Reliability Demonstration Test Plan

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Summary

This procedure creates test plans to demonstrate that a failure time distribution satisfies stated conditions. For example, it may be desired to show with 95% confidence that the reliability of a product equals or exceeds 90% at the end of the warranty period. During the demonstration, *n* units will be tested for a duration equal to *t*. The demonstration will be considered successful if no more than *f* units fail during the test.

The user specifies either the number of units to be tested or the duration of the test. The procedure solves for the other quantity.

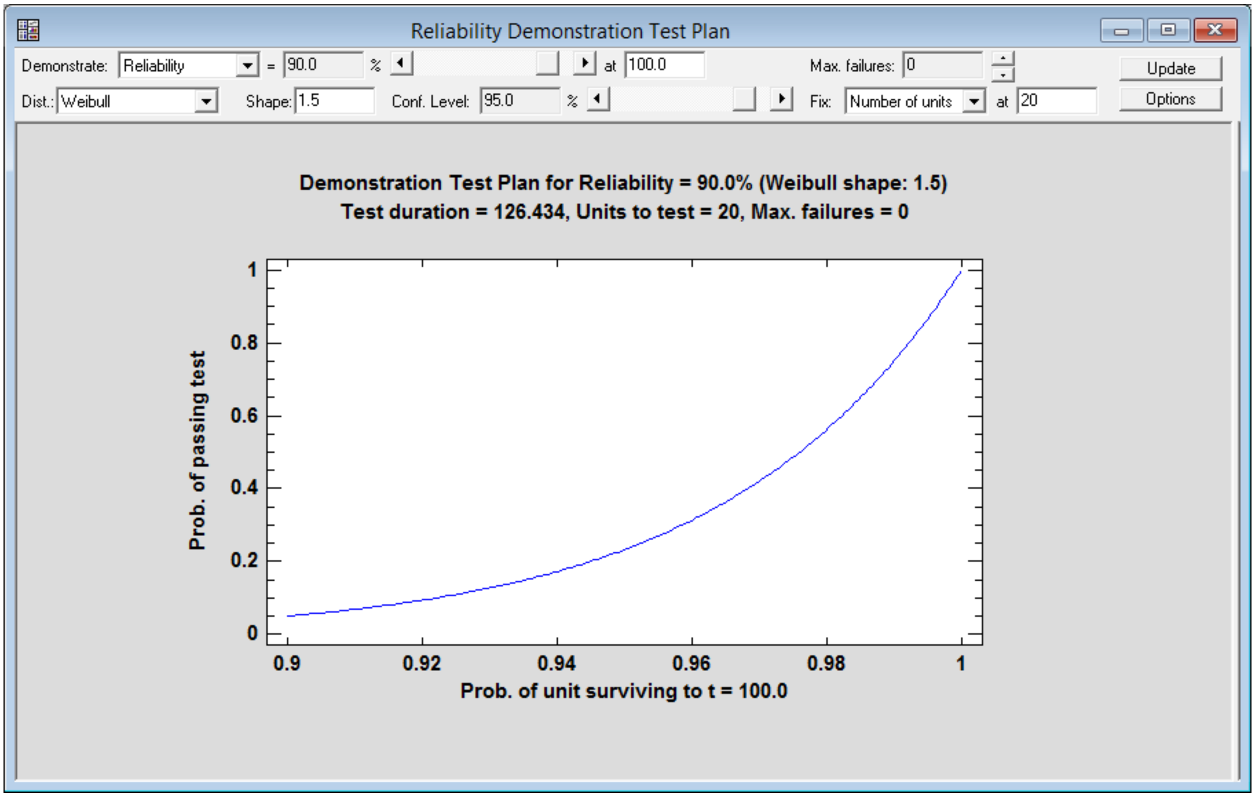
Example

As an example, suppose we wish to solve the following problem:

> "Assuming that the failure times of an item follow a Weibull distribution with a shape parameter equal to 1.5, demonstrate with 95% confidence that the reliability of the item after 1,000 hours equals 90%. Construct a test of duration equal to 250 hours, where the demonstration is deemed successful if none of the test units fail during the test."

Analysis Window

To execute the procedure, select *Describe – Life Data – Reliability Demonstration Test Plan* from the Statgraphics menu. This will display an analysis window similar to that shown below:



The toolbar contains a number of controls that allow you to specify desired options:

- **Demonstrate**: the condition to be demonstrated. There are 4 choices:
    - *Reliability* – the probability that an item has not failed after a duration equal to *t*. Enter the reliability and the value of *t*.
    - *Percentile* - the probability that an item has failed after a duration equal to *t*. Enter the percentile and the value of *t*.
    - *MTTF* – the mean time to failure. Enter the value of the mean.
    - *Parameter* – the unknown parameter of the assumed distribution for failure times (see *parameter to be estimated* below). Enter the value of the parameter.

- **Dist.:** the assumed distribution of failure times and the value of the shape or scale parameter. There are 10 choices:

Distribution	**Specified parameter**	**Parameter to be estimated**
Weibull	Shape α	Scale β
Exponential	None	Mean 1 / λ
Gamma	Shape α	Scale λ
Birnbaum-Saunders	Shape β	Scale θ
Smallest extreme value	Scale β	Mode α
Largest extreme value	Scale β	Mode α
Normal	Standard deviation σ	Mean μ
Lognormal	Standard deviation $\sqrt{e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)}$	Mean $e^{\mu+\sigma^2/2}$
Logistic	Standard deviation σ	Mean μ
Loglogistic	Shape σ	Median exp(μ)

- **Conf. level**: the confidence level for the demonstration.

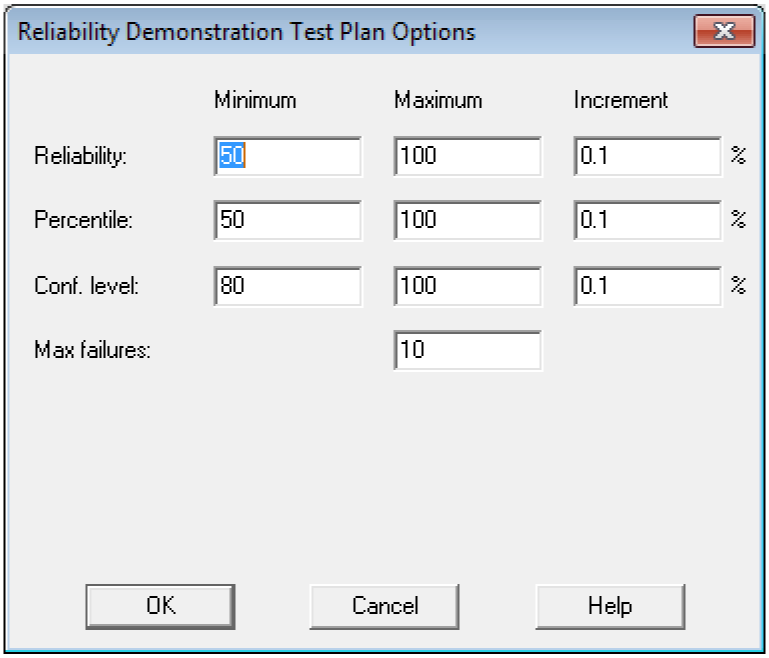
- **Max. failures**: the maximum number of units *f* that are allowed to fail but still result in a successful demonstration.

- **Fix:** the quantity to fix for the text. There are 2 choices:

  - *Number of units*: the number of units *n* to be tested. The procedure will determine the required test duration *t*.

  - *Test duration*: the test duration *t*. The procedure will determine the number of units *n* to be tested.

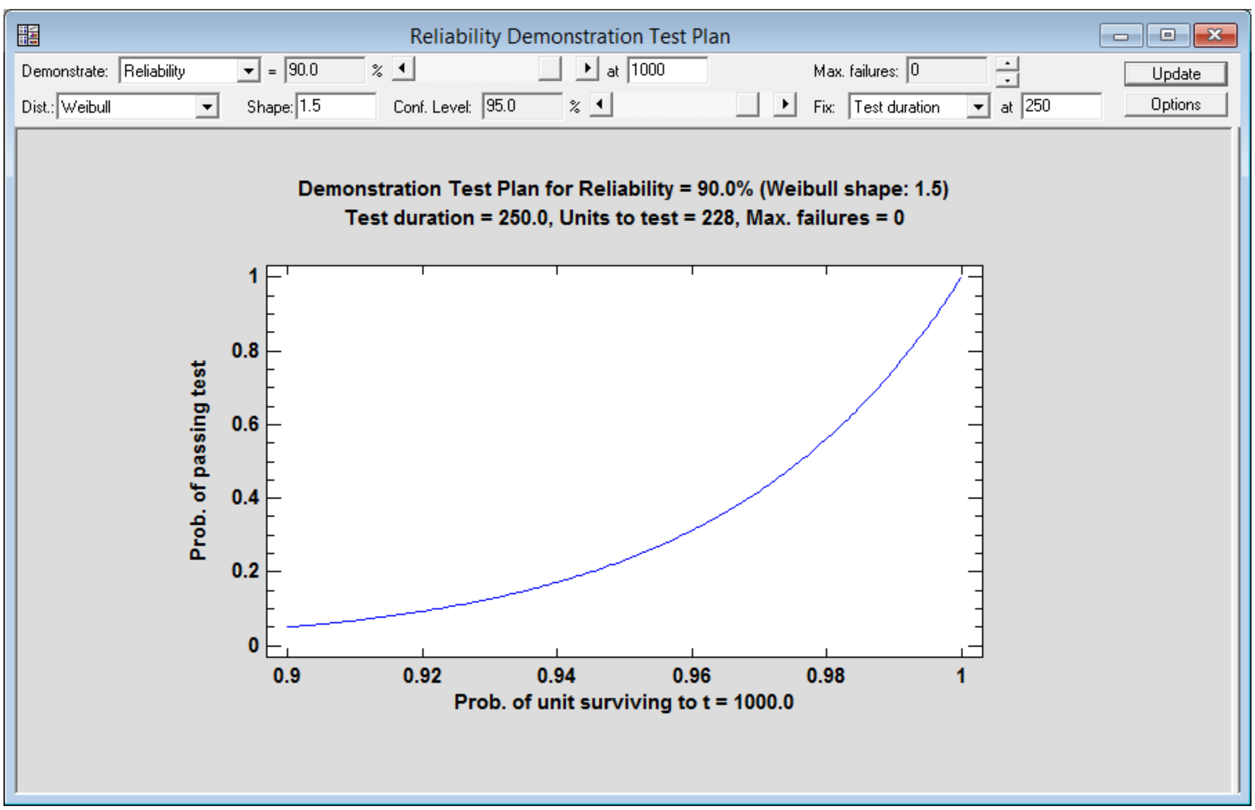
- **Options**: displays a dialog box with additional options:



The *Options* dialog box is used to control the minimum value, maximum value, and increment of the Statlet scrollbars.

Example

To solve the problem stated earlier, set the controls on the toolbar as shown below:



In particular:

1. Set the *Demonstrate* field to demonstrate that *Reliability* equals *90.0%* at a lifetime equal to *1000*.
2. Set the *Max. failures* field to *0*.
3. Set the Dist. field to *Weibull* with a shape parameter equal to *1.5*.
4. Set the confidence level to *95%*.
5. Fix the *Test duration* at t = *250*.
6. Press the *Update* button.

Output

The output of the Statlet shows several important results:

1. The second line of the title shows the duration of the test *t* and the number of units *n*. For the example, *n* = 228 units must be tested for a duration of *t* = 250.

2. The plotted **probability of passing curve** shows the probability of a successful demonstration as a function of the probability that any single unit will survive until the target lifetime of 1000 hours.

The chance of passing the test ranges from about 5% if units survive 1000 hours only 90% of the time, to close to 100% if the probability of surviving 1000 hours is very high. The shape of the curve is most sensitive to the maximum number of allowable failures.

Calculations

The basic procedure is as follows:

- *Step 1*: Given the assumed distribution of failure times, determine the value of any unknown parameters that will result in the specified reliability, percentile, MTTF, or parameter value.

- *Step 2*: Solve for the reliability at the end of the testing period.

- *Step 3*: Determine the values of *n* and *t* that will result in a successful demonstration with probability equal to alpha (1 minus the confidence level) when the statement to be demonstrated is exactly true.

This insures that if the test is passed, the lower confidence bound for the quantity to be demonstrated is no worse than stated.

Distributions

Statgraphics defines the distributions used in this procedure as indicated below.

Birnbaum-Saunders Distribution

Range of X: X > 0

Common use: model for the number of cycles needed to cause a crack to grow to a size that would cause a fracture to occur.

PDF: $f(x) = \dfrac{\sqrt{\frac{x}{\theta}} + \sqrt{\frac{\theta}{x}}}{2\beta x} \phi\left(\frac{1}{\beta}\left(\sqrt{\frac{x}{\theta}} - \sqrt{\frac{\theta}{x}}\right)\right)$ where φ(z) is the standard normal pdf

Parameters: shape β > 0, scale θ > 0

Mean: $\theta\left(1 + \frac{\beta^2}{2}\right)$

Variance: $(\theta\beta)^2\left(1 + \frac{5\beta^2}{4}\right)$

Exponential Distribution

Range of X: X > 0

Common use: time between consecutive arrivals in a Poisson process. Lifetime of items with a constant hazard rate.

PDF: $f(x) = \lambda e^{-\lambda x}$

Parameters: mean λ > 0

Mean: $\frac{1}{\lambda}$

Variance: $\frac{1}{\lambda^2}$

Gamma Distribution

Range of X: X ≥ 0

Common use: model for positively skewed measurements. Time to complete a task, such as a repair.

PDF: $f(x) = \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$

Parameters: shape α >0, scale λ > 0

Mean: $\frac{\alpha}{\lambda}$

Variance: $\frac{\alpha}{\lambda^2}$

Largest Extreme Value Distribution

Range of X: all real X

Common use: distribution of the largest value in a sample from many distributions. Also used for positively skewed measurement data.

PDF: $f(x) = \frac{1}{\beta} \exp\left\{ -\left(\frac{x-\alpha}{\beta} \right) - \exp\left(-\frac{x-a}{\beta} \right) \right\}$

Parameters:

Mean: $\alpha+\beta\Gamma^{-1}(1)$

Variance: $\frac{\beta^2 \pi^2}{6}$

Logistic Distribution

Range of X: all real X

Common use: used as a model for growth and as an alternative to the normal distribution.

PDF: $f(x) = \frac{1}{\sigma} \frac{\exp(z)}{\left[1+\exp(z)\right]^2}$ where $z = \frac{x-\mu}{\sigma}$

Parameters: mean μ, standard deviation σ > 0

Mean: μ

Variance: σ^2

Loglogistic Distribution

Range of X: X > 0

Common use: used for data where the logarithms follow a logistic distribution.

PDF: $f(x) = \frac{1}{\sigma x} \frac{\exp(z)}{\left[1+\exp(z)\right]^2}$ where $z = \frac{\ln(x)-\mu}{\sigma}$

Parameters: median exp(μ), shape σ > 0

Mean: $\exp(\mu)\Gamma(1+\sigma)\Gamma(1-\sigma)$

Variance: $\exp(2\mu)\left[\Gamma(1+2\sigma)\Gamma(1-2\sigma) - \Gamma^2(1+\sigma)\Gamma^2(1-\sigma)\right]$

Lognormal Distribution

Range of X: X > 0

Common use: used for data where the logarithms follow a normal distribution.

PDF: $f(x) = \frac{1}{x\sqrt{2\pi}\sigma} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$

Parameters: location μ, scale σ > 0

Mean: $e^{\mu+\sigma^2/2}$

Variance: $e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right)$

Normal Distribution

Range of X: all real X

Common use: widely used for measurement data, particularly when variability is due to many sources.

PDF: $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Parameters: mean μ, standard deviation σ > 0

Mean: μ

Variance: σ^2

Smallest Extreme Value Distribution

Range of X: all real X

Common use: distribution of the smallest value in a sample from many distributions. Also used for negatively skewed measurement data.

PDF: $f(x) = \frac{1}{\beta}\exp\left\{\left(\frac{x-\alpha}{\beta}\right)-\exp\left(\frac{x-a}{\beta}\right)\right\}$

Parameters: mode α, scale $\beta > 0$

Mean: $\alpha-\beta\Gamma^{-1}(1)$

Variance: $\frac{\beta^2\pi^2}{6}$

Weibull Distribution

Range of X: $X \geq 0$

Common use: widely used in reliability analysis to model product lifetimes.

PDF: $f(x) = \frac{\alpha}{\beta^\alpha}x^{\alpha-1}e^{-(x/\beta)^\alpha}$

Parameters: shape $\alpha > 0$, scale $\beta > 0$

Mean: $\frac{\beta}{\alpha}\Gamma\left(\frac{1}{\alpha}\right)$

Variance: $\frac{\beta^2}{\alpha}\left[2\Gamma\left(\frac{2}{\alpha}\right)-\frac{1}{\alpha}\Gamma\left(\frac{1}{\alpha}\right)^2\right]$

