Median and Range Charts

Summary

The **Median and Range Charts** procedure creates control charts for a single numeric variable where the data have been collected in subgroups. It creates both a Median chart to monitor the subgroup medians and an R chart to monitor the subgroup ranges. Out-of-control signals are highlighted, including both points beyond the control limits and any unusual runs in the data. The charts may be constructed in either *Initial Study* (Phase 1) mode, where the current data determine the control limits, or in *Control to Standard* (Phase 2) mode, where the limits come from either a known standard or from prior data.

The output and options available in the *Median and Range Charts* procedure are similar to those in the *X-Bar and R Charts* procedure. This document will highlight only the differences between the two procedures. For a detailed discussion of the common options, refer to the documentation for *X-Bar and R Charts*.

Sample StatFolio: *medianrchart.sgp*

Sample Data:

The file *wafers.sgd* contains measurements made on the flow width of wafers, taken from Montgomery (2005). The data consist of $m = 45$ samples of 5 wafers each. The table below shows a partial list of the data in that file:

Sample	X1	X2	X3	X4	X5
1	1.3235	1.4128	1.6744	1.4573	1.6914
2	1.4314	1.3592	1.6075	1.4666	1.6109
3	1.4284	1.4871	1.4932	1.4324	1.5674
4	1.5028	1.6352	1.3841	1.2831	1.5507
5	1.5604	1.2735	1.5265	1.4363	1.6441
6	1.5955	1.5451	1.3574	1.3281	1.4198
7	1.6274	1.5064	1.8366	1.4177	1.5144
8	1.419	1.4303	1.6637	1.6067	1.5519
9	1.3884	1.7277	1.5355	1.5176	1.3688
10	1.4039	1.6697	1.5089	1.4627	1.522
11	1.4158	1.7667	1.4278	1.5928	1.4181
12	1.5821	1.3355	1.5777	1.3908	1.7559

STATGRAPHICS refers to each row of the file as a *subgroup*. The subgroup size $n = 5$.

The first 25 rows of the file will be used in a Phase 1 study to establish control limits for the process.

Data Input

In entering data for this procedure, you may enter either:

1. the original measurements.
2. the subgroup means and ranges.

Case #1: Entering Original Measurements
In this case, the data to be analyzed consist of the original measurements taken on a single variable.

- **Observations:** one or more numeric columns. If more than one column is entered, each row of the file is assumed to represent a subgroup with subgroup size *n* equal to the number of columns entered. If only one column is entered, then the *Date/Time/Labels or Size* field is used to form the groups.
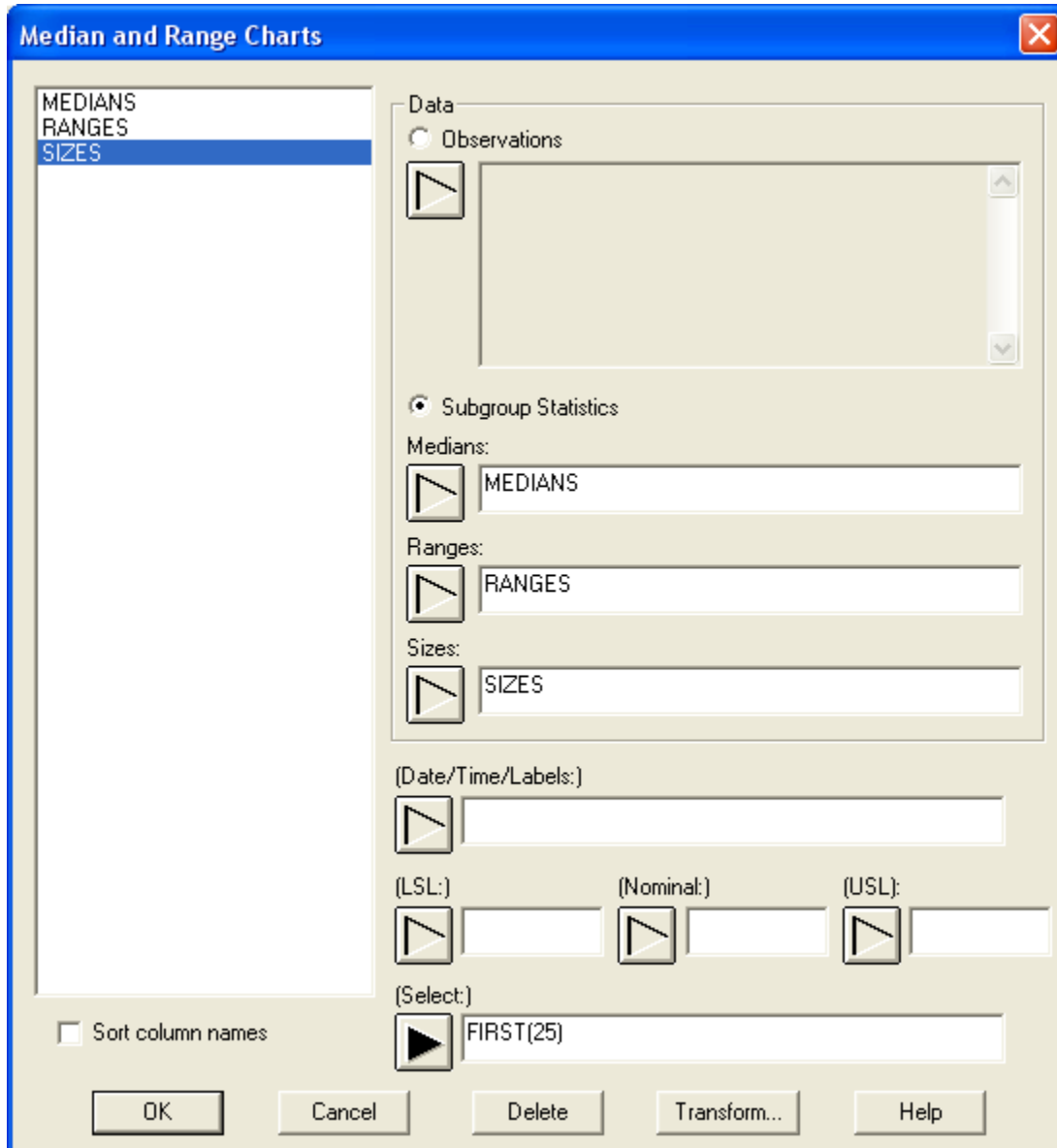
- **Date/Time/Labels or Size**: If each set of *n* rows represents a group, enter the single value *n*. For example, entering a 5 implies that the data in rows 1-5 form the first group, rows 6-10 form the second group, and so on. If the subgroup sizes are not equal, enter the name of an additional numeric or non-numeric column containing group identifiers. The program will scan this column and place sequential rows with identical codes into the same group.

- **LSL, Nominal, USL**: optional lower specification limit, nominal (target) value, and upper specification limit.

- **Select:** subset selection.

Note the use of the FIRST operator to select only the first *m* = 25 rows for the Phase 1 analysis.

Case #2: Entering Subgroup Statistics
In this case, the statistics for each subgroup have been computed elsewhere and entered into the datasheet, as in the table below:
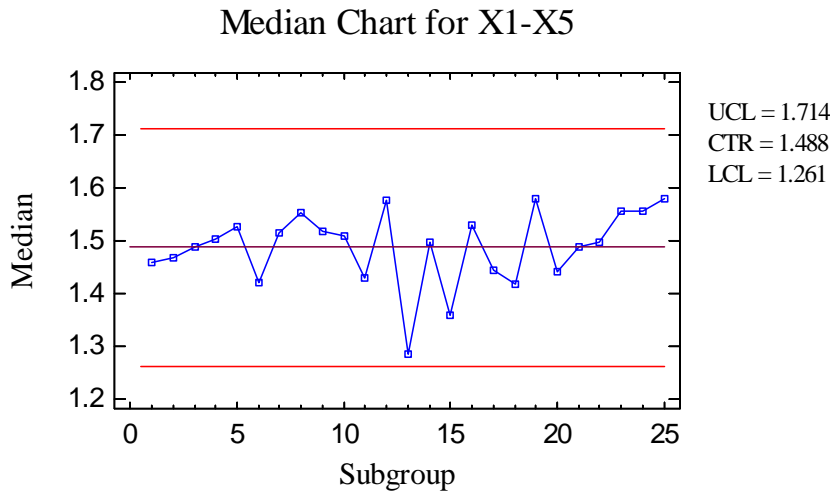
Sample	Medians	Ranges	Sizes
1	1.4573	0.3679	5
2	1.4666	0.2517	5
3	1.4871	0.139	5
4	1.5028	0.3521	5
5	1.5265	0.3706	5
6	1.4198	0.2674	5
7	1.5144	0.4189	5
8	1.5519	0.2447	5
9	1.5176	0.3589	5
10	1.5089	0.2658	5
11	1.4278	0.3509	5
12	1.5777	0.4204	5

- **Subgroup Statistics**: the names of the column containing the subgroup medians, subgroup ranges, and subgroup sizes.

- **Date/Time/Labels:** optional labels for each subgroup.

- **LSL, Nominal, USL**: optional lower specification limit, nominal (target) value, and upper specification limit.

- **Select:** subset selection.

Median Chart

This chart plots the subgroup medians $\tilde{x}_j$.

Median Chart for X1-X5

In Phase 1 (*Initial Studies*) mode, the centerline and control limits are determined from the data. The centerline is located at the weighted average of the subgroup medians:

$$CL = \frac{\sum_{j=1}^{m} n_j \tilde{x}_j}{\sum_{j=1}^{m} n_j} \qquad (1)$$

The control limits are placed above and below the centerline at:

$$CL \pm kf\, \frac{\hat{\sigma}}{\sqrt{n}} \qquad (2)$$

where *k* is the sigma multiple specified on the *Control Charts* tab of the *Preferences* dialog box (*k* = 3 except in rare cases), $\hat{\sigma}$ is the estimate of the process sigma, *n* is the subgroup size, and *f* is the ratio of the standard error of the subgroup medians divided by the standard error of the subgroup means. If the subgroup sizes are not equal, then depending on *Analysis Options*, *n* is replaced by either:
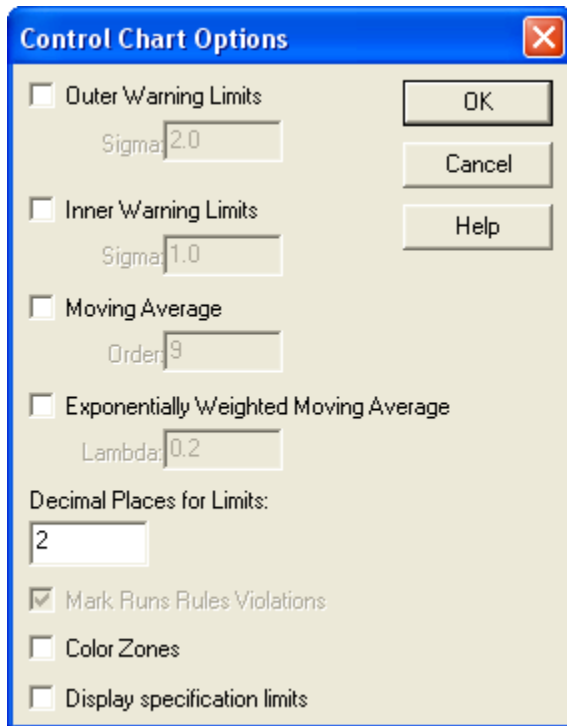
    (1) $\bar{n}$, the average subgroup size. In this case, the control limits are the same for all subgroups.

    (2) n_j, the individual subgroup sizes. In this case, the control limits are step functions.

The method for estimating the process sigma also depends on the settings on the *Control Charts* tab of the *Preferences* dialog box, as discussed in the *Analysis Summary* section below.

Any points beyond the control limits will be flagged using a special point symbol. Any point excluded from the analysis, usually by clicking on a point on the chart and pressing the

Exclude/Include button, will be indicated by an X. If so indicated on the *Pane Options* dialog box, unusual sequences of points may also be flagged. In the current chart, no unusual points or out-of-control signals are indicated.
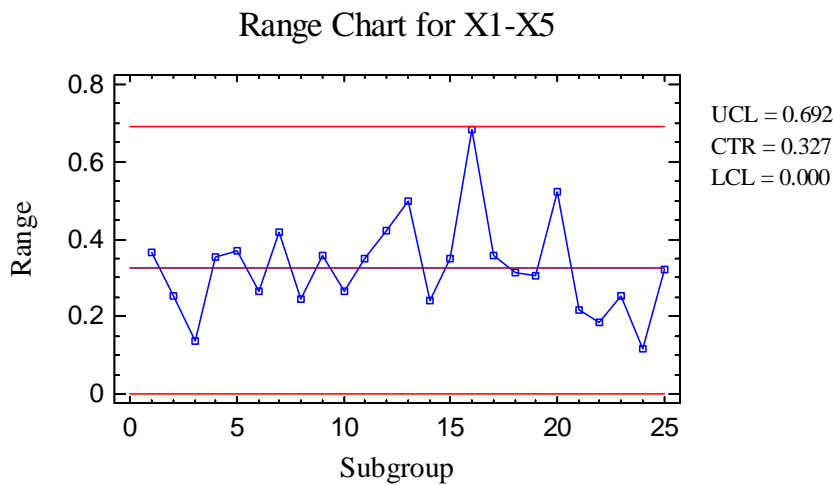
Pane Options

- **Outer Warning Limits**: check this box to add warning limits at the specified multiple of sigma, usually at 2 sigma.

- **Inner Warning Limits**: check this box to add warning limits at the specified multiple of sigma, usually at 1 sigma.

- **Moving Average:** check this box to add a moving average smoother to the chart. In addition to the subgroup means, the average of the most recent q points will also be displayed, where q is the *order* of the moving average. The default value $q = 9$ since the 1-sigma inner warning limits for the original subgroup means are equivalent to the 3-sigma control limits for that order moving average.

- **Exponentially Weighted Moving Average:** check this box to add an EWMA smoother to the chart. In addition to the subgroup means, an exponentially weighted moving average of the subgroup means will also be displayed, where λ is the smoothing parameter of the EWMA. The default value $\lambda = 0.2$ since the 1-sigma inner warning limits for the original subgroup means are equivalent to the 3-sigma control limits for that EWMA.

- **Decimal Places for Limits:** the number of decimal places used to display the control limits.

- **Mark Runs Rules Violations:** flags with a special point symbol any unusual sequences or runs. The runs rules applied by default are specified on the *Runs Tests* tab of the *Preferences* dialog box.

- **Color Zones:** check this box to display green, yellow and red zones.

- **Display Specification Limits**: whether to add horizontal lines to the chart displaying the location of the specification limits (if any).

R Chart

This chart the subgroup ranges R_j.

Range Chart for X1-X5

UCL = 0.692
CTR = 0.327
LCL = 0.000

In Phase 1 (*Initial Studies*) mode, the centerline and control limits are determined from the data. The centerline is located at:

$$CL = d_2(\bar{n})\hat{\sigma} \qquad (3)$$

If sigma is estimated from the average range, this equals $\bar{R}$. The control limits are placed above and below the centerline at the following locations:

$$CL \pm d_3(n)\hat{\sigma} \qquad (4)$$

where *k* is the sigma multiple specified on the *Control Charts* tab of the *Preferences* dialog box (*k* = 3 except in rare cases), $\hat{\sigma}$ is the estimate of the process sigma, and *n* is the subgroup size. If the subgroup sizes are not equal, *Analysis Options* specifies whether to use the average subgroup size or the individual subgroup sizes.

The R chart for the sample data shows no unusual signals.

Pane Options
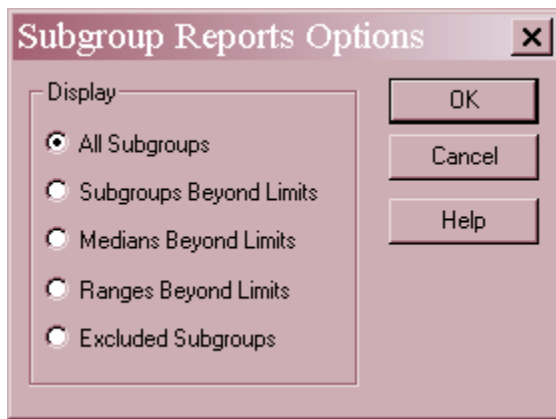The same options exist as for the Median chart.

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Subgroup Reports

This pane tabulates the values plotted on the control charts:

Subgroup Reports
All Subgroups
X = Excluded   * = Beyond Limits

Subgroup	Size	Median	Range
1	5	1.4573	0.3679
2	5	1.4666	0.2517
3	5	1.4871	0.139
4	5	1.5028	0.3521
5	5	1.5265	0.3706
6	5	1.4198	0.2674
7	5	1.5144	0.4189
8	5	1.5519	0.2447
9	5	1.5176	0.3589
10	5	1.5089	0.2658
11	5	1.4278	0.3509
12	5	1.5777	0.4204
13	5	1.2856	0.4992
14	5	1.4969	0.2422
15	5	1.3589	0.3499
16	5	1.5301	0.6823
17	5	1.4449	0.3589
18	5	1.4163	0.3153
19	5	1.5796	0.3062
20	5	1.4412	0.524
21	5	1.488	0.2185
22	5	1.4973	0.1863
23	5	1.5551	0.2533
24	5	1.5563	0.1156
25	5	1.5797	0.3224

Out-of-control points are indicated by an asterisk. Points excluded from the calculations are indicated by an X.

Pane Options

- **Display:** specify the subgroups to display in the report.

Analysis Summary

The *Analysis Summary* summarizes the data and the control charts.

Median and Range Charts - X1-X5 (FIRST(25))
Selection variable: FIRST(25)
Number of subgroups = 25
Subgroup size = 5.0
0 subgroups excluded

Distribution: Normal
Transformation: none

Median Chart

Period	#1-25
UCL: +3.0 sigma	1.71362
Centerline	1.48753
LCL: -3.0 sigma	1.26145

0 beyond limits

Range Chart

Period	#1-25
UCL: +3.0 sigma	0.692064
Centerline	0.327296
LCL: -3.0 sigma	0.0

0 beyond limits

Estimates

Period	#1-25
Process mean	1.48753
Process sigma	0.140712
Average range	0.327296

Sigma estimated from average range

Included in the table are:

- **Subgroup Information**: the number of subgroups *m* and the average subgroup size

$$\bar{n} = \frac{\sum_{j=1}^{m} n_j}{m} \qquad (9)$$

  If any subgroups have been excluded from the calculations, that number is also displayed.

- **Distribution:** the assumed distribution for the data. By default, the data are assumed to follow a normal distribution. However, one of 26 other distributions may be selected using *Analysis Options*.

- **Transformation:** any transformation that has been applied to the data. Using *Analysis Options*, you may elect to transform the data using either a common transformation such as a square root or optimize the transformation using the Box-Cox method.

- **Median Chart:** a summary of the centerline and control limits for the median chart, which plots the subgroup medians $\tilde{x}_j$. Using *Analysis Options*, separate limits may be calculated for different periods (sets of subgroups).
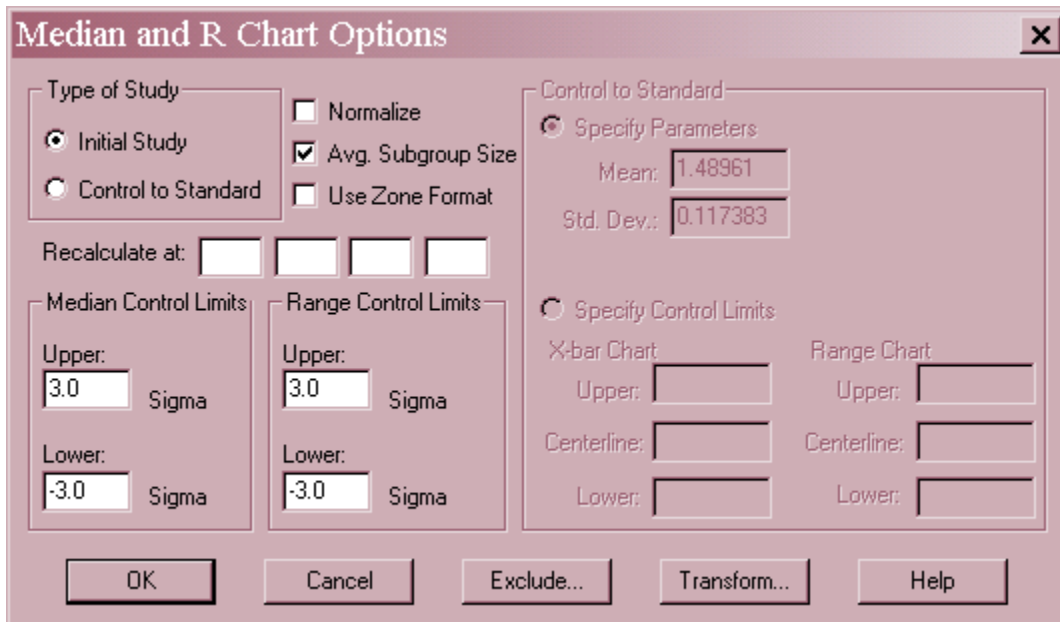
- **R Chart:** a summary of the centerline and control limits for the R chart, which plots the subgroup ranges R_j.

- **Estimates:** estimates of the process mean μ and the process standard deviation σ. The process mean is estimated from the weighted average of the subgroup medians. The process sigma may be estimated in any of 3 ways, depending upon the settings on the *Control Charts* tab of the *Preferences* dialog box, accessible through the *Edit* menu. There are 3 options:

   (1) *From average range*: the process sigma is estimated from a weighted average of the subgroup ranges. This is the estimator used in most SPC textbooks.
   (2) *From pooled s with no bias correction*: the process sigma is estimated from the pooled within group variance, as in a oneway ANOVA. In this case, s^2 is an unbiased estimate of σ^2 but s is a biased estimator for σ.
   (3) *From pooled s with bias correction*: the process sigma is estimated from the pooled within group variance and then multiplied by a factor that makes the result an unbiased estimate of σ.

- **Average Range:** the average of the subgroup ranges:

$$\overline{R} = \frac{\sum_{j=1}^{m} R_j}{m} \tag{10}$$

Analysis Options

- **Type of Study:** determines how the control limits are set. For an *Initial Study* (Phase 1) chart, the limits are estimated from the current data. For a *Control to Standard* (Phase 2)

chart, the control limits are determined from the information in the *Control to Standard* section of the dialog box.

- **Normalize:** if selected, all statistics will be normalized by calculating Z-scores and the Z-scores plotted on the charts. The centerline on such a chart is always located at 0, and the control limits are always located at $\pm k$.

- **Avg. Subgroup Size:** if checked, the control limits will be horizontal lines based on the average subgroup size. If not checked, the individual subgroup sizes will be used, resulting in step function control limits if the subgroups sizes are not all equal.

- **Use Zone Format:** if checked, the statistics will be plotted using a zone chart rather than the usual format. See the example below.

- **Recalculate at:** the control limits of the chart may be recalculated at up to 4 locations along the X axis by specifying the subgroup numbers at which new estimates are to be initiated. Separate estimates of the process mean and sigma will be obtained using the data in the different sections. In such cases, the control limits will be adjusted at the start of each new period.

- **Median Control Limits:** specify the multiple k to use in determining the upper and lower control limits on the median chart. To suppress a limit completely, enter 0.

- **Range Control Limits:** specify the multiple k to use in determining the upper and lower control limits on the R chart. To suppress a limit completely, enter 0.

- **Control to Standard:** to perform a Phase 2 analysis, select *Control to Standard* for the *Type of Study* and then enter either of the following:

  Specify Parameters: specify the established standard process mean and sigma (or other parameters if not assuming a normal distribution). These values will then be used to position the centerlines and control limits on the charts.

  Specify Control Limits: specify the location of the centerlines and control limits exactly where you wish them to be placed.

- **Exclude button:** Use this button to exclude specific subgroups from the calculations.

- **Transform Button**: Use this button to specify a transformation or non-normal distribution.

For examples of the use of transformations and non-normal distributions, see the documentation for the *Individuals Control Charts*.

Save Results

The following results can be saved to the datasheet:

1. *Medians* – the subgroup medians.
2. *Ranges* – the subgroup ranges.
3. *Sizes* – the subgroup sizes.
4. *Labels* – the subgroup labels.
5. *Process Mean* – the estimated process mean.
6. *Process Sigma* – the estimated process standard deviation.