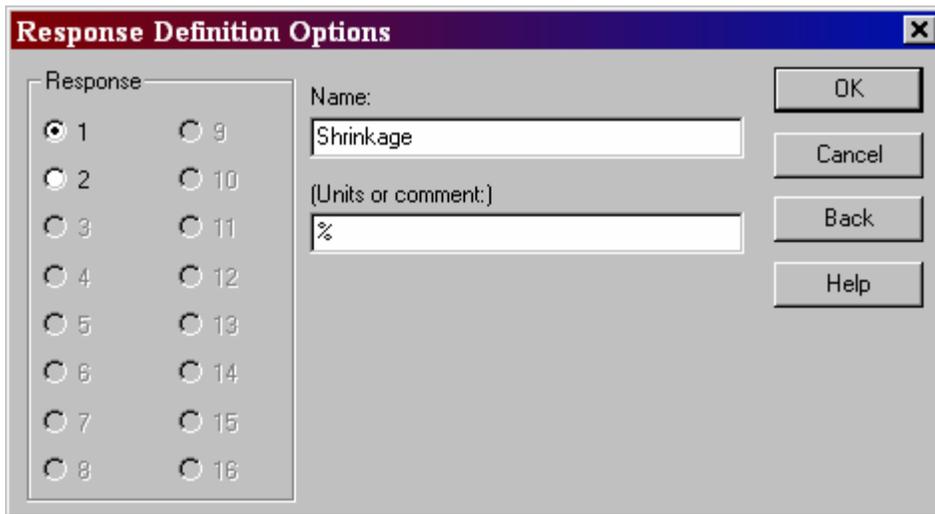


Figure 9: Third Design Creation Dialog Box

The fourth dialog box contains a pull-down list of all screening designs with 8 experimental factors:

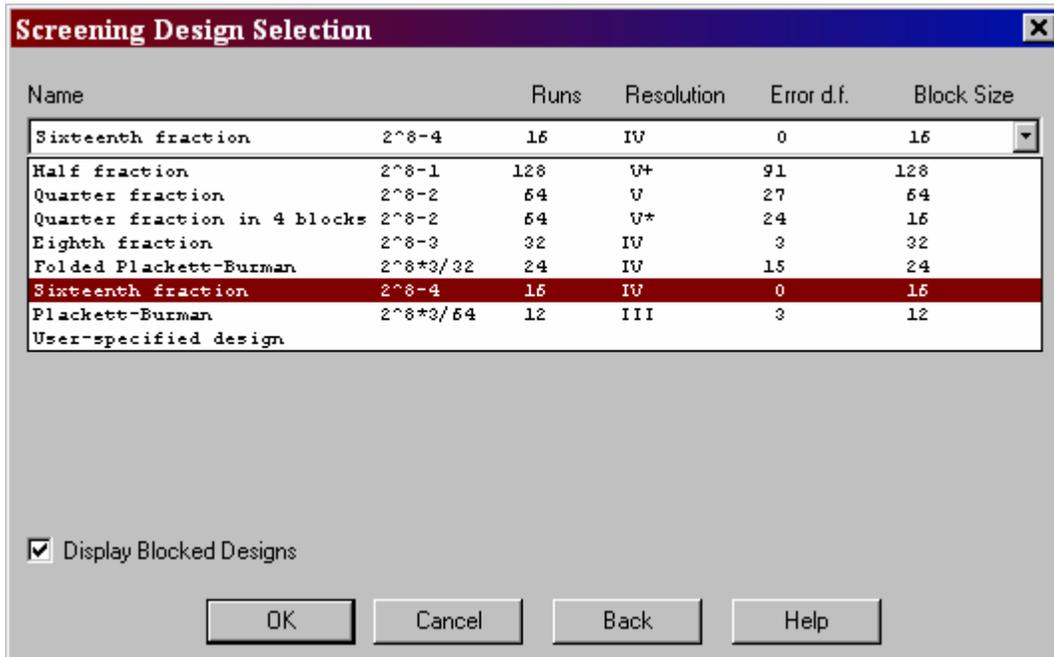


Figure 10: Fourth Design Creation Dialog Box

The design chosen earlier is the *Sixteenth fraction*, which contains 16 runs at carefully chosen combinations of the low and high levels of each factor. Note that 0 degrees of freedom are available from which to estimate the experimental error. This will be rectified on the next dialog box when centerpoints are added to the design.

The fifth and final dialog box specifies options for the selected design:

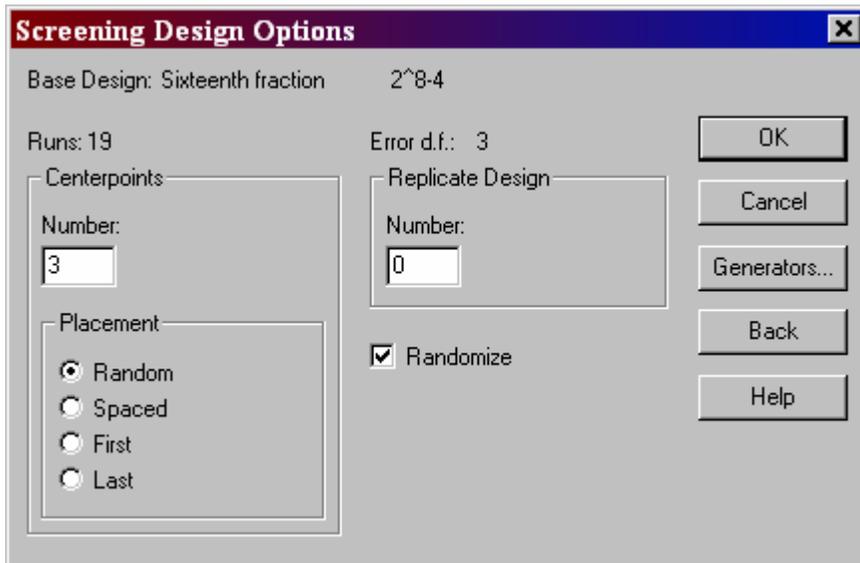


Figure 11: Fifth Design Creation Dialog Box

To estimate the experimental error, 3 centerpoints have been requested, bringing the total number of experimental runs to 19. It has also been requested that all of the runs be done in random order.

After the final dialog box has been completed, an analysis window entitled *Screening Design Attributes* will be created. This window summarizes the design:

Screening Design Attributes
 Design class: Screening
 Design name: Sixteenth fraction 2⁸⁻⁴
 File name: howto9.sfx
 Comment: Injection molding design

Base Design
 Number of experimental factors: 8
 Number of blocks: 1
 Number of responses: 2
 Number of runs: 19, including 3 centerpoints per block
 Error degrees of freedom: 3
 Randomized: Yes

<i>Factors</i>	<i>Low</i>	<i>High</i>	<i>Units</i>	<i>Continuous</i>
Mold Temperature	175	200	degrees C	Yes
Moisture Content	.1	.2	%	Yes
Holding Pressure	50	60	Mpa	Yes
Cavity Thickness	2	3	mm	Yes
Booster Pressure	65	70	Mpa	Yes
Cycle Time	35	40	seconds	Yes
Gate Size	.80	.95	mm	Yes
Screw Speed	250	500	rpm	Yes

<i>Responses</i>	<i>Units</i>
Shrinkage	%
Warpage	%

Figure 12: Screening Design Attributes Summary

Each of the experimental factors and response variables is summarized.

It is also instructive to press the *Tables* button  on the analysis toolbar and request the *Alias Structure*:

Alias Structure	
<i>Contrast</i>	<i>Estimates</i>
1	A
2	B
3	C
4	D
5	E
6	F
7	G
8	H
9	AB+CG+DH+EF
10	AC+BG+DF+EH
11	AD+BH+CF+EG
12	AE+BF+CH+DG
13	AF+BE+CD+GH
14	AG+BC+DE+FH
15	AH+BD+CE+FG

The StatAdvisor
 The alias structure shows which main effects and interactions are confounded with each other. Since this design is resolution IV, the main effects will be clear of the two-factor interactions. However, at least one two-factor interaction will be confounded with another two-factor interaction or a block effect. You will not be able to estimate these interactions. Check the table to determine which interactions are confounded.

Figure 13: Confounding Pattern for the Design

Each line of this table shows an effect or combination of effects that the design will be capable of estimating. A single letter such as “A” represents the main effect of a factor. Since each main effect appears alone on a separate line, they can be estimated clear of any other effects in the design. The terms such as “AB” represent interactions between pairs of factors. In this case, each two-factor interaction is confounded with 3 other such interactions. That implies that we will not be able to separately estimate each of the interactions, which makes sense, since there are not enough runs to separately estimate all of those effects.

The final experimental design, which consists of 19 runs, will have been loaded into datasheet A:

	BLOCK	Mold Temperature	Moisture Content	Holding Pressure	Cavity Thickness	Booster pressure	Cycle Time	Gate Size	Screw Speed	Shrinkage	Warpage
		degrees C	%	Mpa	mm	Mpa	seconds	mm	rpm	%	%
1	1	175	0.2	50	3	65	40	0.95	250		
2	1	200	0.1	50	3	70	35	0.95	250		
3	1	200	0.1	60	2	70	35	0.8	500		
4	1	175	0.1	60	2	70	40	0.95	250		
5	1	200	0.2	50	2	70	40	0.8	250		
6	1	200	0.2	60	3	70	40	0.95	500		
7	1	175	0.2	60	2	65	40	0.8	500		
8	1	200	0.2	50	3	65	35	0.8	500		
9	1	175	0.1	50	2	65	35	0.8	250		
10	1	187.5	0.15	55	2.5	67.5	37.5	0.875	375		
11	1	187.5	0.15	55	2.5	67.5	37.5	0.875	375		
12	1	200	0.1	60	3	65	40	0.8	250		
13	1	175	0.2	50	2	70	35	0.95	500		
14	1	175	0.1	50	3	70	40	0.8	500		
15	1	200	0.1	50	2	65	40	0.95	500		
16	1	175	0.1	60	3	65	35	0.95	500		
17	1	175	0.2	60	3	70	35	0.8	250		
18	1	187.5	0.15	55	2.5	67.5	37.5	0.875	375		
19	1	200	0.2	60	2	65	35	0.95	250		

Figure 14: Final Design Loaded into Datasheet A

The datasheet contains:

1. One row for each experimental run to be performed.
2. A column labeled *Block* that identifies which block each run is assigned to. This is only relevant when the runs have been grouped into blocks according to an additional nuisance factor. In this case, all of the runs are contained in a single block.
3. A column for each experimental factor.
4. A column for each response variable.

Step 3: Perform the Screening Experiment

The 19 runs in the experimental design would then be performed and the values of the response variables entered into the appropriate columns of the datasheet. If more than one measurement is taken during each run, perhaps from multiple samples, then the entry in the cell would usually be the average value of the measurements. Additional response variables could also be added to the end of the datasheet to contain standard deviations or other sample statistics that the experimenter wished to analyze.

The data for this example have been saved in the file *howto9.sfx*. Note that experiment files in STATGRAPHICS Centurion have a special extension, since they contain not only the data but also additional information about the type of experimental design that was created.

Step 4: Analyze the Results

To analyze the results of the experiment:

- If using the Classic menu, select: *DOE – Design Analysis – Analyze Design*.
- If using the Six Sigma menu, select: *Improve – Experimental Design Analysis – Analyze Design*.

A data input dialog box will appear, listing each of the response variables:

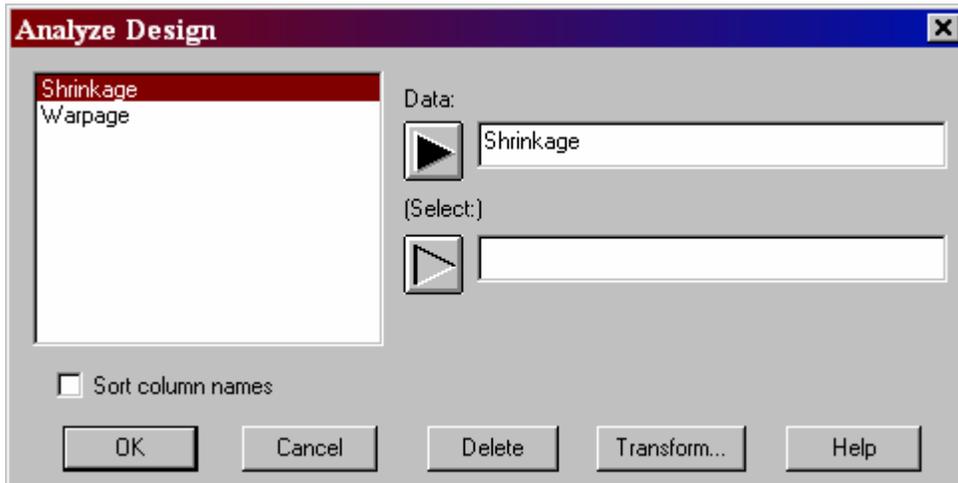


Figure 15: Analyze Design Data Input Dialog Box

A separate analysis will be done on each response, which may be affected by different experimental factors.

Analysis of Shrinkage

The first step in analyzing a screening design is to determine which factors have a significant impact on the response variables. This is most easily done using a *Pareto Chart*, which is generated by default when the analysis window is created:

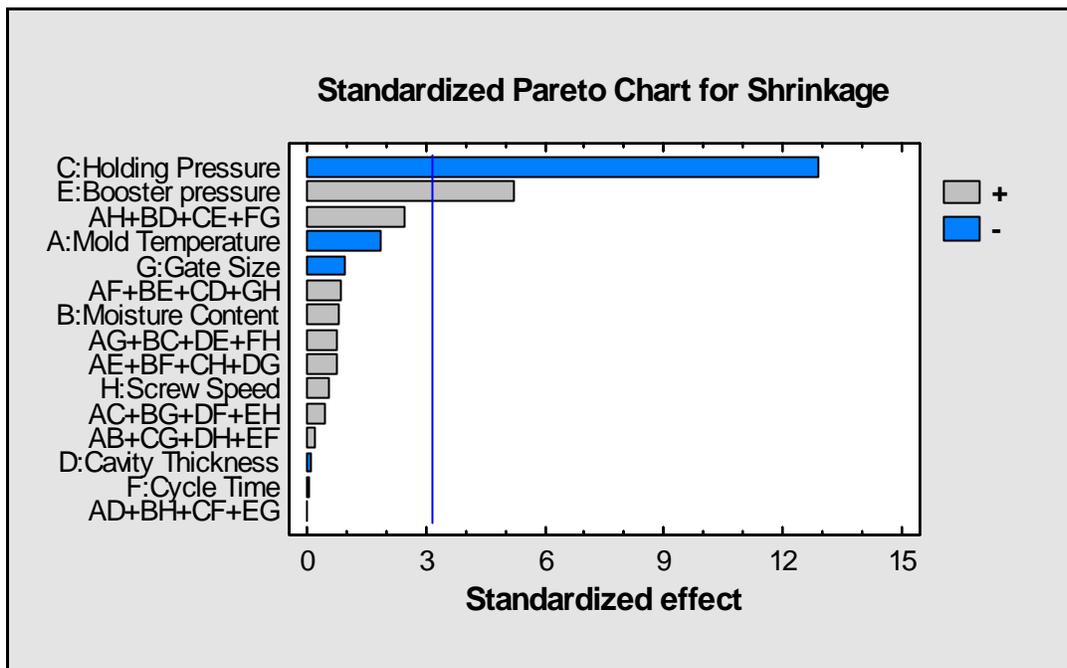


Figure 16: Standardized Pareto Chart for Shrinkage

The standardized Pareto Chart contains a bar for each effect, sorted from most significant to least significant. The length of each bar is proportional to the standardized effect, which equals the magnitude of the t-statistic that would be used to test the statistical significance of that effect. A vertical line is drawn at the location of the 0.05 critical value for Student's t. Any bars that extend to the right of that line indicate effects that are statistically significant at the 5% significance level.

Exact P-values may also be obtained from the ANOVA Table:

Analysis of Variance for Shrinkage - Injection molding example

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
A:Mold Temperature	0.393756	1	0.393756	3.42	0.1614
B:Moisture Content	0.0770063	1	0.0770063	0.67	0.4732
C:Holding Pressure	19.1188	1	19.1188	166.18	0.0010
D:Cavity Thickness	0.00075625	1	0.00075625	0.01	0.9405
E:Booster pressure	3.14176	1	3.14176	27.31	0.0136
F:Cycle Time	0.00030625	1	0.00030625	0.00	0.9621
G:Gate Size	0.107256	1	0.107256	0.93	0.4055
H:Screw Speed	0.0351563	1	0.0351563	0.31	0.6189
AB+CG+DH+EF	0.00455625	1	0.00455625	0.04	0.8550
AC+BG+DF+EH	0.0217562	1	0.0217562	0.19	0.6930
AD+BH+CF+EG	0.00000625	1	0.00000625	0.00	0.9946
AE+BF+CH+DG	0.0637562	1	0.0637562	0.55	0.5106
AF+BE+CD+GH	0.0885062	1	0.0885062	0.77	0.4450
AG+BC+DE+FH	0.0637562	1	0.0637562	0.55	0.5106
AH+BD+CE+FG	0.701406	1	0.701406	6.10	0.0901
Total error	0.345148	3	0.115049		
Total (corr.)	24.1636	18			

Figure 17: Analysis of Variance Table for Shrinkage

Note that there are two effects with P-values below 0.05: the main effect of *Holding Pressure* and the main effect of *Booster Pressure*. Another effect, consisting of the sum of four interactions AH+BD+CE+FG, has a P-value of approximately 0.09. Since we decided to run an experiment with only three degrees of freedom for the error term, P-values in the neighborhood

of 0.1 may be large enough to be interesting. As with all resolution IV designs, it is impossible to determine which of the four interactions is responsible for the large effect. However, since the effect contains the interaction of the two significant main effects (factors C and E), a large CE interaction is the most likely explanation.

If we use *Analysis Options* to exclude all effects other than C, E, and CE, the following *Interaction Plot* may be obtained:

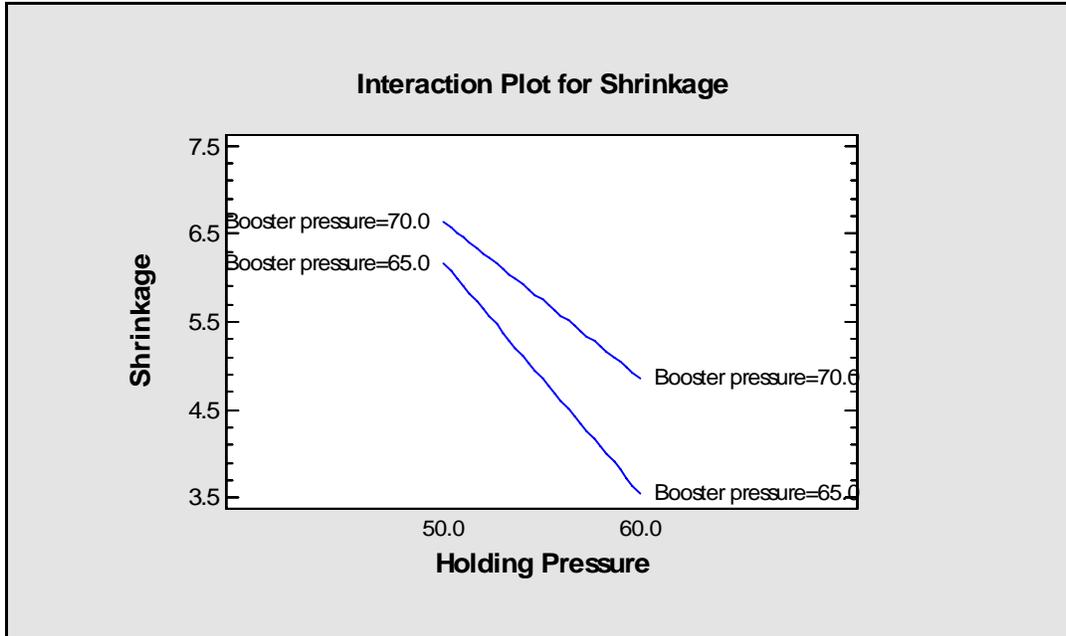


Figure 18: Interaction Plot for Holding Pressure and Booster Pressure

Evidently, increasing *Holding Pressure* reduces *Shrinkage*. In addition, the effect is more pronounced at lower *Booster Pressure*.

The effect of the two factors is also well displayed by a contour plot:

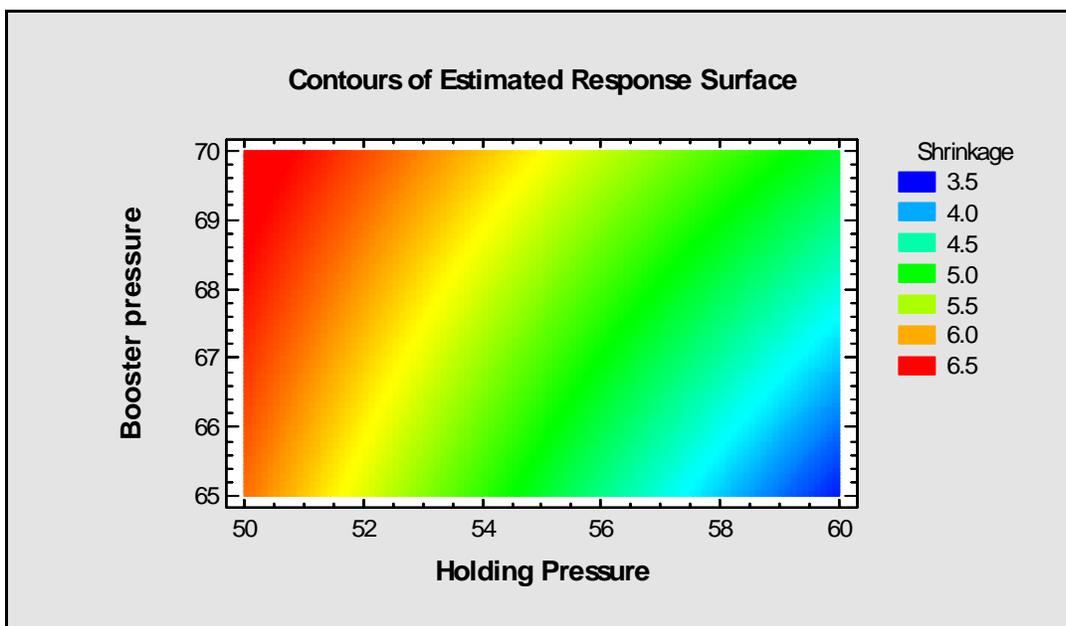


Figure 19: Contour Plot for Holding Pressure and Booster Pressure

The region of dark blue toward the right bottom indicates combinations of pressure that would result in relatively low shrinkage.

Analysis of Warpage

When the same analysis is performed on *Warpage*, there are two significant main effects shown on the Pareto Chart, *Holding Pressure* and *Cycle Time* :

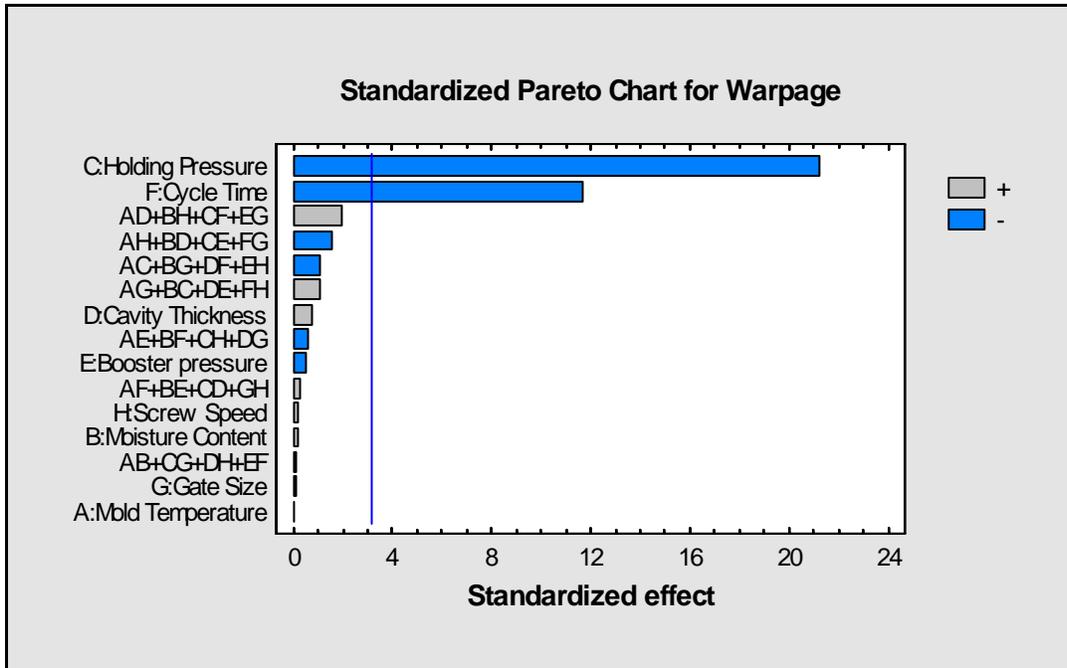


Figure 20: Standardized Pareto Chart for Warpage

Excluding all other factors from the model yields the following *Contour Plot*:

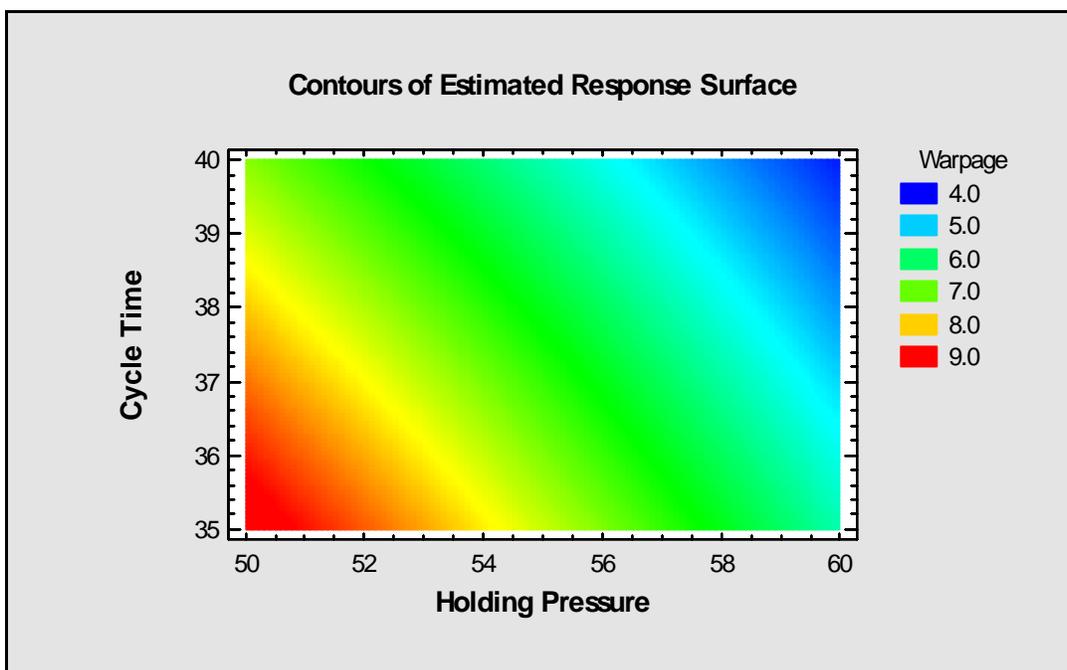


Figure 21: Contour Plot for Warpage

Evidently, low *Warpage* is achieved at high values of *Holding Pressure* and low values of *Cycle Time*.

Step 5: Follow the Path of Steepest Ascent/Descent

It appears that both *Shrinkage* and *Warpage* can be decreased by increasing the *Holding Pressure*. At the same time, decreasing *Booster Pressure* should reduce *Shrinkage*, while increasing *Cycle Time* should reduce *Warpage*.

In order to confirm these results, the researchers decided to do some experiments along the path of steepest descent. This is the path that is predicted to decrease the responses most quickly for the smallest changes in the input factors. We will have to generate two paths: one for each response. To generate the path for *Shrinkage*, we will first collapse the design by removing all variables except *Holding Pressure* and *Boasting Pressure*. This is done as follows:

1. Close the *Howto9.sgp* StatFolio.
2. Reopen the *Howto9.sfx* experiment file in datasheet A.
3. Go the top menu and select:
 - If using the Classic menu, select: *DOE – Design Creation – Augment Existing Design*.
 - If using the Six Sigma menu, select: *Improve – Experimental Design Creation – Augment Existing Design*.

When the first dialog box appears, indicate that you wish to collapse the design:

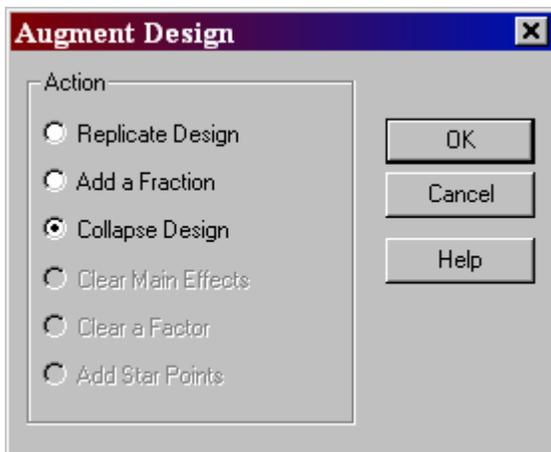


Figure 22: First Augment Design Dialog Box

One the second dialog box, indicate that you wish to remove *Mold Temperature* from the design:



Figure 23: Second Augment Design Dialog Box

When you press OK, the column for *Mold Temperature* will be deleted from datasheet A. Now repeat this process until only *Holding Pressure* and *Booster Pressure* remain in the design. Then use *File – Save As* to save the design with a new name.

4. Select *Analyze Design* and refit the model for *Shrinkage*. It should contain three significant effects, now labeled A, B and AB, as shown in the Pareto chart below:

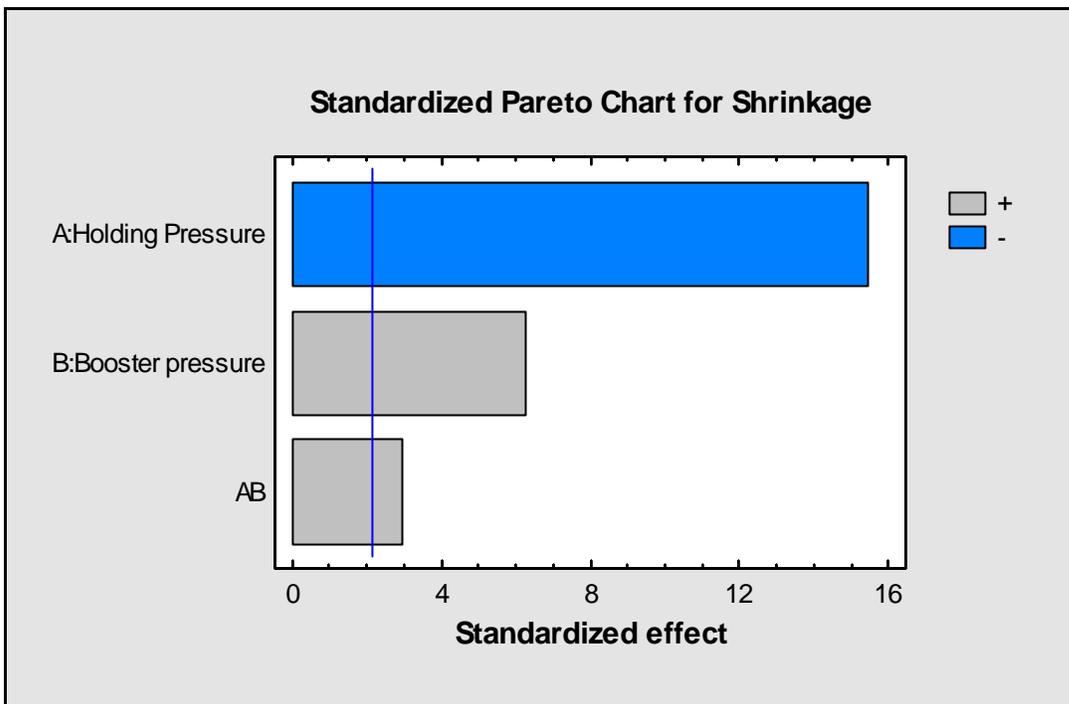


Figure 24: Pareto Chart for Shrinkage After Collapsing Design

5. Finally, select *Path of Steepest Ascent* from the list of *Tables* available in the *Analyze Design* window. Before examining the results, press *Pane Options* and set the options as shown below:

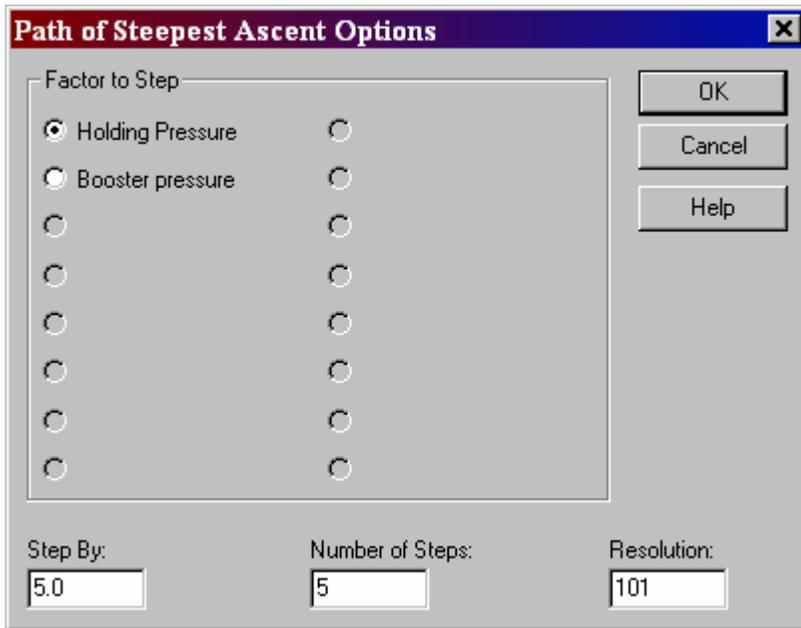


Figure 25: Pane Options Dialog Box for Path of Steepest Ascent

The dialog box above requests 5 steps along the path of steepest ascent or descent, each changing *Holding Pressure* by 5 Mpa. The program will then calculate and display the values of all other factors so that you will move along the path:

Path of Steepest Ascent for Shrinkage

		<i>Predicted</i>
<i>Holding Pressure</i>	<i>Booster Pressure</i>	<i>Shrinkage</i>
(Mpa)	(Mpa)	(%)
55.0	67.5	5.30368
60.0	66.3038	3.89835
65.0	64.7984	2.18605
70.0	63.0635	0.12325
75.0	61.1606	-2.31619
80.0	59.1345	-5.1478

Figure 26: Calculated Points Along the Path of Steepest Ascent

As *Holding Pressure* is increased, *Booster Pressure* is decreased. Note that the predicted *Shrinkage* falls rapidly as one moves along the path. Eventually, extrapolation of the model leads to unrealistic negative predictions. While such models can not be expected to predict well too far outside the experimental region, they do give a good suggestion of the direction to look for improved results.

To generate the path of steepest descent for *Warpage*, you must now reload the original design. This time, collapse out all factors except for *Holding Pressure* and *Cycle Time*. Then select *Analyze Design* to fit a model with the remaining two factors. The standardized Pareto chart should appear as illustrated below:

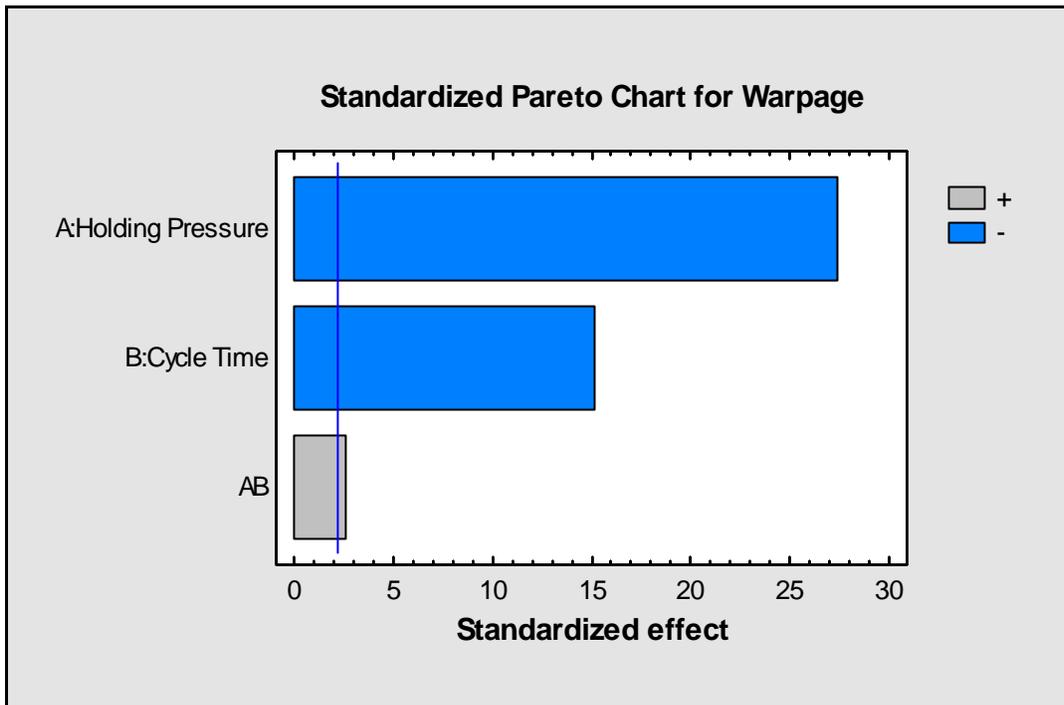


Figure 27: Pareto Chart for Warpage After Collapsing Design

Now generate the new path of steepest descent as before:

Path of Steepest Ascent for Warpage

		Predicted
Holding Pressure	Cycle Time	Warpage
(Mpa)	(seconds)	(%)
55.0	37.5	6.53789
60.0	38.7931	4.47599
65.0	39.8994	2.6193
70.0	40.7927	0.937993
75.0	41.4484	-0.604398
80.0	41.8456	-2.0507

Figure 28: Calculated Points Along the Path of Steepest Descent

The predicted responses at points along the path of steepest descent suggest a direction in which to look for better results. Being fairly cautious, the researchers next decided to do a sequence of experiments along the suggested path to verify the predictions. The table below shows the outcome of 5 experiments along that path:

Holding Pressure	Booster Pressure	Cycle Time	Shrinkage	Warpage
(Mpa)	(Mpa)	(seconds)	(%)	(%)
60.0	66.3	38.8	3.88	4.59
65.0	64.8	39.9	3.26	3.61
70.0	63.1	40.8	2.81	3.10
75.0	61.2	41.4	2.73	3.39
80.0	59.1	41.8	2.98	4.23

Figure 29: Results of Experiments Along the Path of Steepest Ascent

Notice that the first few steps along the path decreased both *Shrinkage* and *Warpage*, although not as dramatically as the models had predicted. Eventually, both responses began to rise again. This is evidence that the first-order models fit by the screening designs failed to capture the curvature in the response surfaces. This is not surprising, since the primary task of the screening experiment was to select the most important factors from amongst the initial 8. The fact that the

screening experiment gave an indication of the direction in which to look for improved performance is an added benefit.

Step 6: Construct an Optimization Experiment

Now that the number of experimental factors has been reduced to a manageable number, an optimization experiment can be constructed. Based on the results of the experiments along the path of steepest descent, it was decided to construct a second experiment covering the following region:

Holding Pressure: 65 – 80 Mpa
Booster Pressure: 60 – 65 Mpa
Cycle Time: 40 – 45 seconds

To construct this experiment, the StatFolio was cleared and *Create Design* selected from the main menu:

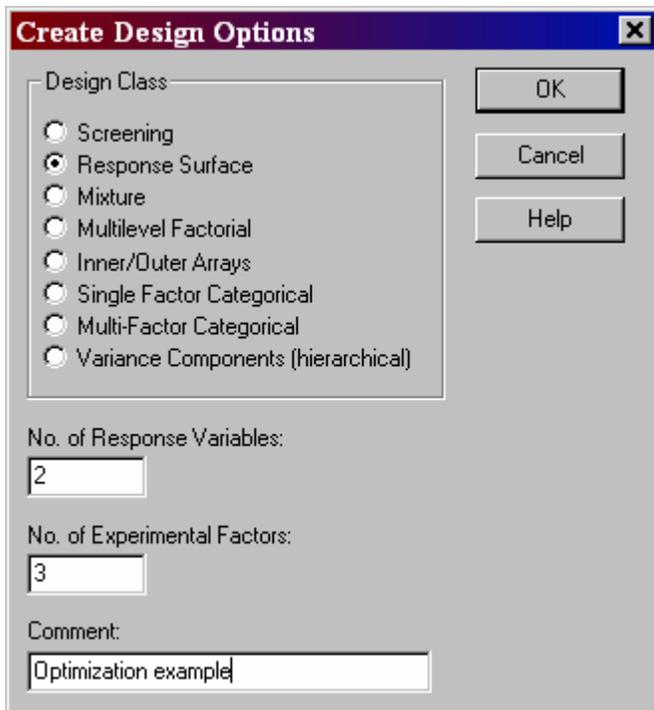


Figure 30: Initial Dialog Box for Creation of a Response Surface Design

On the next two dialog boxes, the factors and experimental region were defined as listed above. On the design selection dialog box, a central composite design was selected:

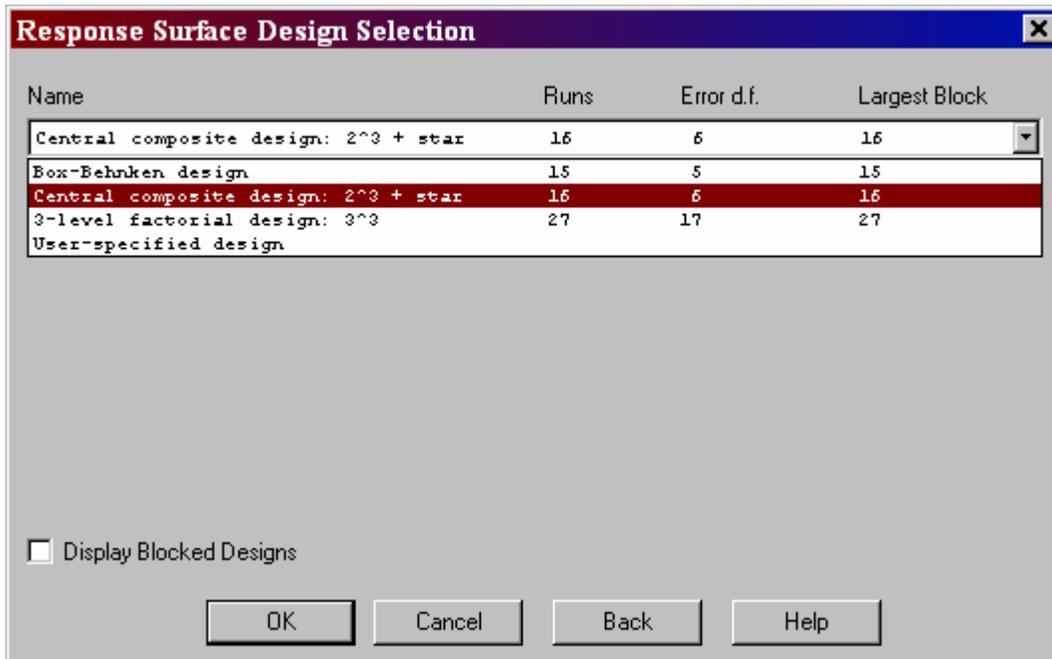


Figure 31: Response Surface Design Selection Dialog Box

This design consists of 16 runs:

1. 8 runs at all combinations of the high and low levels of the 3 factors. When plotted in 3 dimensions, these points form a cube.
2. 6 runs at star points, located at the end of radial lines extending out through the 6 faces of the cube.
3. 2 runs at the center of the design.

On the design options dialog box, all of the default settings were accepted:

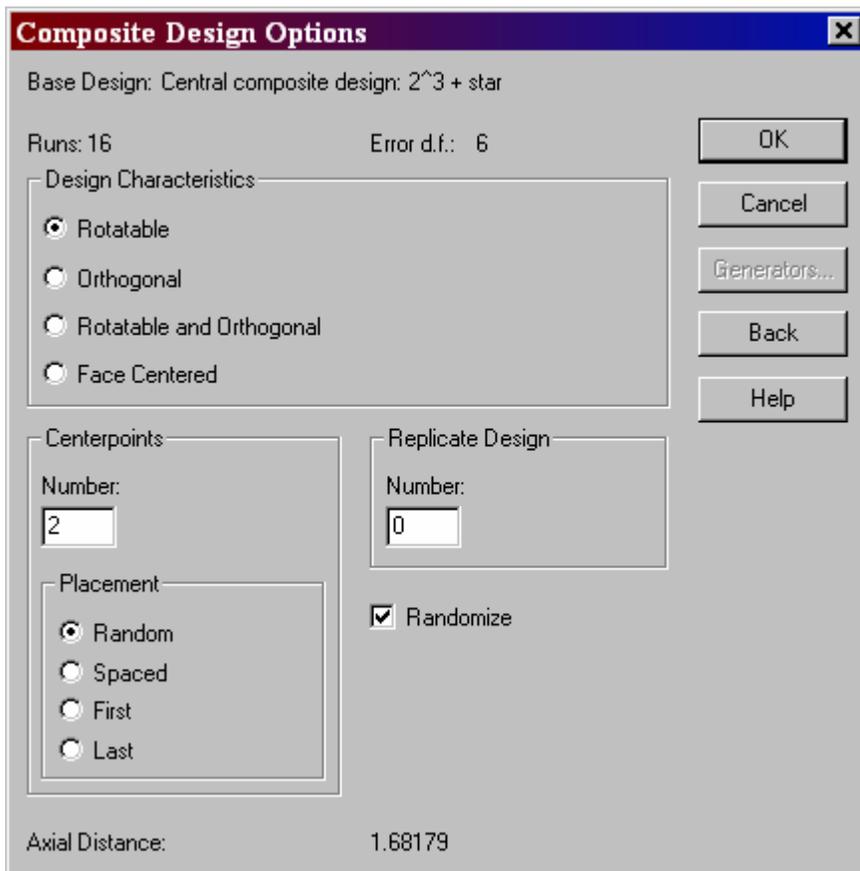


Figure 32: Response Surface Design Options Dialog Box

The final design is shown below:

	BLOCK	Holding Pressure	Booster Pressure	Cycle Time	Shrinkage	Warpage
1	1	85	62.5	42.5		
2	1	60	62.5	42.5		
3	1	80	65	45		
4	1	72.5	67	42.5		
5	1	65	65	40		
6	1	72.5	62.5	38		
7	1	80	60	45		
8	1	72.5	62.5	47		
9	1	65	65	45		
10	1	65	60	45		
11	1	80	60	40		
12	1	72.5	58	42.5		
13	1	72.5	62.5	42.5		
14	1	65	60	40		
15	1	80	65	40		
16	1	72.5	62.5	42.5		

Figure 33: Design Matrix for Optimization Experiment

Note: the star points generated by STATGRAPHICS were placed at locations to make the design perfectly rotatable, which is a property that insures equal predictive power in all directions. Once entered into the datasheet, the levels were rounded slightly by hand.

Step 7: Analyze the Optimization Experiment

The 16 runs were then performed and the values of *Shrinkage* and *Warpage* were measured. The results are contained in the file Howto9A.sfx.

Analysis of Shrinkage

The standardized Pareto chart for *Shrinkage* is shown below:

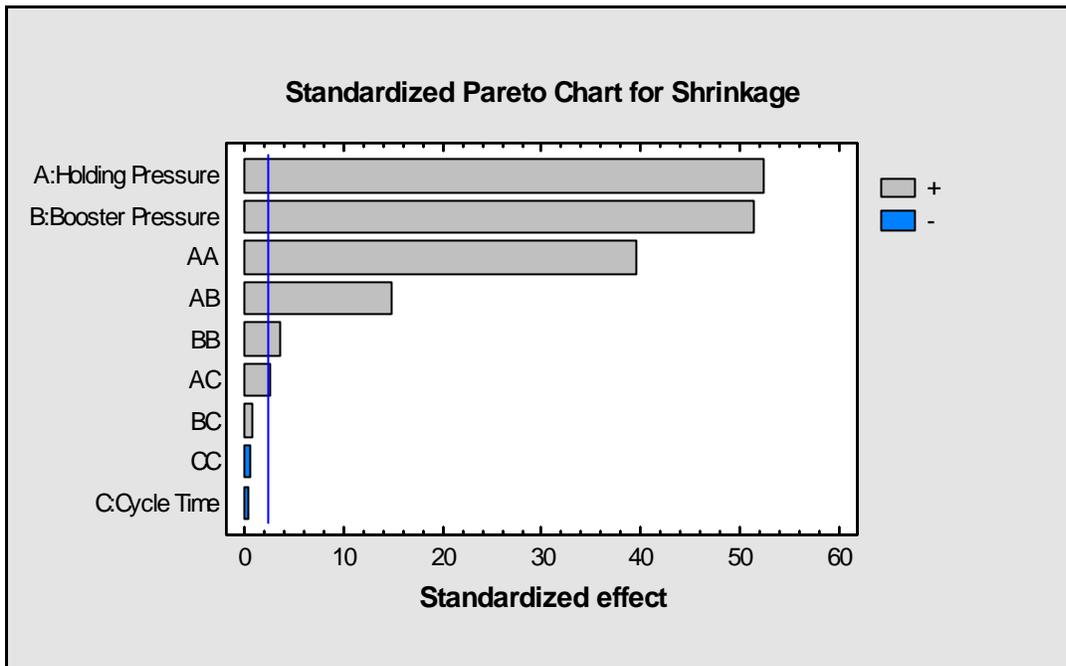


Figure 34: Standardized Pareto Chart for Shrinkage

None of the terms involving factor C appear to be statistically significant, so *Cycle Time* was dropped from the model. The resulting contour plot is shown below:

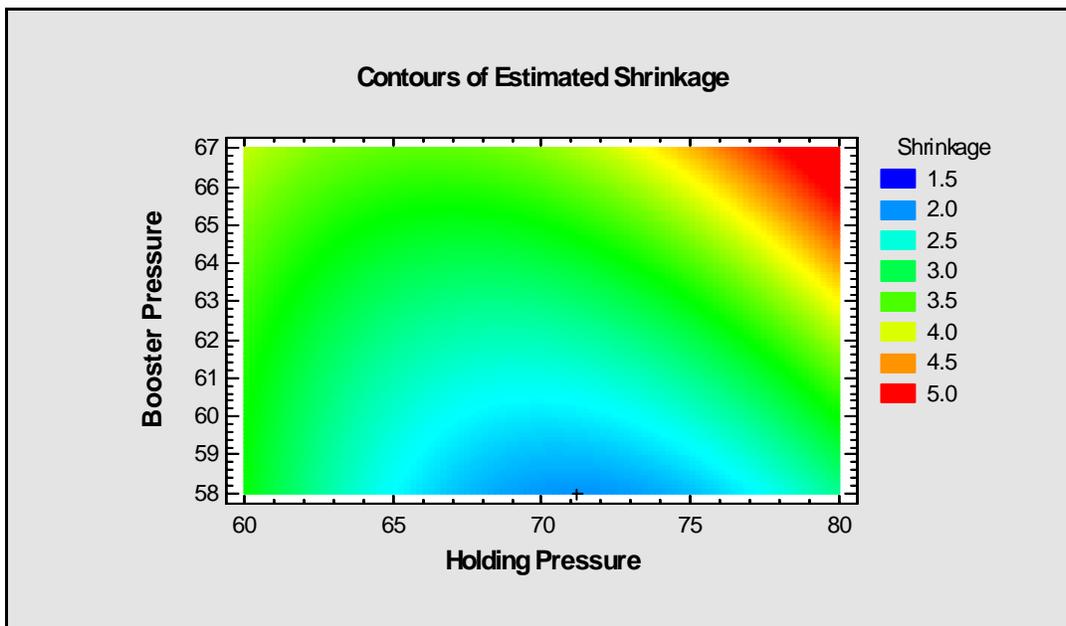


Figure 35: Contour Plot for Shrinkage

Minimum *Shrinkage* is achieved at low *Booster Pressure*, with *Holding Pressure* around 71 Mpa. Decreasing *Booster Pressure* below 58 might reduce *Shrinkage* even more.

Analysis of Warpage

The standardized Pareto chart for *Warpage* shows that only *Holding Pressure* and *Cycle Time* have a significant effect:

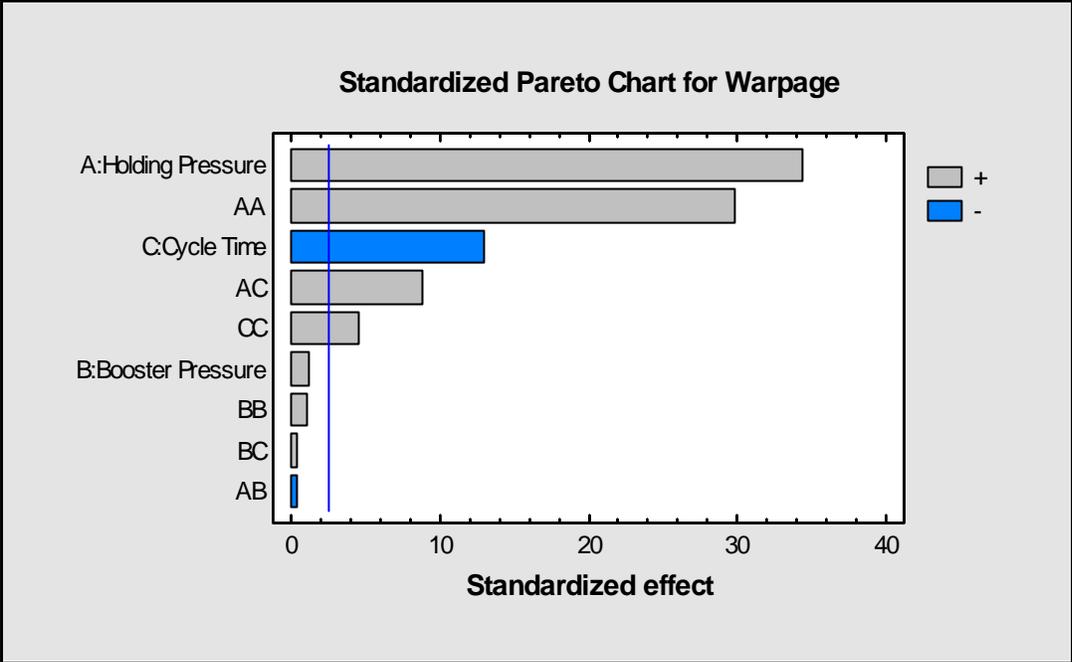


Figure 36: Standardized Pareto Chart for Warpage

Dropping *Booster Pressure* from the model yields the following contour plot:

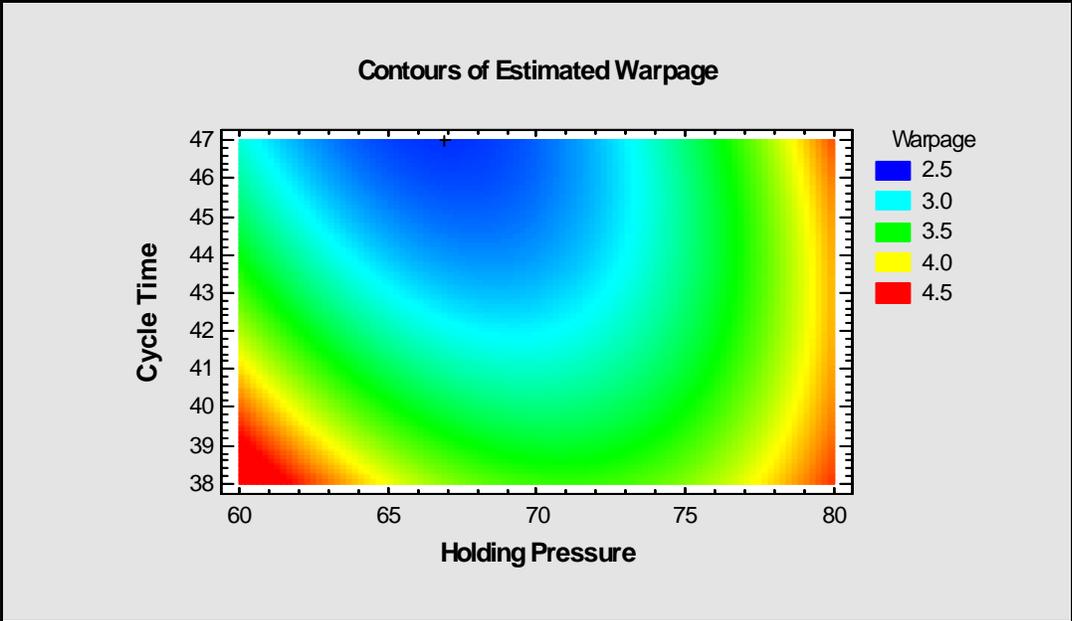


Figure 37: Contour Plot for Warpage

Minimum *Warpage* is achieved at high *Cycle Time*, with *Holding Pressure* around 67 Mpa. Increasing *Cycle Time* above 47 might reduce *Warpage* even more.

Step 8: Perform a Multiple Response Optimization

The optimal settings for each response variable, obtained from the *Optimization* pane in each separate analysis, are summarized below:

<i>Factor</i>	<i>Low</i>	<i>High</i>	<i>Optimum Shrinkage =</i> 1.98714 at	<i>Optimum Warpage =</i> 2.5815 at
Holding Pressure	60.0	85.0	71.1878	66.8891
Booster Pressure	58.0	67.0	58.0	Any
Cycle Time	38.0	47.0	Any	47.0

Figure 38: *Optimization Results for Individual Responses*

Since *Booster Pressure* and *Cycle Time* affect only one response, no tradeoff is necessary for those factors. However, *Holding Pressure* affects both responses, and the optimal setting is somewhat different for each response.

To find a level of *Holding Pressure* that achieves a good tradeoff between the two response variables, you can use the *Multiple Response Optimization* procedure. Be sure that you have the *Analyze Design* analysis window open for each of the two responses, since the *Multiple Response Optimization* procedure will search for those windows to obtain the fitted model for each response. Then:

- If using the Classic menu, select: *DOE – Design Analysis – Multiple Response Optimization*.
- If using the Six Sigma menu, select: *Improve – Experimental Design Creation – Multiple Response Optimization*.

On the data input dialog box, specify the names of both response variables:

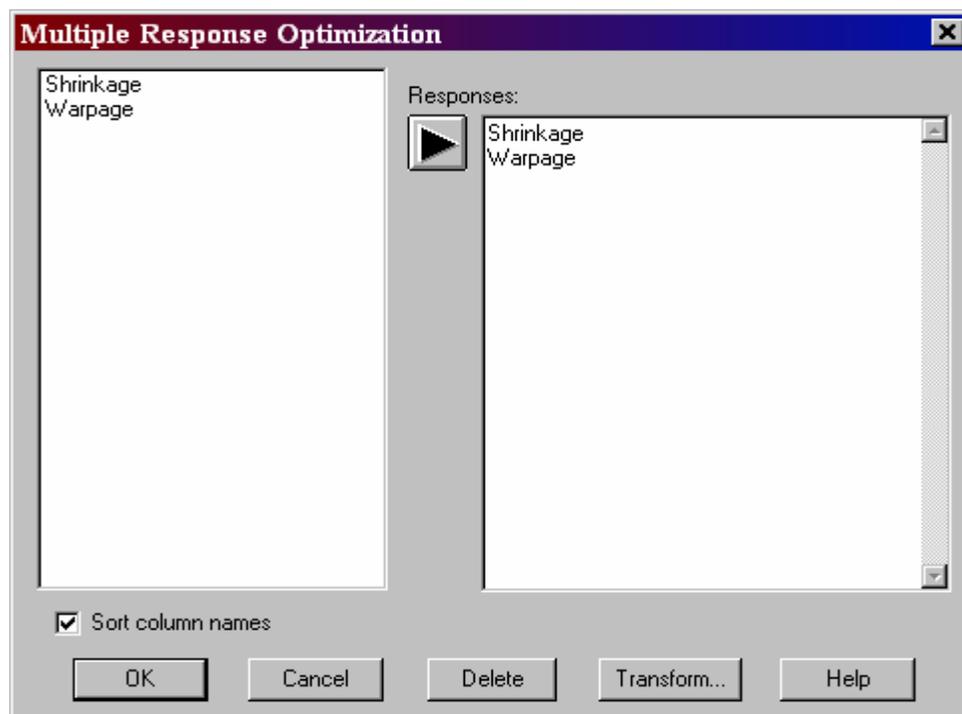


Figure 39: *Multiple Response Optimization Data Input Dialog Box*

The procedure will then find the settings of the experimental factors that maximize a combined *desirability function*, which is a function that expresses the desirability of a solution involving m responses through the function of the form

$$D = \{d_1^{I_1} d_2^{I_2} \dots d_m^{I_m}\}^{1/\left(\sum_{j=1}^m I_j\right)}$$

where d_j is the calculated desirability of the j^{th} response and I_j is an impact coefficient that ranges between 1 and 5. Setting the impact coefficient of one response higher than another will give it more weight in determining the final solution.

When a response is to be minimized, the desirability of a predicted response equal to \hat{y}_j is defined as

$$d_j = \begin{cases} 1 & \hat{y}_j < low_j \\ \left(\frac{high_j - \hat{y}_j}{high_j - low_j}\right)^s & low_j \leq \hat{y}_j \leq high_j \\ 0 & \hat{y}_j > high_j \end{cases},$$

The user defines the values for *low* and *high*, as well as the shape parameter s , which may range between 0.1 and 10. The graph below illustrates the shape of the desirability function for different values of s :

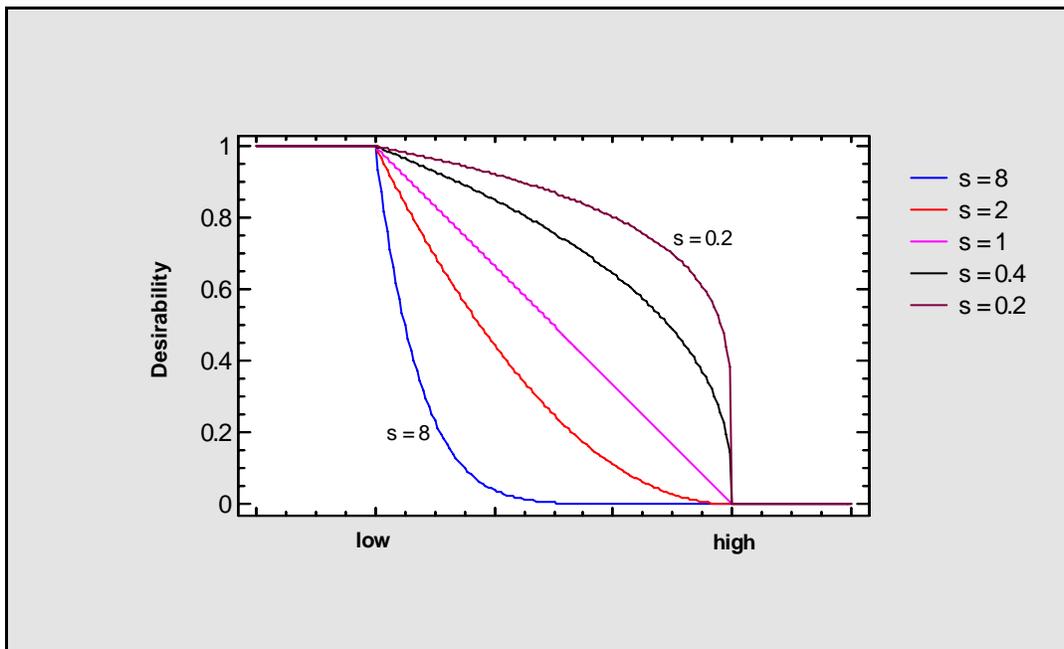


Figure 40: Desirability Functions with Different Shape Parameters

For $s = 1$, the desirability decreases linearly from 1 at the *low* value to 0 at the *high* value. For $s > 1$, it falls quickly at first and then levels off. For $s < 1$, it decreases slowly at first and then speeds up. The analyst can set s large if it is very important to be close to the minimum level.

Once the *Multiple Response Optimization* window opens, select *Analysis Options*. This displays the following dialog box:

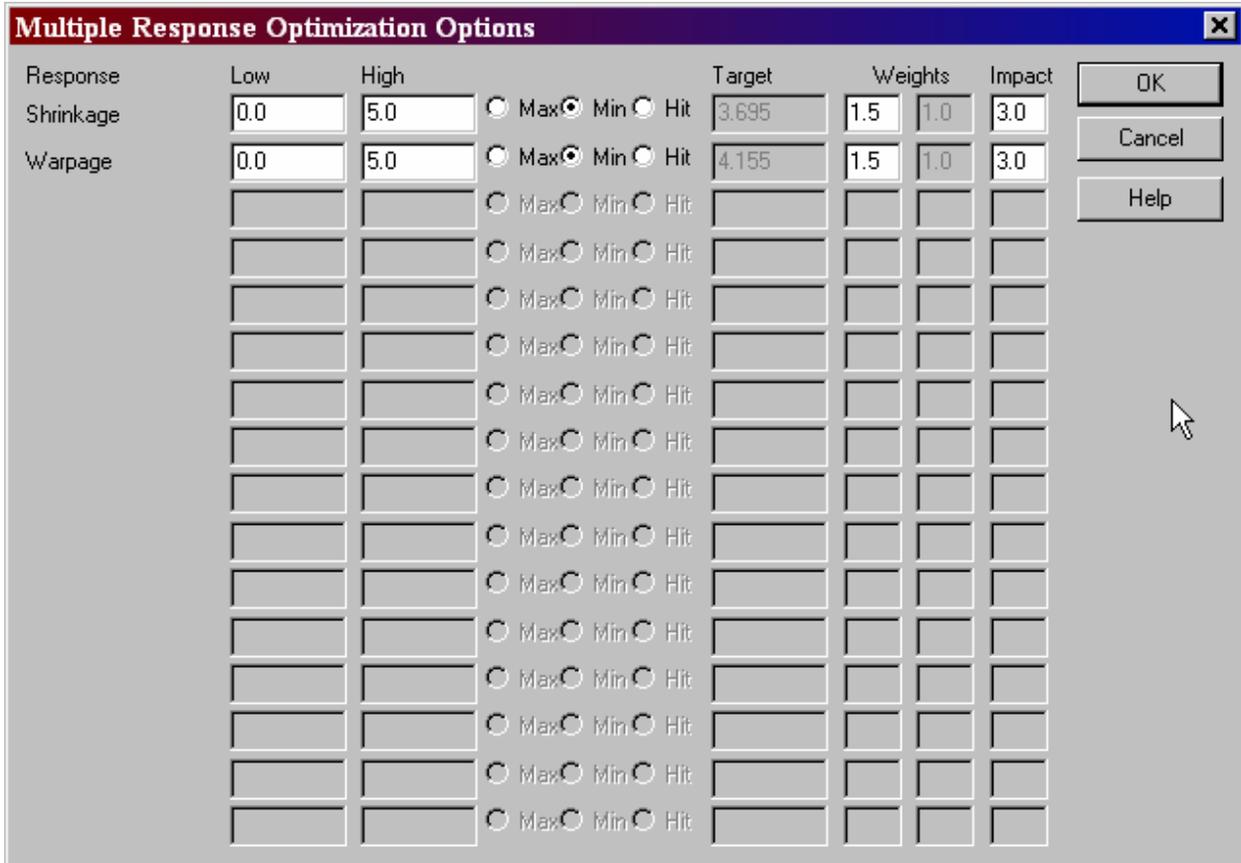


Figure 41: Multiple Response Optimization Analysis Options Dialog Box

The settings on the dialog box above specify equal *Impact* values for each response, which implies that *Shrinkage* and *Warpage* are equally important. It also sets the low and high values for each response to 0 and 5, respectively. s is set to 1.5, which causes the desirability function to decrease more rapidly than linearly.

The *Optimization* pane displays the final solution:

Optimize Desirability			
Optimum value = 0.386613			
Factor	Low	High	Optimum
Holding Pressure	60.0	85.0	68.7636
Booster Pressure	58.0	67.0	58.0
Cycle Time	38.0	47.0	47.0
Response	Optimum		
Shrinkage	2.0447		
Warpage	2.61746		

Figure 42: Optimal Solution for Multiple Responses

As expected, *Booster Pressure* is set low, while *Cycle Time* is set high. The optimal setting for *Holding Pressure* is 68.8, which lies between the solutions for each response when optimized separately. Note also that while neither response variable is quite as small as when it is optimized separately, both responses are quite reasonable compared to the variation observed over the experimental region.

Conclusion

This guide has demonstrated a typical optimization experiment. The study began by performing a screening experiment involving 8 experimental factors and 2 response variables. It was found that only 3 of the 8 factors appeared to have a significant effect on the responses. The design was then collapsed and the path of steepest descent was calculated for each response. Experiments along that path suggested that some improvement was possible by moving outside the current experimental region, although the simple linear models appeared to break down when extrapolated far from the original experimental region.

An optimization experiment was then performed in the neighborhood of the best solutions found while moving along the path of steepest descent. Since curvature had been observed, a full second-order central composite design was performed. Models were fit and each response was optimized separately. The *Multiple Response Optimization* procedure was then used to find a tradeoff between the best solutions for each response.

To further improve the process, additional experiments could be performed along the new path of steepest descent, until an acceptable solution is found or until no further improvement is possible.

Note: The author welcomes comments about this guide. Please address your responses to neil@statgraphics.com.