

Moving Average (MA) Charts

Summary

The **Moving Average Charts** procedure creates control charts for a single numeric variable where the data have been collected either individually or in subgroups. In contrast to traditional X-Bar or X charts, a point plotted on the MA chart represents not just the last observation or subgroup but a moving average of current and past data. Such charts are very useful in Phase 2 monitoring of processes against established control limits, since they have a shorter average run length in detecting small process shifts.

The procedure creates both an MA chart and an R chart, S chart, or MR(2) chart. The charts may be constructed in either *Initial Study* (Phase 1) mode, where the current data determine the control limits, or in *Control to Standard* (Phase 2) mode, where the limits come from either a known standard or from prior data.

Sample StatFolio: *machart.sgp*

Sample Data:

The file *process shift.sgd* contains a sample of random numbers described by Montgomery (2005). The data consist of $m = 30$ observations, to be treated as individuals. A partial list of the data in that file is shown below:

<i>Sample</i>	<i>X</i>
1	9.45
2	7.99
3	9.29
4	11.66
5	12.16
6	10.18
7	8.04
8	11.46
9	9.20
10	10.34

The first 20 observations were randomly generated from a normal distribution with $\mu = 10$ and $\sigma = 1$. The last 10 observations were randomly generated from a normal distribution with $\mu = 11$ and $\sigma = 1$, representing a 1-sigma shift in the process mean. Montgomery uses this example to illustrate the properties of various types of time-weighted charts.

Data Input

There are two menu selections that create moving average charts, one for individuals data and one for grouped data. In the case of grouped data, the original observations may be entered, or subgroup statistics may be entered instead.

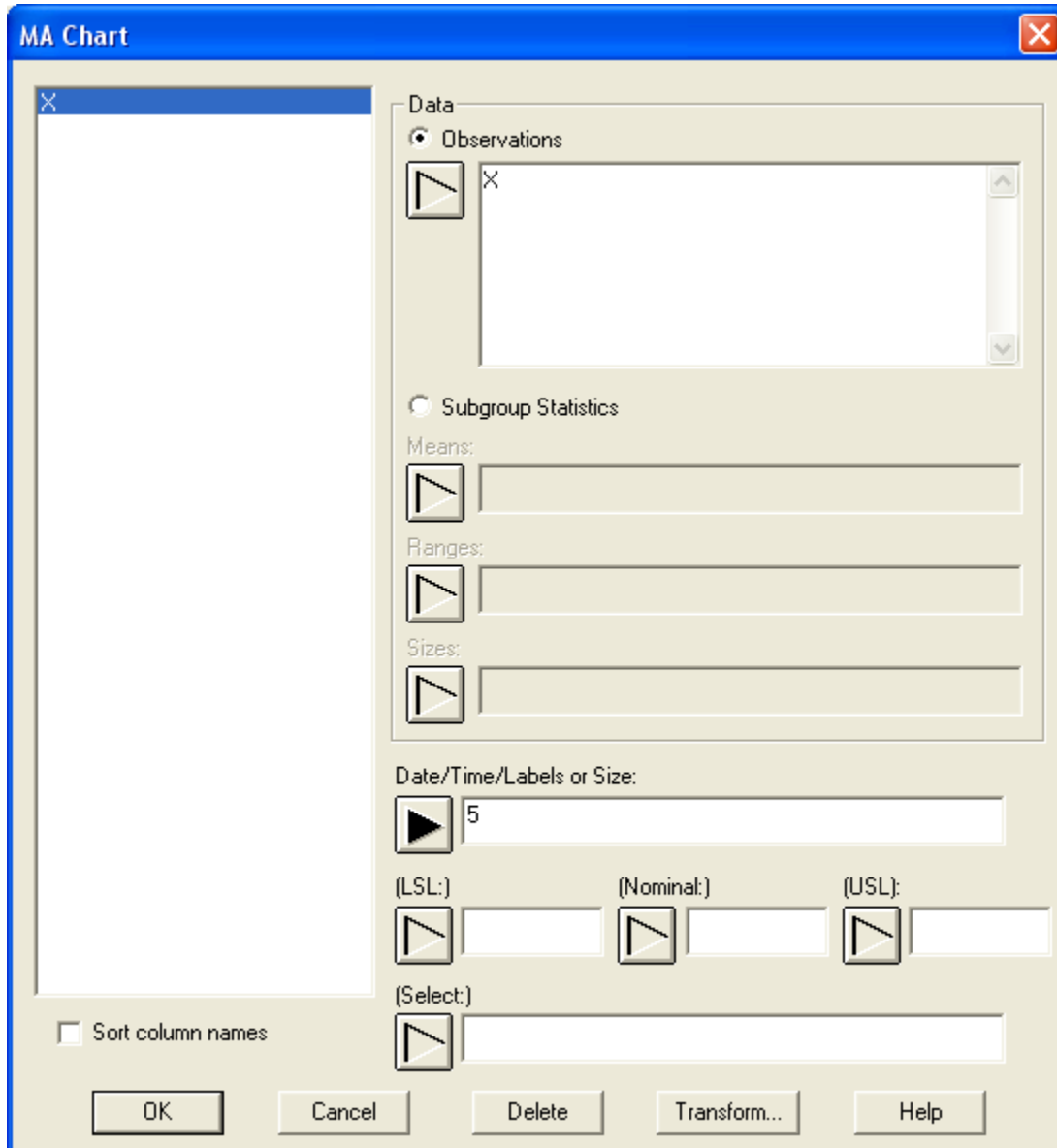
Case #1: Individuals

The data to be analyzed consist of a single numeric column containing n observations. The data are assumed to have been taken one at a time.

- **Observations:** numeric column containing the data to be analyzed.
- **Date/Time/Labels:** optional labels for each observation.
- **LSL, Nominal, USL:** optional lower specification limit, nominal (target) value, and upper specification limit.
- **Select:** subset selection.

Case #2: Grouped Data – Original Observations

The data to be analyzed consist of one or more numeric columns. The data are assumed to have been taken in groups, in sequential order by rows.

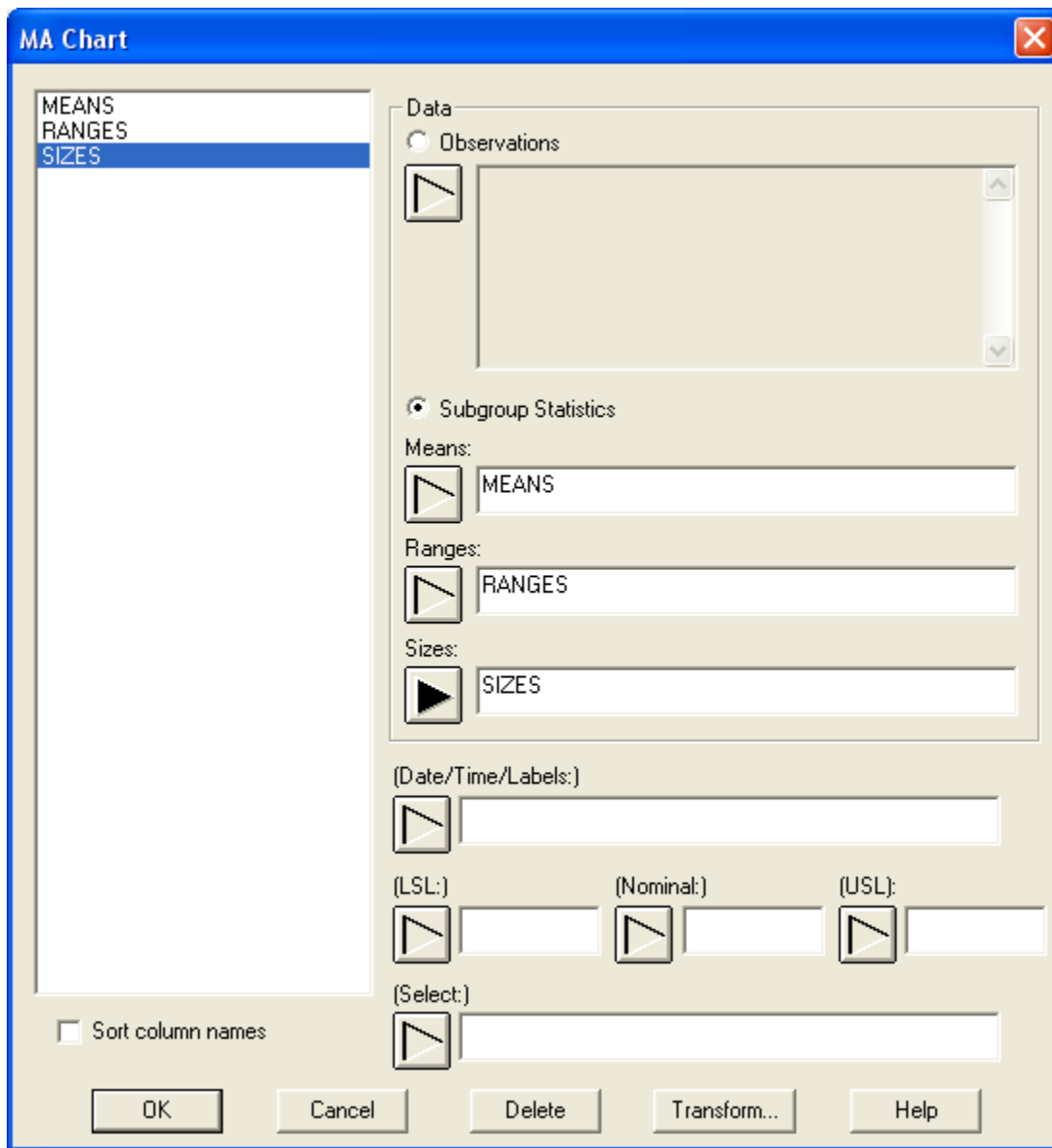


- **Observations:** one or more numeric columns. If more than one column is entered, each row of the file is assumed to represent a subgroup with subgroup size m equal to the number of columns entered. If only one column is entered, then the *Date/Time/Labels or Size* field is used to form the groups.
- **Date/Time/Labels or Size:** If each set of m rows represents a group, enter the single value m . For example, entering a 5 as in the example above implies that the data in rows 1-5 form the first group, rows 6-10 form the second group, and so on. If the subgroup sizes are not equal, enter the name of an additional numeric or non-numeric column containing group identifiers. The program will scan this column and place sequential rows with identical codes into the same group.
- **LSL, Nominal, USL:** optional lower specification limit, nominal (target) value, and upper specification limit.
- **Select:** subset selection.

Case #3: Grouped Data – Subgroup Statistics

In this case, the statistics for each subgroup have been computed elsewhere and entered into the datasheet, as in the table below:

<i>Sample</i>	<i>Means</i>	<i>Ranges</i>	<i>Sizes</i>
1	10.11	4.17	5
2	9.844	3.42	5
3	10.098	2.44	5
4	9.932	2.32	5
5	10.924	2.96	5

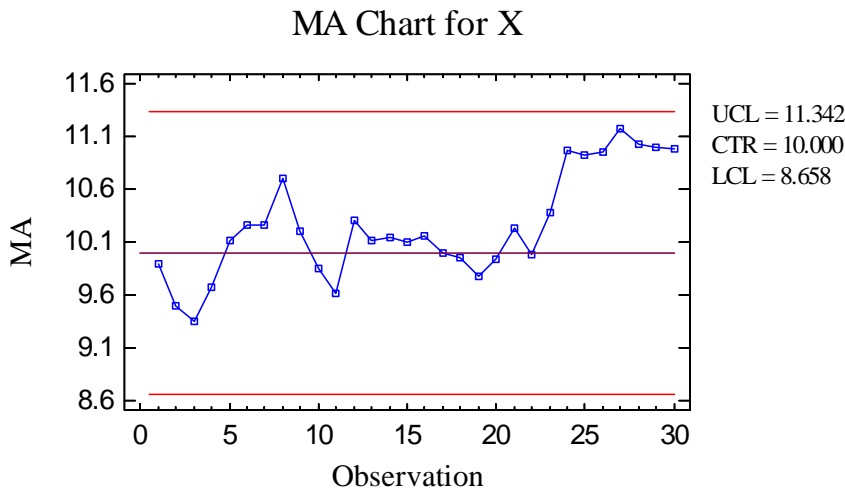


- **Subgroup Statistics:** the names of the column containing the subgroup means, subgroup ranges, and subgroup sizes.

- **Date/Time/Labels:** optional labels for each subgroup.
- **LSL, Nominal, USL:** optional lower specification limit, nominal (target) value, and upper specification limit.
- **Select:** subset selection.

MA Chart

This chart plots the average of the last *s* observations or subgroups, where *s* is called the *order* or *span* of the moving average.



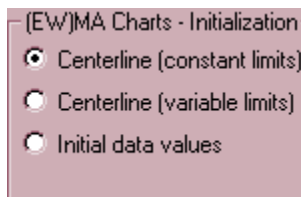
The moving average at location *j* is defined by:

$$MA_j = \frac{\sum_{i=j-s+1}^j x_i}{s} \tag{1}$$

for individuals data or by

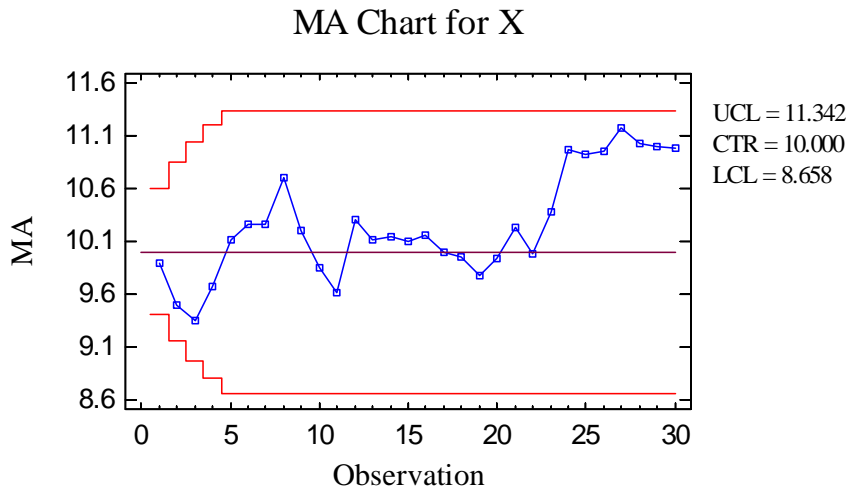
$$MA_j = \frac{\sum_{i=j-s+1}^j \bar{x}_i}{s} \tag{2}$$

for data collected in subgroups. For $j < s$, an assumption must be made about the initial values needed to compute the moving average. The *Control Charts* tab of the *Preferences* dialog box, accessible from the *Edit* menu, provides 3 choices:

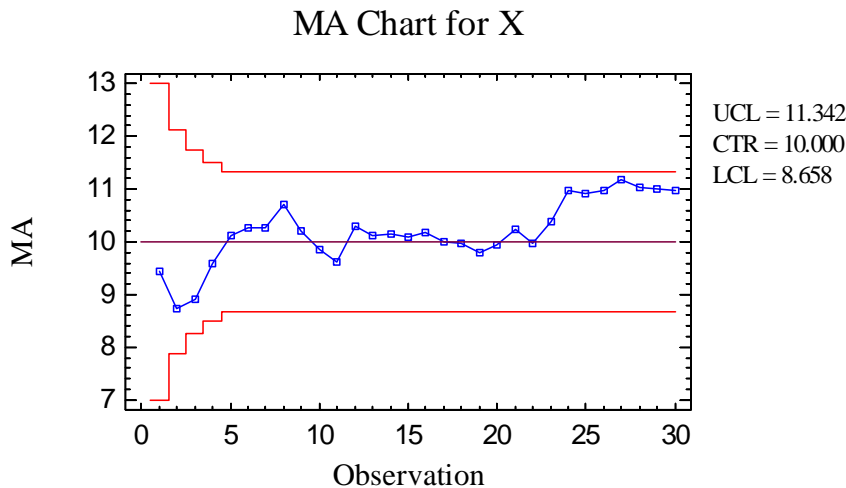


- (1) **Centerline (constant limits):** Values of x_j or \bar{x}_j for $j < 1$ are set equal to the centerline. Although the initial values of the moving average then vary less about the centerline than for $j \geq s$, this decreased variability is ignored when plotting the control limits so that they are horizontal lines. This is the format of the chart shown above.
- (2) **Centerline (variable limits):** Values of x_j or \bar{x}_j for $j < 1$ are set equal to the centerline. The control limits are drawn as step functions, reflecting the decreased variability of the moving averages for $j < s$.
- (3) **Initial data values:** Values of x_j or \bar{x}_j for $j < 1$ are set equal x_1 or \bar{x}_1 . The control limits are drawn as step functions, reflecting the increased variability of the moving averages for $j < s$.

Example: Initializing at Centerline with Variable Limits



Example: Initializing using First Data Value



In Phase 1 (*Initial Studies*) mode, the centerline and control limits are determined from the data. The centerline is located at the average of the subgroup means or observations:

$$\bar{x} = \frac{\sum_{j=1}^m \bar{x}_j}{m} \tag{3}$$

The control limits are placed above and below the centerline at:

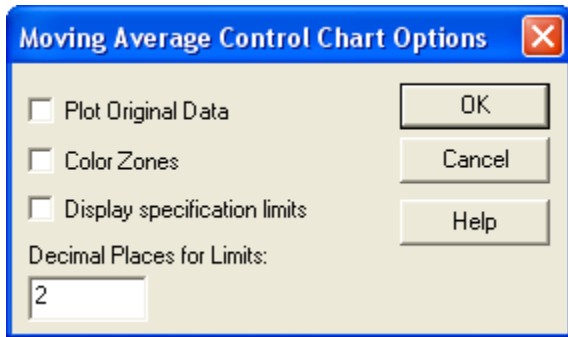
$$\bar{x} \pm k \frac{\hat{\sigma}}{\sqrt{ns}} \tag{4}$$

where k is the sigma multiple specified on the *Control Charts* tab of the *Preferences* dialog box ($k = 3$ except in rare cases), n is the subgroup size, and $\hat{\sigma}$ is the estimate of the process sigma.

The method for estimating the process sigma also depends on the settings on the *Control Charts* tab of the *Preferences* dialog box, as discussed in the *Analysis Summary* section below.

Any points beyond the control limits will be flagged using a special point symbol. Any point excluded from the analysis, usually by clicking on a point on the chart and pressing the *Exclude/Include* button, will be indicated by an X. In the current chart, no out-of-control signals are indicated.

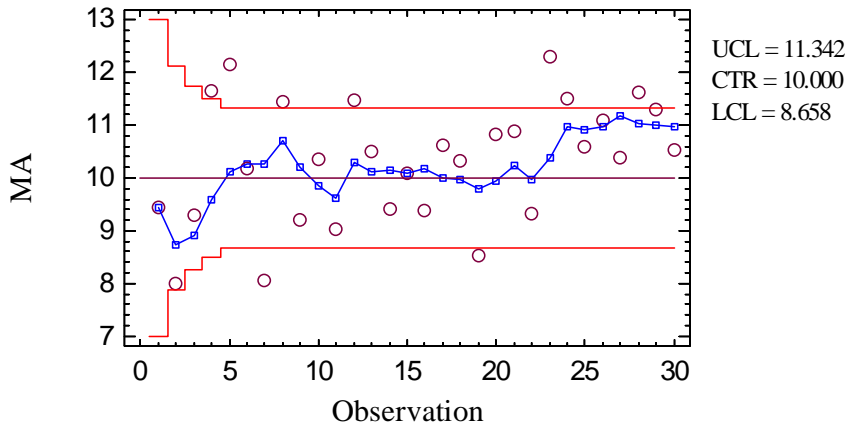
Pane Options



- **Plot Original Data:** check this box to plot the original observations or subgroups in addition to the moving averages.
- **Decimal Places for Limits:** the number of decimal places used to display the control limits.
- **Color Zones:** check this box to display green, yellow and red zones.
- **Display Specification Limits:** whether to add horizontal lines to the chart displaying the location of the specification limits (if any).

Example: Plotting Original Data

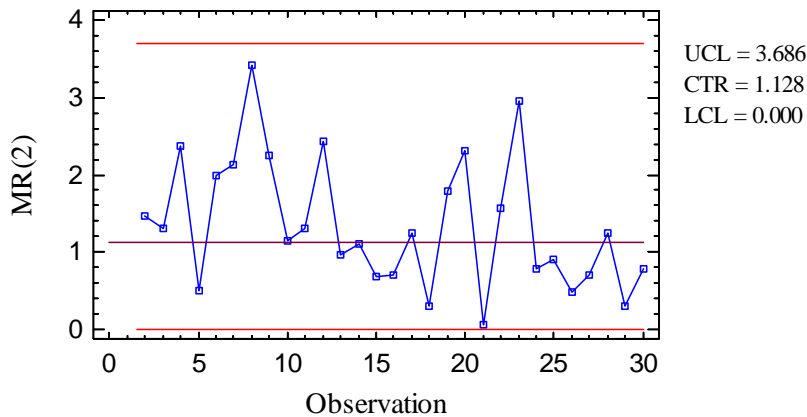
MA Chart for X



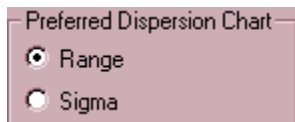
MR(2)/R/S Chart

A second chart is also included to monitor the process variability.

MR(2) Chart for X



For individuals data, the chart displayed is an MR(2) chart, described in the *Individuals Control Charts* documentation. For grouped data, either an R chart or an S chart is plotted, depending on the setting on the *Control Charts* tab of the *Preferences* dialog box:



These charts are described in the *X-Bar and R Charts* and the *X-Bar and S Charts* documents.

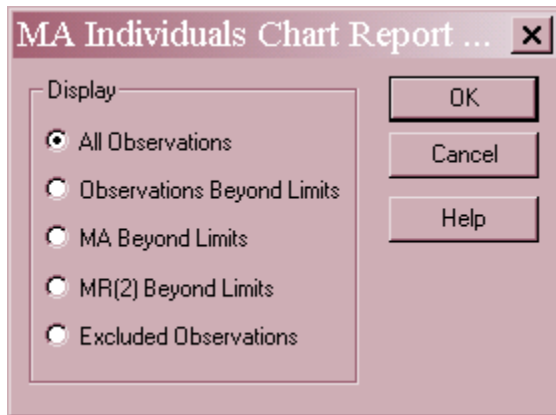
MA Chart Report

This pane tabulates the values plotted on the control charts:

MA Individuals Chart Report		
All Observations		
X = Excluded * = Beyond Limits		
Observation	MA	MR(2)
1	9.45	
2	8.72	1.46
3	8.91	1.3
4	9.5975	2.37
5	10.11	0.5
6	10.256	1.98
7	10.266	2.14
8	10.7	3.42
9	10.208	2.26
10	9.844	1.14

Out-of-control points are indicated by an asterisk. Points excluded from the calculations are indicated by an X.

Pane Options



- **Display:** specify the observations or subgroups to display in the report.

Analysis Summary

The *Analysis Summary* summarizes the data and the control charts.

<u>Moving Average Individuals Chart - X</u>		
Number of observations = 30		
0 observations excluded		
Distribution: Normal		
Transformation: none		
<u>Moving Average Chart - Order = 5</u>		
<i>Period</i>	#1-30	
UCL: +3.0 sigma	11.3416	
Centerline	10.0	
LCL: -3.0 sigma	8.65836	
0 beyond limits		
<u>MR(2) Chart</u>		
<i>Period</i>	#1-30	
UCL: +3.0 sigma	3.6855	
Centerline	1.128	
LCL: -3.0 sigma	0.0	
0 beyond limits		
<u>Estimates</u>		
<i>Period</i>	<i>#1-30</i>	<i>Standard</i>
Process mean	10.315	10.0
Process sigma	1.19987	1.0
Mean MR(2)	1.35345	
Sigma estimated from average moving range		

Included in the table are:

- Subgroup Information:** the number of observations or subgroups m and the subgroup size n (if not individuals). If any observations or subgroups have been excluded from the calculations, that number is also displayed.
- Distribution:** the assumed distribution for the data. By default, the data are assumed to follow a normal distribution. However, one of 26 other distributions may be selected using *Analysis Options*.
- Transformation:** any transformation that has been applied to the data. Using *Analysis Options*, you may elect to transform the data using either a common transformation such as a square root or optimize the transformation using the Box-Cox method.
- Moving Average Chart:** a summary of the centerline and control limits for the MA chart.
- MR(2)/R/S Chart:** a summary of the centerline and control limits for the dispersion chart.
- Estimates:** estimates of the process mean μ and the process standard deviation σ . The methods for estimating the process sigma depending upon the settings on the *Control*

Charts tab of the *Preferences* dialog box, accessible through the *Edit* menu. The options are described in the *Individuals Control Charts* and *X-bar and R Charts* documentation.

- **Mean MR(2), Mean Range, or Mean S:** the average of the values plotted on the dispersion chart.

Analysis Options

The screenshot shows the 'MA Individuals Chart Options' dialog box. It is divided into several sections:

- Type of Study:** Radio buttons for 'Initial Study' and 'Control to Standard' (selected).
- Order:** A text box containing the value '5'.
- Initialization:** Radio buttons for 'Centerline (constant limits)', 'Centerline (variable limits)', and 'Initial data values' (selected).
- ARL at 1:** A text box containing '1', with 'sigma = 12.2' displayed next to it.
- MA Control Limits:** Text boxes for 'Upper:' (3.0) and 'Lower:' (-3.0), each followed by 'Sigma'.
- MR(2) Control Limits:** Text boxes for 'Upper:' (3.0) and 'Lower:' (-3.0), each followed by 'Sigma'.
- Control to Standard Specify Parameters:** Text boxes for 'Mean:' (10.0) and 'Std. Dev.:' (1.0).
- Buttons:** OK, Cancel, Exclude..., Transform..., and Help.

- **Type of Study:** determines how the control limits are set. For an *Initial Study* (Phase 1) chart, the limits are estimated from the current data. For a *Control to Standard* (Phase 2) chart, the control limits are determined from the information in the *Control to Standard* section of the dialog box.
- **Order:** the order or span s of the moving average. The higher the order, the more influence past data has on the plotted values. The value of s is often selected so as to achieve a desired sensitivity or average run length in the presence of a small process shift.
- **ARL:** the approximate average run length for an MA chart with the selected order if the process mean shifts by the indicated multiple of sigma. If you enter a different multiple, the ARL will be updated instantaneously.
- **Initialization:** method for initializing the moving average for $j < s$. If *Centerline (constant limits)*, the values of x prior to $j = 1$ are set equal to the centerline, but the decreased variability is not reflected in the control limits. If *Centerline (variable limits)*, the values of x prior to $j = 1$ are set equal to the centerline and the control limits are tightened to reflect the decreased variability. If *Initial data values*, the values of x prior to $j = 1$ are set equal the value at $j = 1$ and the control limits are loosened to reflect the increased variability.
- **MA Control Limits:** specify the multiple k to use in determining the upper and lower control limits on the MA chart. To suppress a limit completely, enter 0.

- **MR(2) Control Limits:** specify the multiple k to use in determining the upper and lower control limits on the MR(2), R, or S chart. To suppress a limit completely, enter 0.
- **Control to Standard:** to perform a Phase 2 analysis, select *Control to Standard* for the *Type of Study* and then enter the established standard process mean and sigma (or other parameters if not assuming a normal distribution).
- **Exclude button:** Use this button to exclude specific subgroups from the calculations.
- **Transform Button:** Use this button to specify a transformation or non-normal distribution.

For a discussion of the *Exclude* and *Transform* features, see the documentation for *Individuals Control Charts*.

Capability Indices

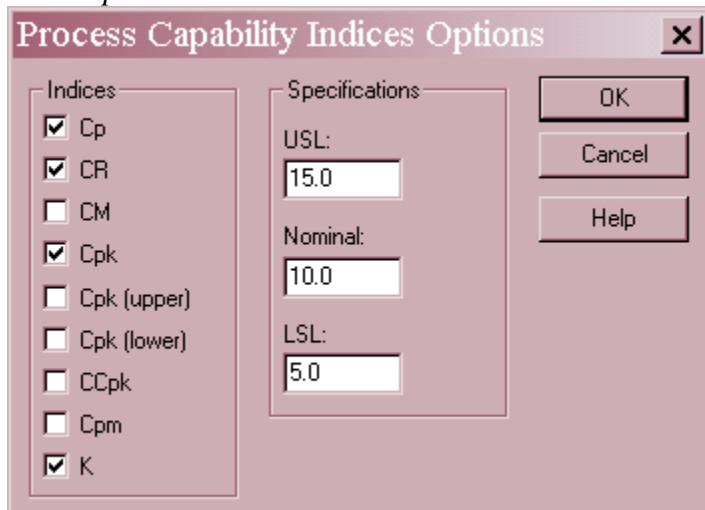
The *Capability Indices* pane displays the values of selected indices that measure how well the data conform to the specification limits.

Capability Indices for X		
Specifications		
USL = 15.0		
Nom = 10.0		
LSL = 5.0		
	<i>Short-Term</i>	<i>Long-Term</i>
	<i>Capability</i>	<i>Performance</i>
Sigma	1.19987	1.15354
Cp/Pp	1.38904	1.44483
CR/PR	71.9919	69.2122
Cpk/Ppk	1.30153	1.35381
K		0.063

Based on 6 sigma limits. Short-term sigma estimated from average moving range.

The indices displayed by default depend on the settings of the *Capability* tab on the *Preferences* dialog box. A detailed discussion of these indices may be found in the documentation for *Process Capability (Variables)*.

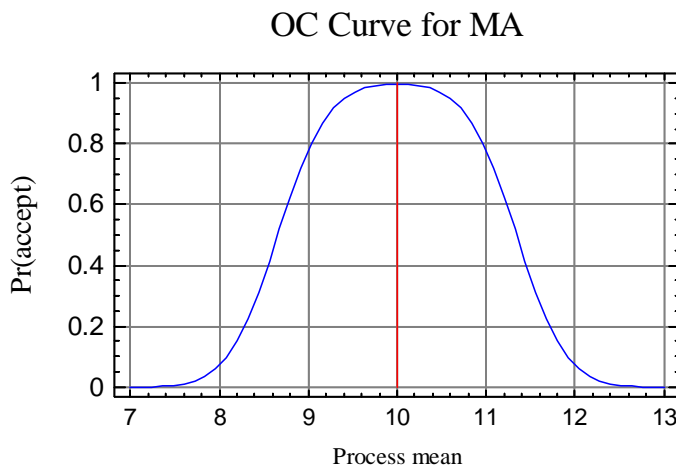
Pane Options



- **Indices:** select the indices to be displayed.
- **Specifications:** the upper specification limit, nominal or target value, and lower specification limit. Any of these entries may be left blank if not relevant.

OC Curve

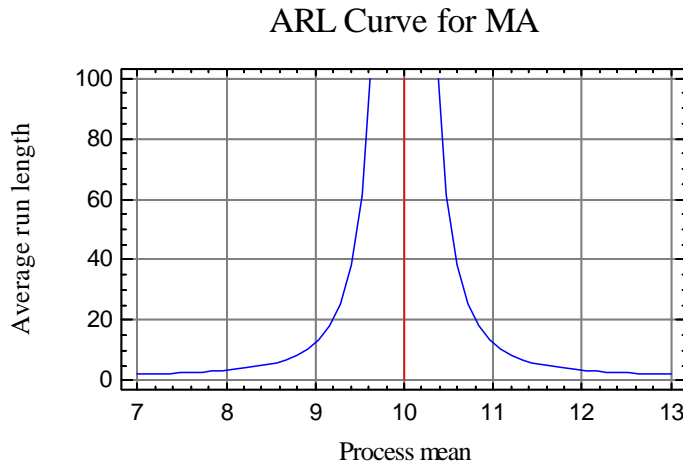
The *OC (Operating Characteristic) Curve* is designed to illustrate the properties of a Phase 2 control chart.



The chart displays the probability that a plotted value will be within the control limits on the MA chart, as a function of the true process mean. For example, if the process mean were to shift to 11, the probability that the next moving average will be within the control limits is approximately 80%. The probability decreases as the order of the moving average *s* increases.

ARL Curve

The *ARL Curve* is another way to view the performance of a Phase 2 chart.



The ARL curve plots the average run length (average number of values plotted up to and including the first point beyond the control limits) as a function of the true process mean. Assuming that the process mean suddenly shifts to a new value, the chart shows how long it takes on average until an out-of-control signal is generated. For very small shifts, it can take in excess of 350 points on average to detect the shift. At a shift to $\mu = 11$, the ARL is approximately 12.

Save Results

The following results can be saved to the datasheet, depending on whether the data are individuals or grouped:

1. *Moving Averages* – the values plotted on the moving average chart.
2. *Ranges, sigmas, or moving ranges* – the values plotted on the dispersion chart.
3. *Sizes* – the subgroup sizes.
4. *Labels* – the subgroup labels.
5. *Process Mean* – the estimated process mean.
6. *Process Sigma* – the estimated process standard deviation.
7. *Included Observations* – a column of 0's and 1's for excluded and included observations, respectively. This column can then be used in the *Select* field on other data input dialog boxes.