Robust Parameter Design Using Statgraphics Centurion

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Robust Parameter Designs

Experimental designs containing 2 types of factors:

- *controllable factors* that the experimenter can manipulate both during the experiment and during production.
- *noise factors* that can be manipulated during the experiment but are normally uncontrollable.

Goal: To find *robust operating conditions*, i.e., levels of the controllable factors where the values of the response variables are both desirable and relatively insensitive to variation in the noise factors.

Two Approaches

1. Crossed arrays (Genichi Taguchi)
   - 2 designs are created, 1 for the controllable factors and 1 for the noise factors.
   - Taguchi called these designs inner and outer arrays.
   - An inner array is created for the controllable factors.
   - At each location of the inner array, the outer array is performed.

2. Combined arrays (Doug Montgomery's response surface method)
   - A single design is created for both the controllable and noise factors.
   - Interactions between controllable factors and noise factors reveal location of robust operating conditions.

Example #1 – Crossed Arrays

- Myers, Montgomery, and Anderson-Cook (2009) provide an example aimed at minimizing the rate of soldering defects.

- Response: Y = defects per million joints

- Controllable factors:
  - X1 = temperature
  - X2 = speed
  - X3 = flux density
  - X4 = preheat temperature
  - X5 = wave height

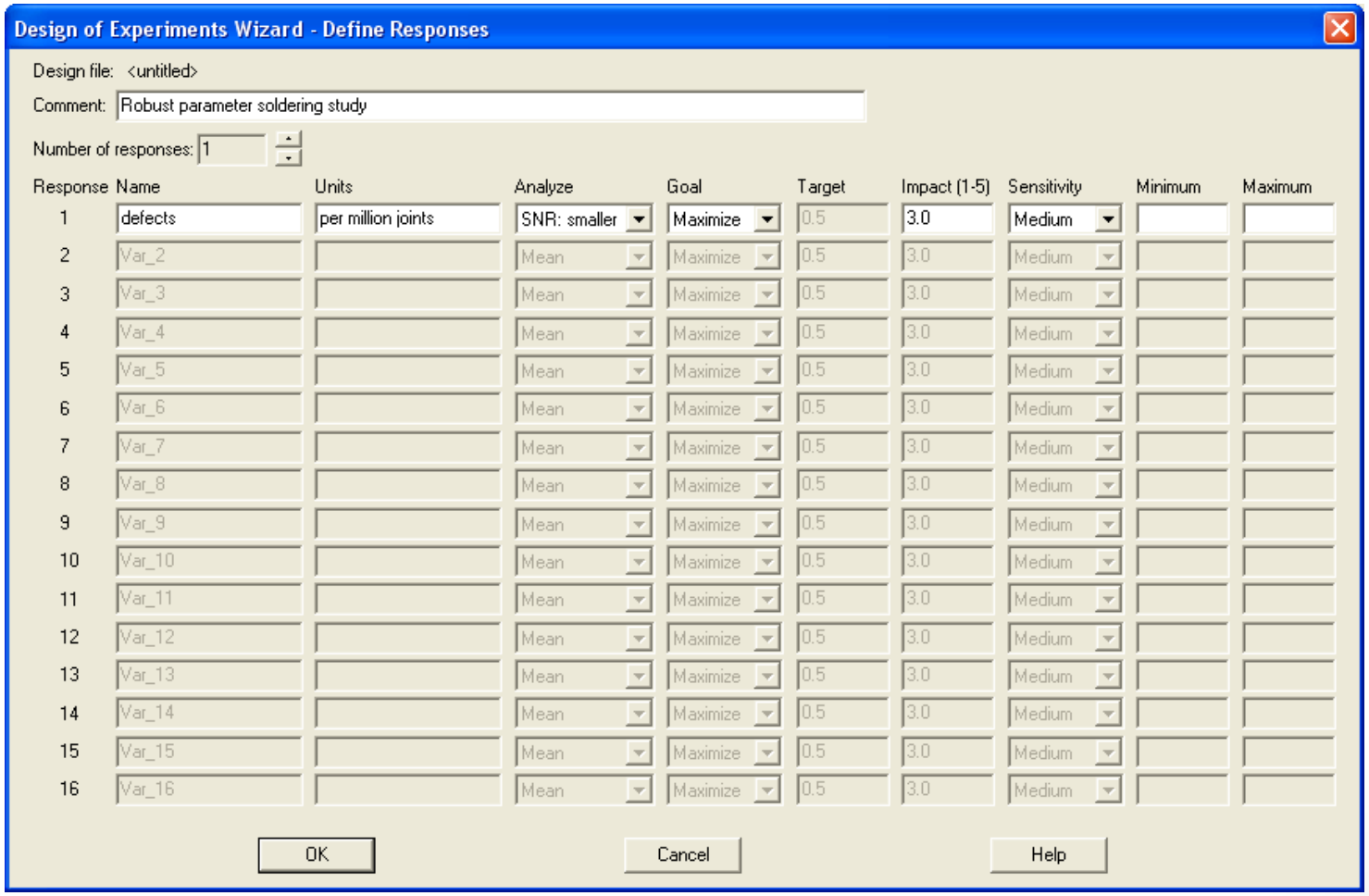
Noise factors:

Z1 = variation in temperature

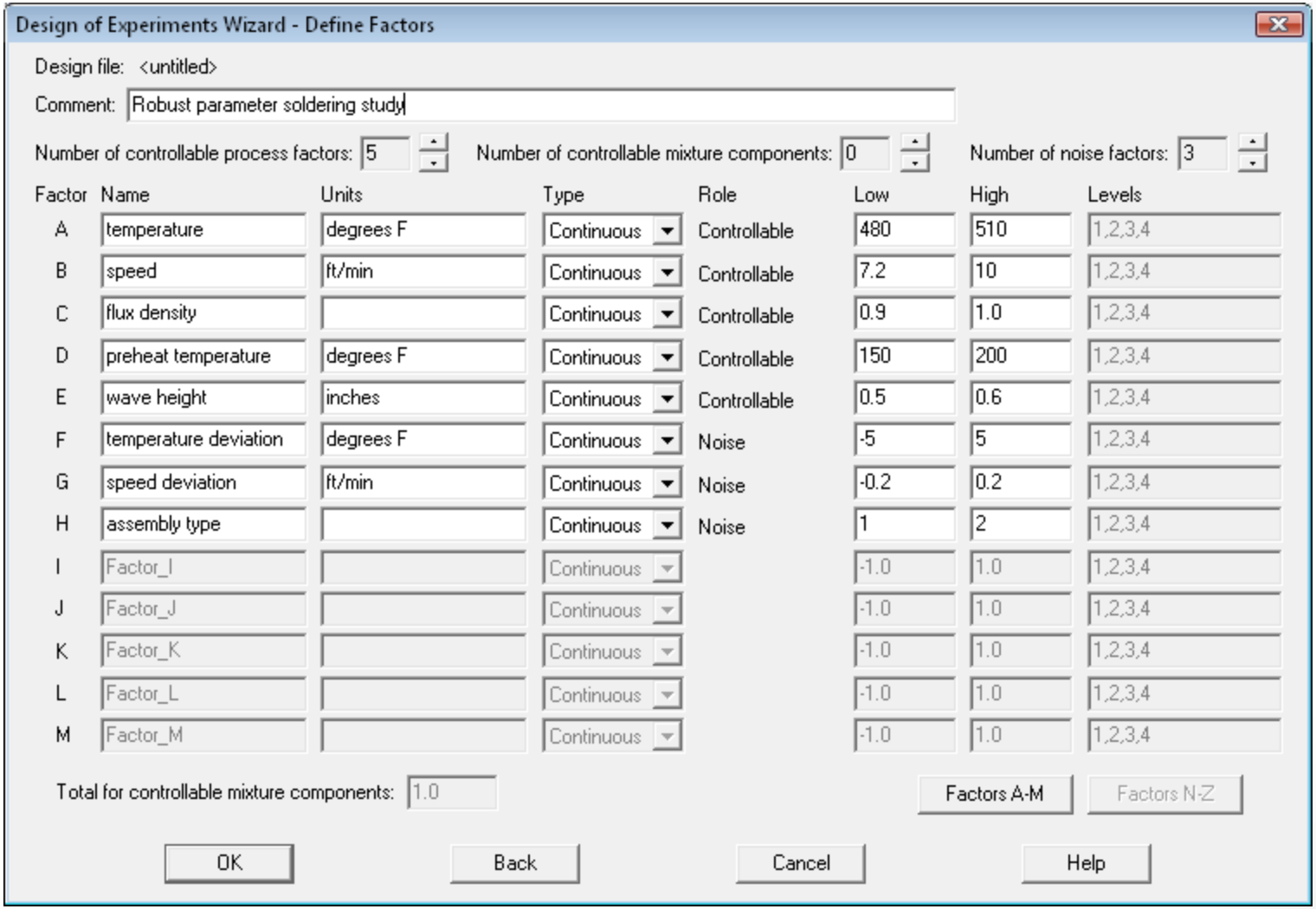
Z2 = variation in conveyor speed

Z3 = type of assembly

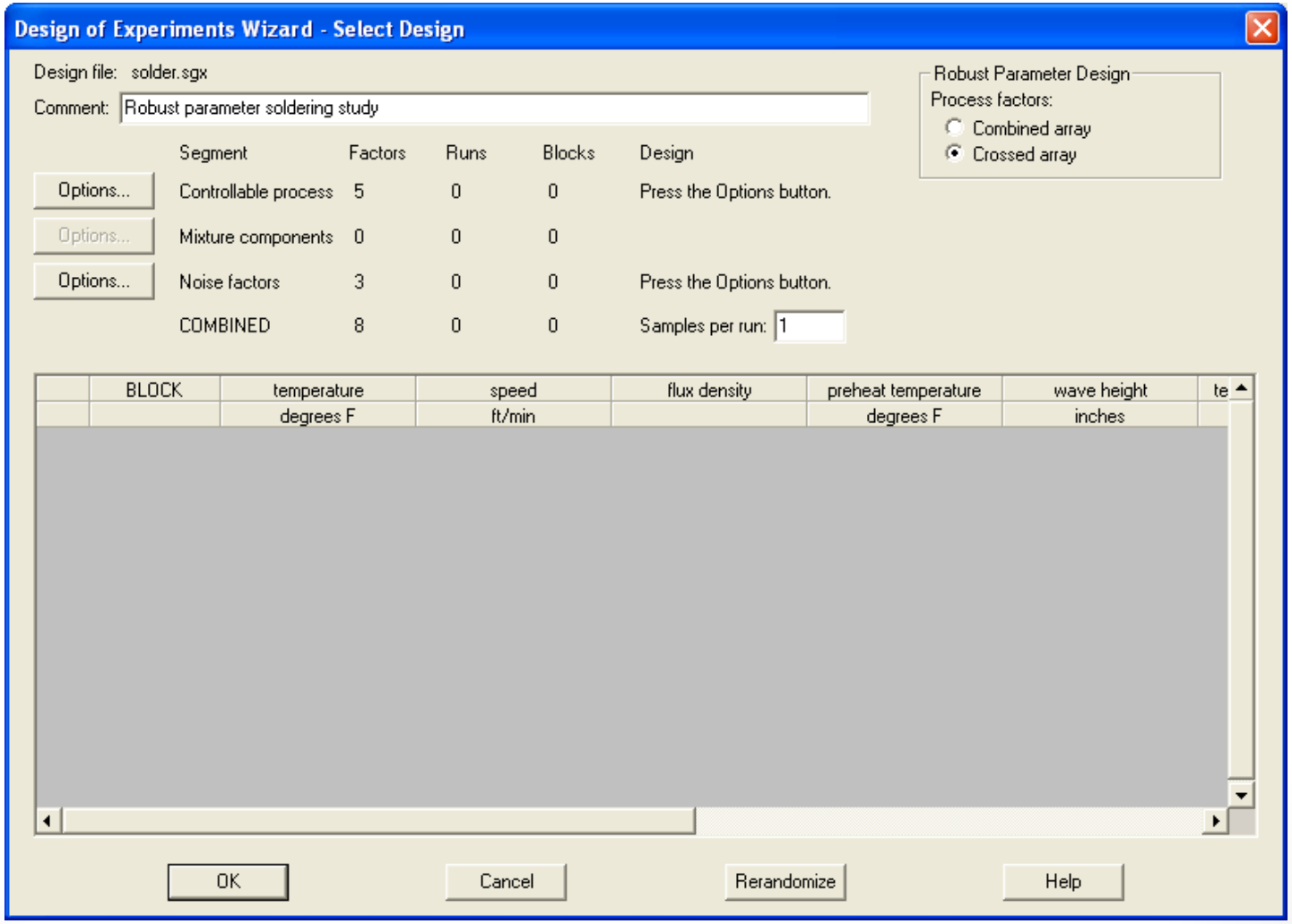
Step 1: Specify response



Step 2: Specify factors

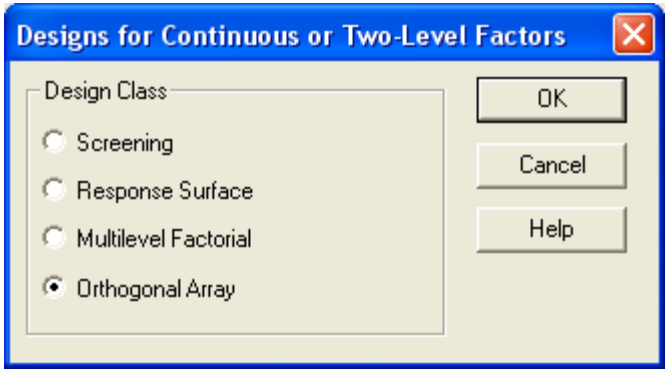


Step 3: Select design

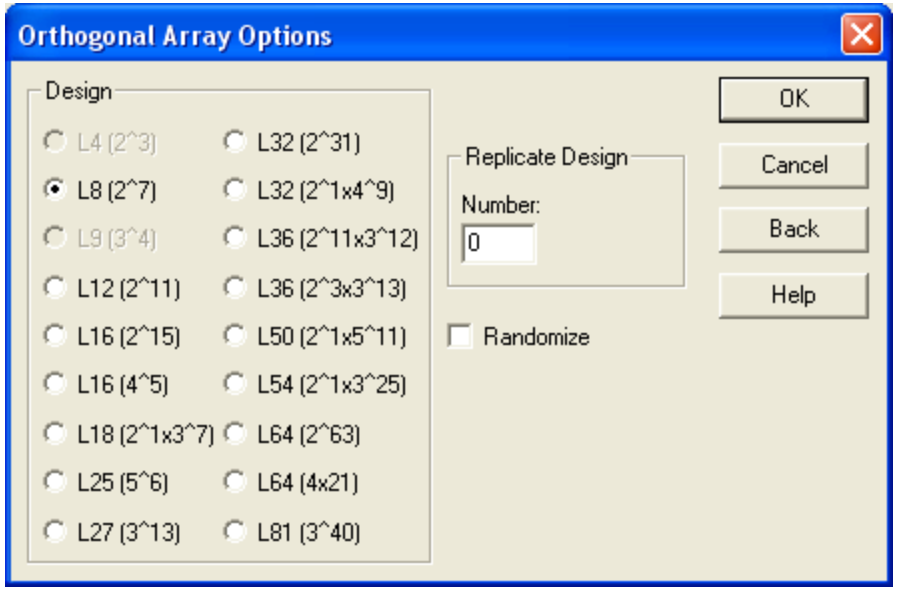


Step 3: Select design (cont.)

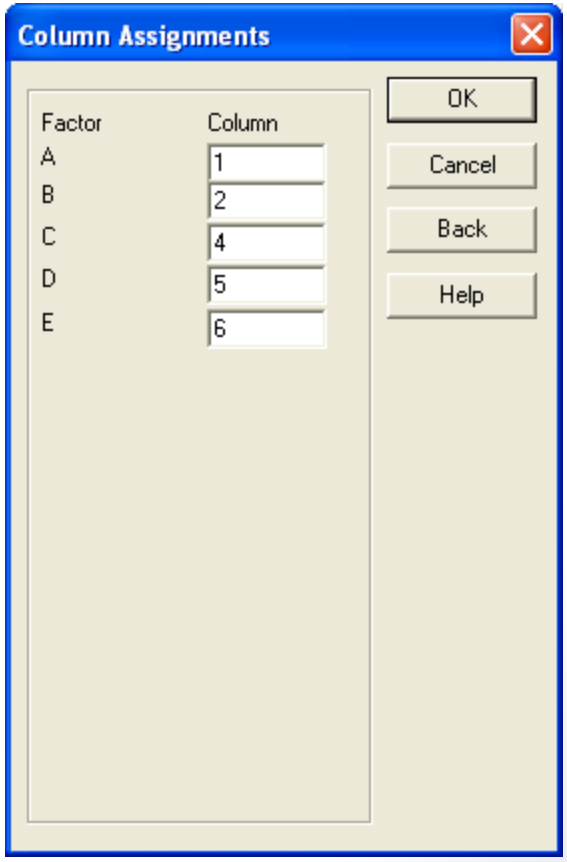
First select the design for the controllable factors.



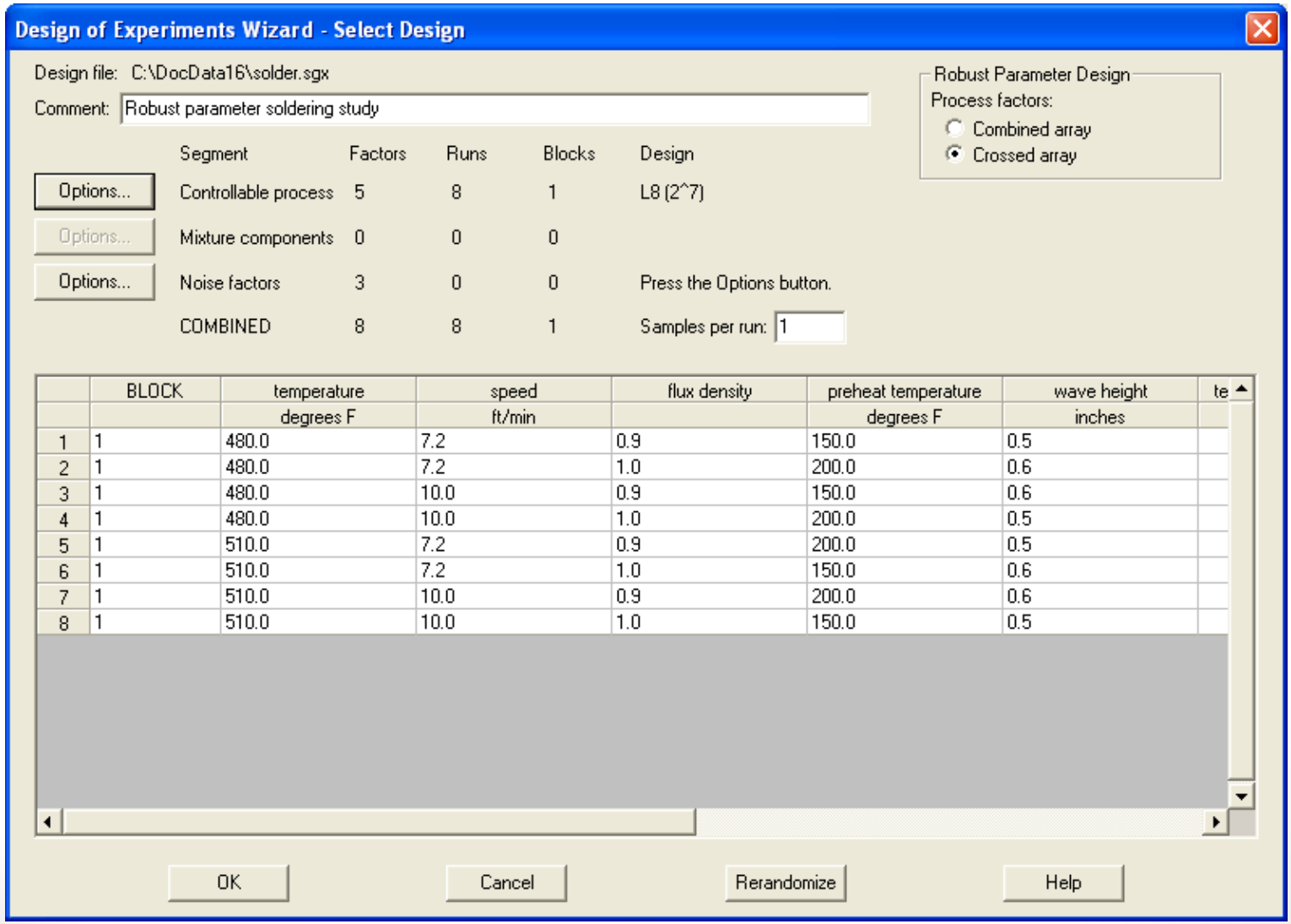
Step 3: Select design (cont.)



Column Run	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

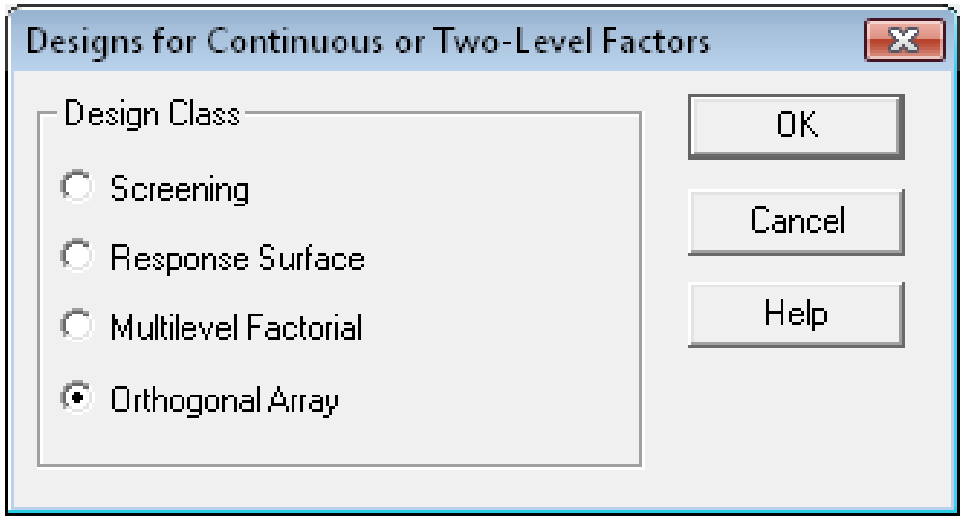


Step 3: Select design (cont.)

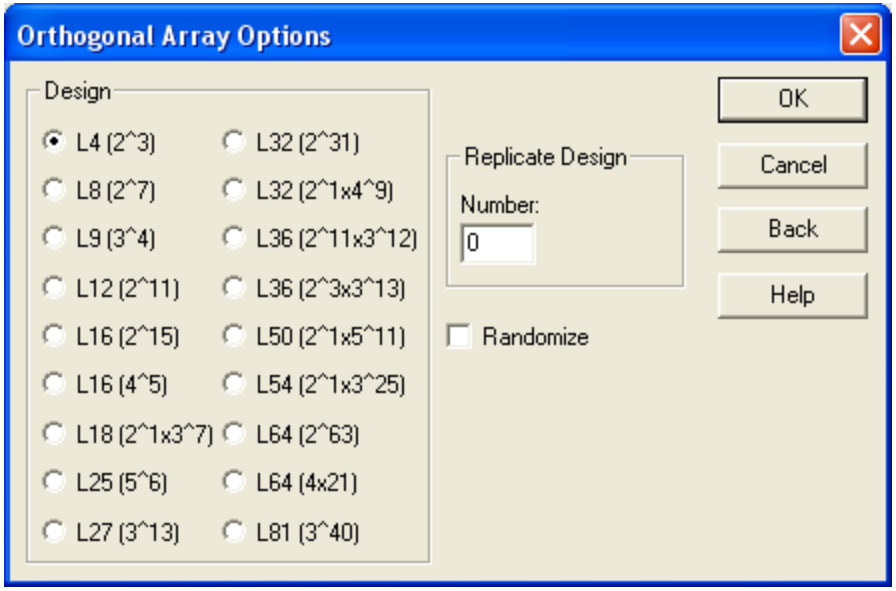


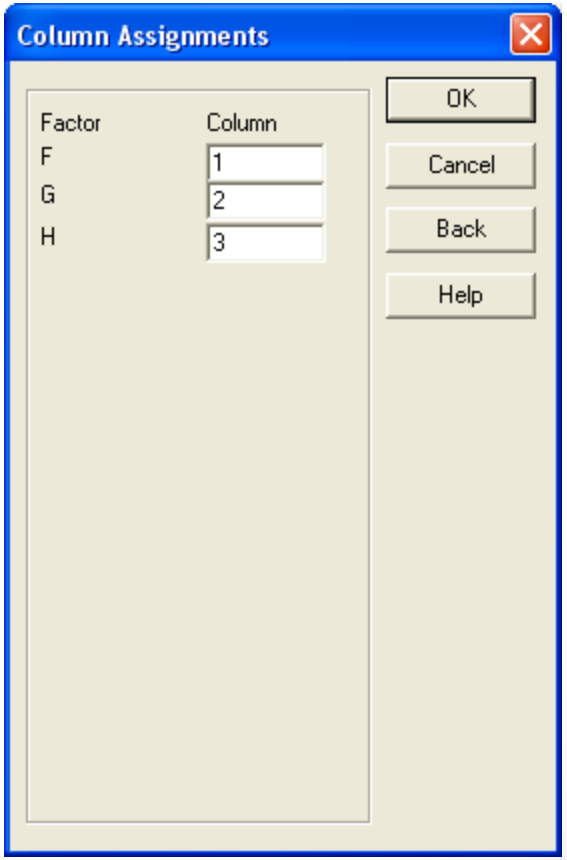
Step 3: Select design (cont.)

Next select the design for the noise factors.



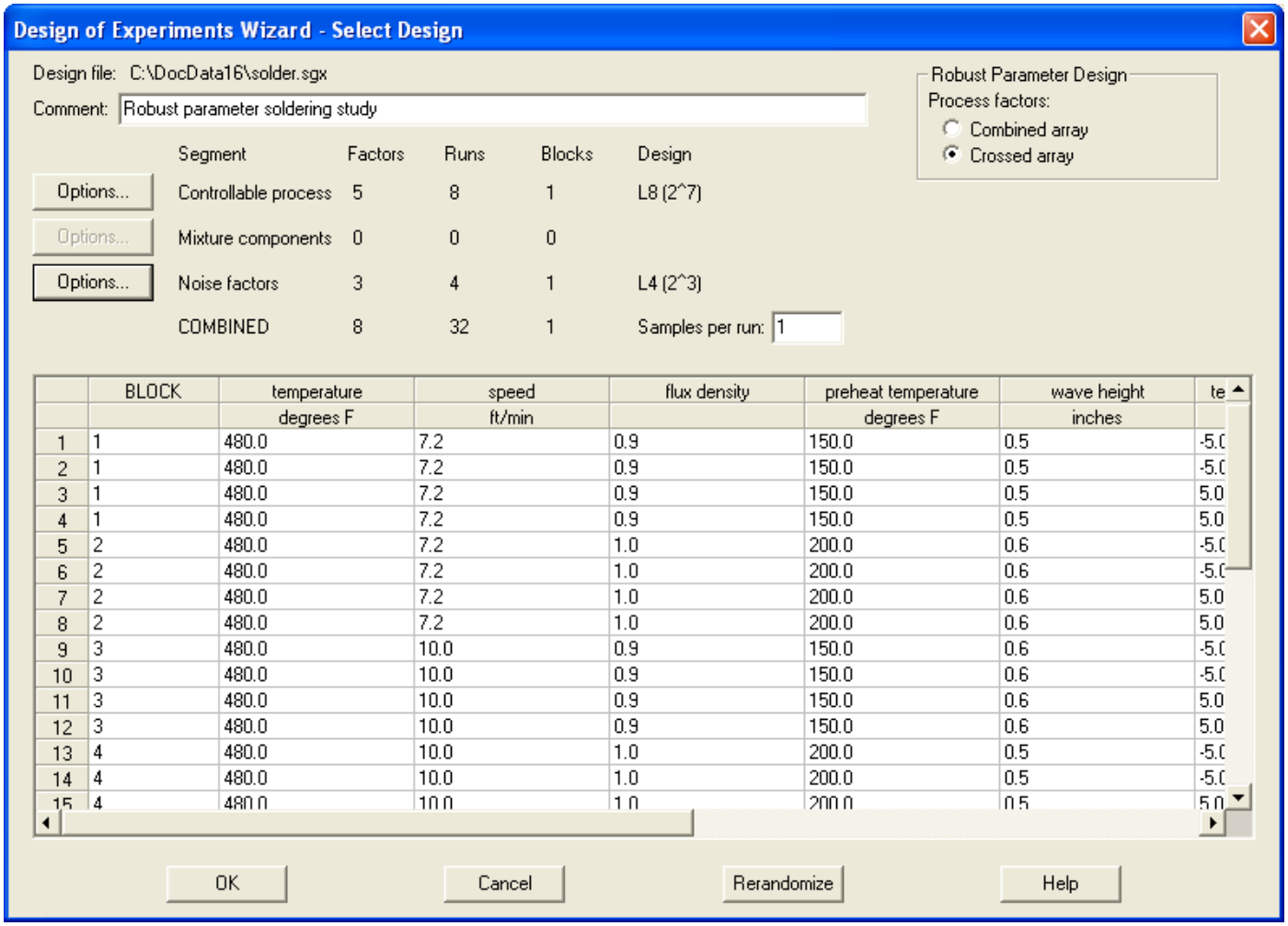
Step 3: Select design (cont.)





Column Run	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Step 3: Select design (cont.)



Step 8: Analyze data

Signal-to-Noise Ratio (Smaller the Better) - For situations in which the response is to be minimized, Taguchi proposed analyzing

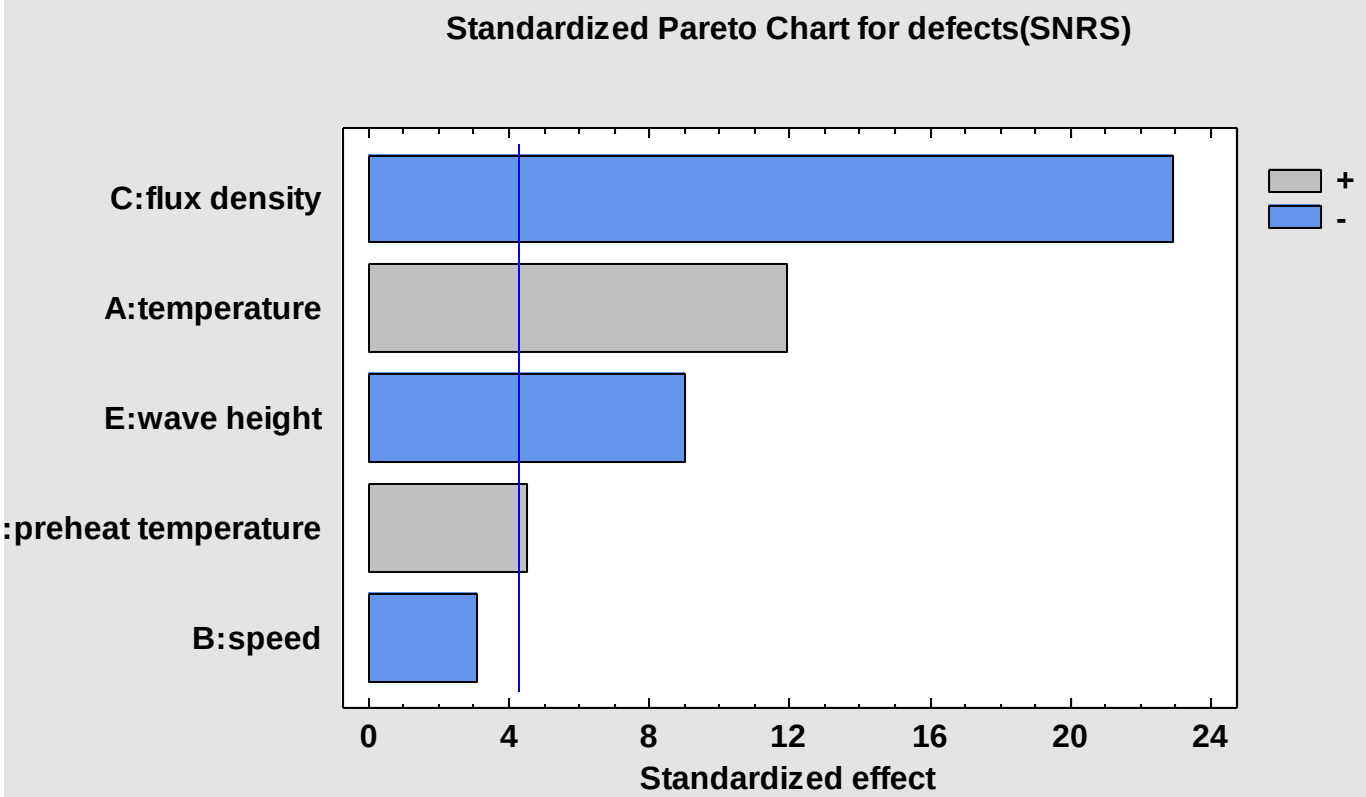
$$SNR_S = -10 \log \sum_{i=1}^{n} \left(\frac{y_i^2}{n} \right)$$

n = number of rows in outer array

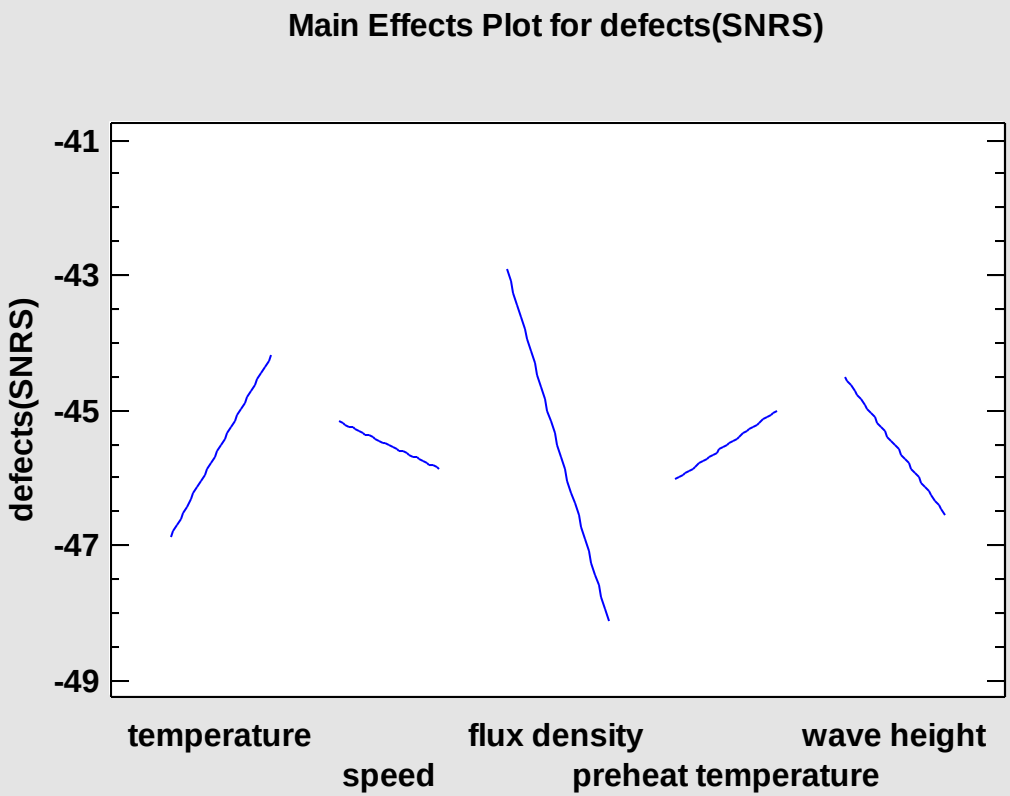
$$\sum_{i=1}^{n} \left(\frac{y_i^2}{n} \right) = \bar{y}^2 + \left(1 - \frac{1}{n} \right) s^2$$

Filename: solder2.sgx

Step 8: Analyze data (cont).



Step 8: Analyze data (cont.)



Step 9: Optimize response

Step 9: Optimize the responses

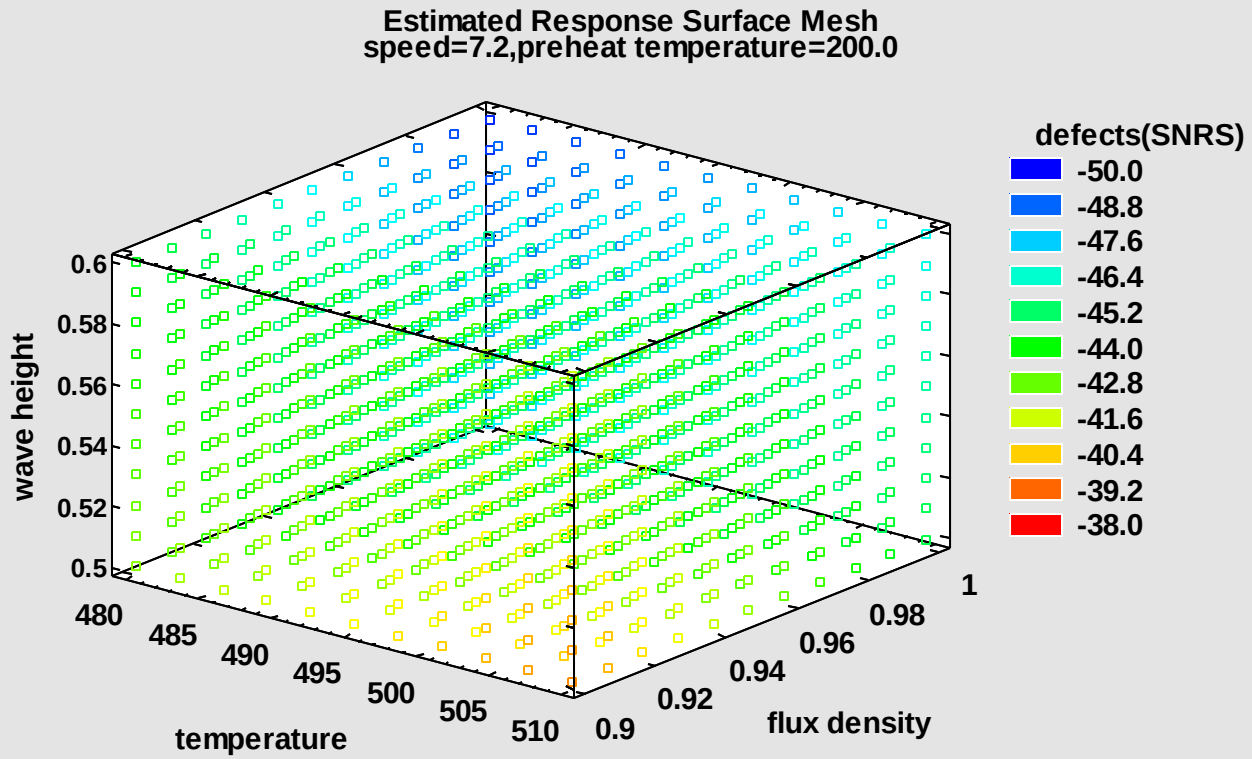
Response Values at Optimum

Response	*Prediction*	*Lower 95.0% Limit*	*Upper 95.0% Limit*	*Desirability*
defects(SNRS)	-39.6893	-40.8836	-38.4951	0.984796

Factor Settings at Optimum

Factor	*Setting*
temperature	510.0
speed	7.2
flux density	0.9
preheat temperature	200.0
wave height	0.5

Step 9: Optimize response (Cont.)



Example #2 – Combined Array

- Myers, Montgomery, and Anderson-Cook (2009) provide an example optimizing the quality of color television images.

- Response: Y = reception quality in decibels

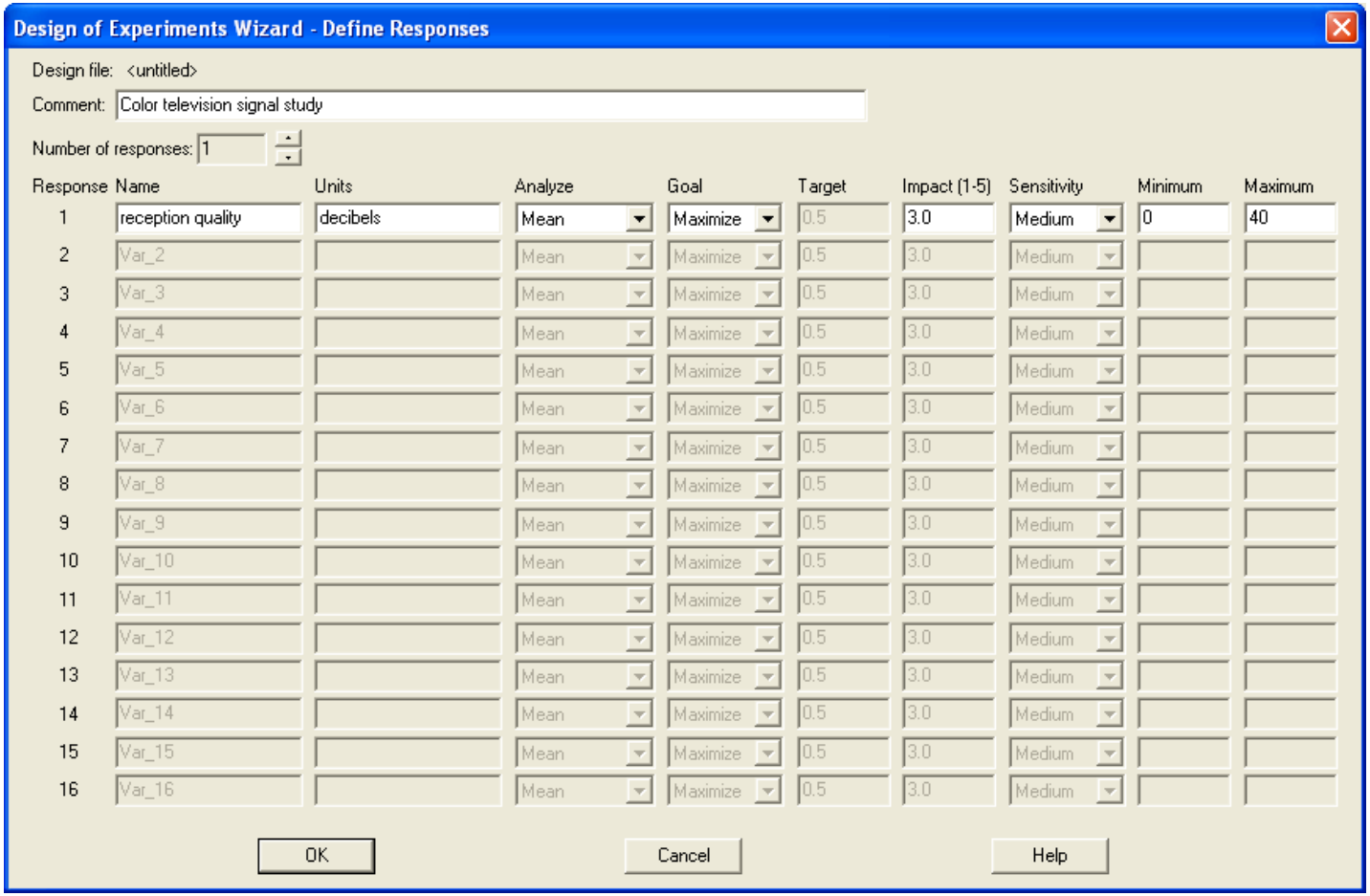
- Controllable factors:
  - X1 = filter tabs
  - X2 = sampling frequency

Noise factors:

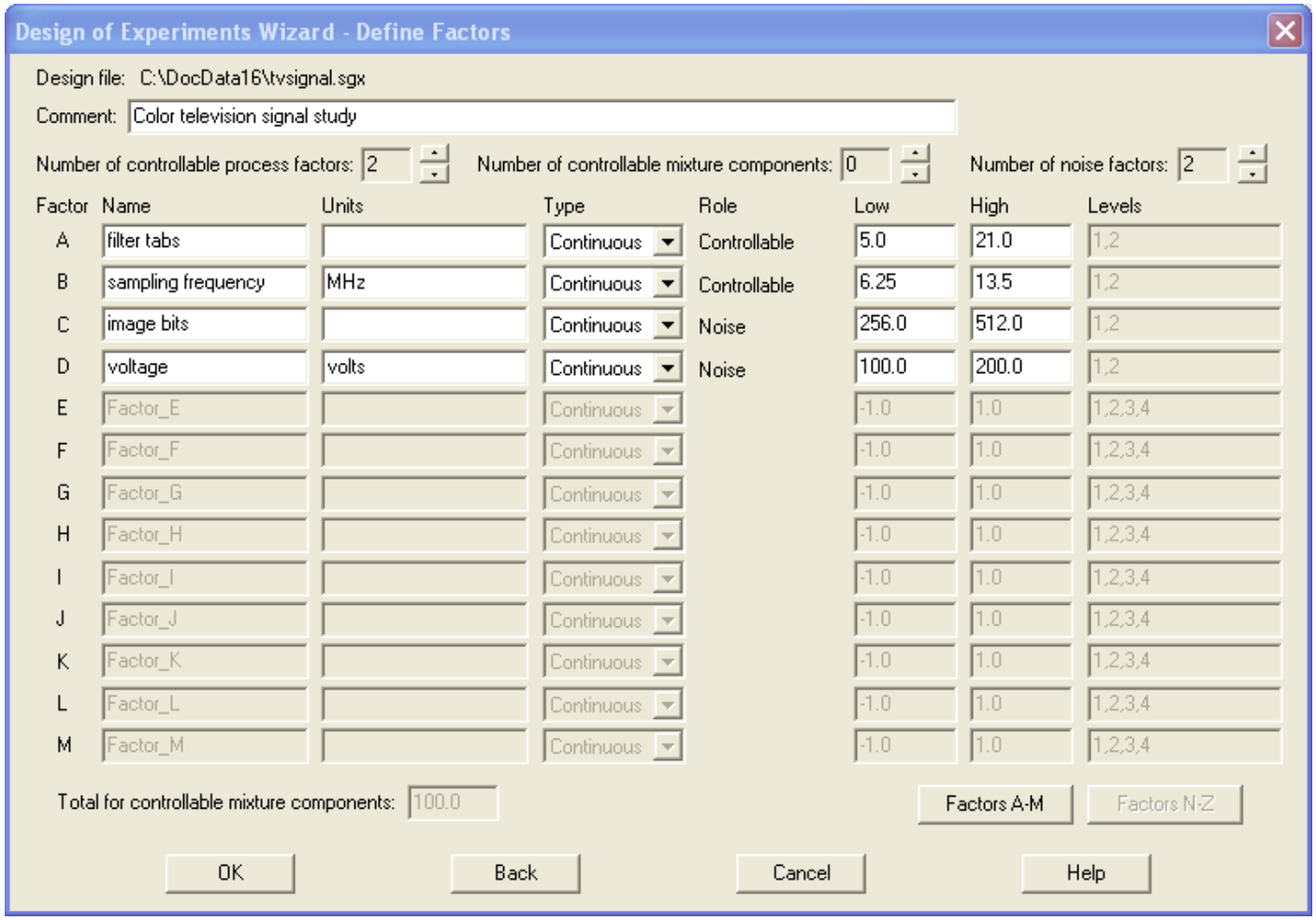
Z1 = image bits

Z2 = voltage

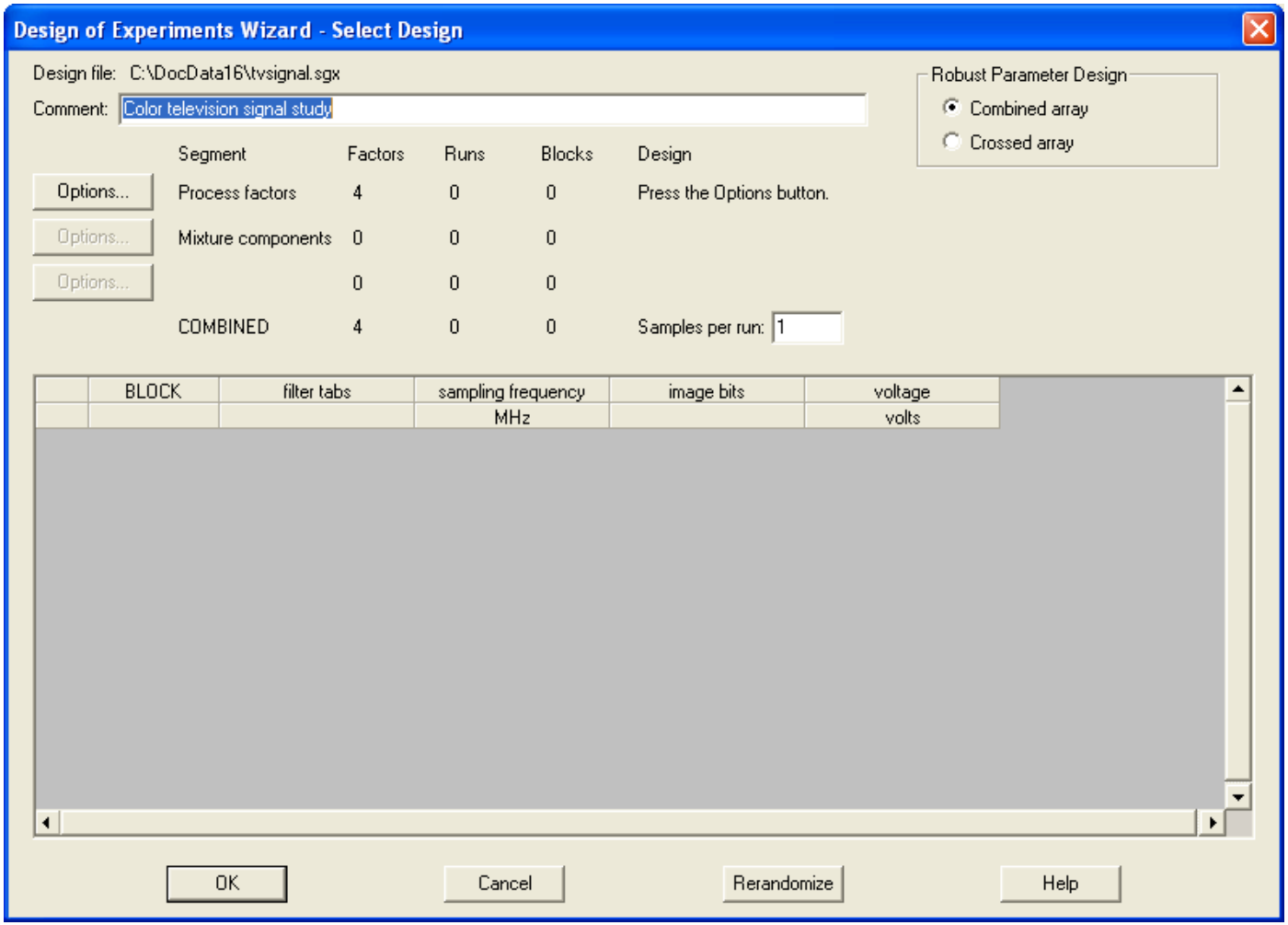
Step 1: Specify response



Step 2: Specify factors

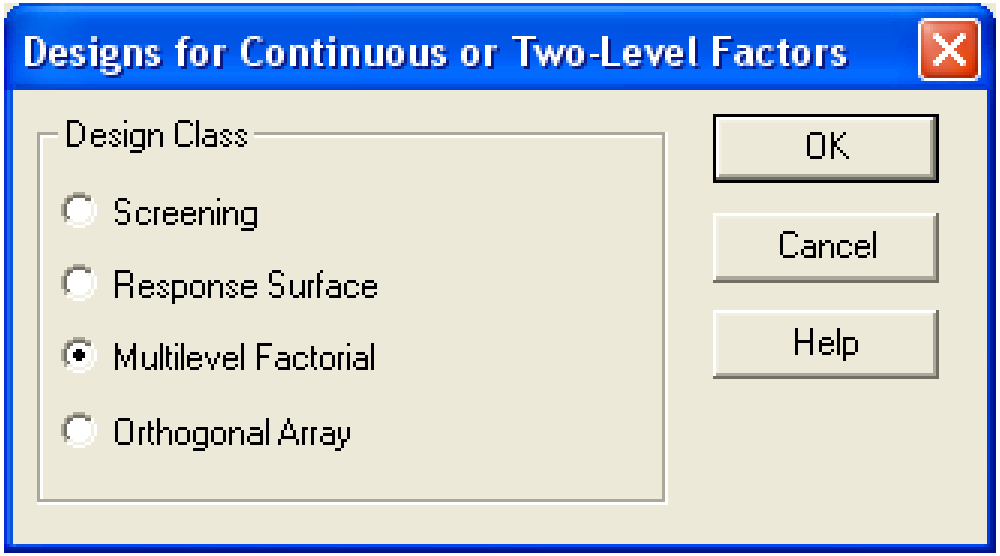


Step 3: Select design

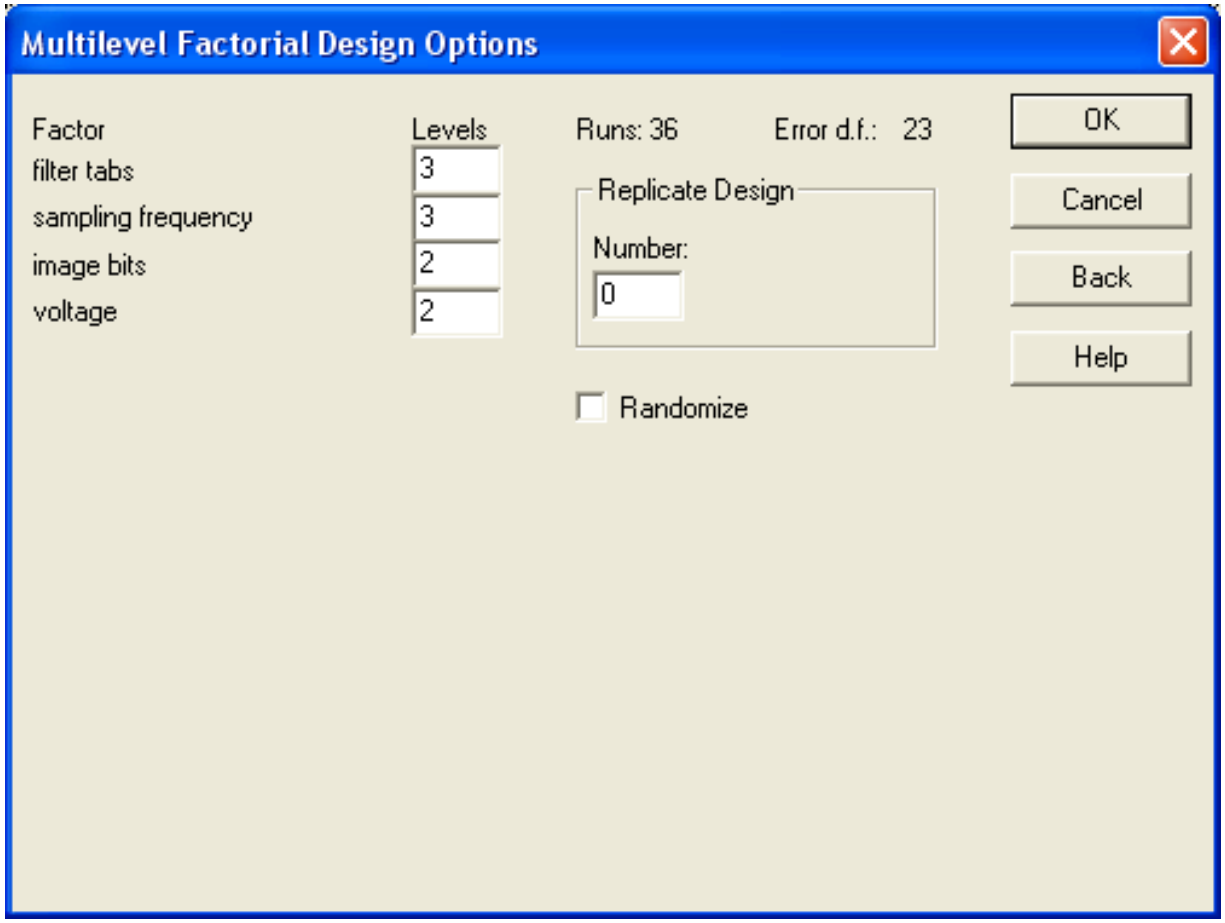


Step 3: Select design (cont.)

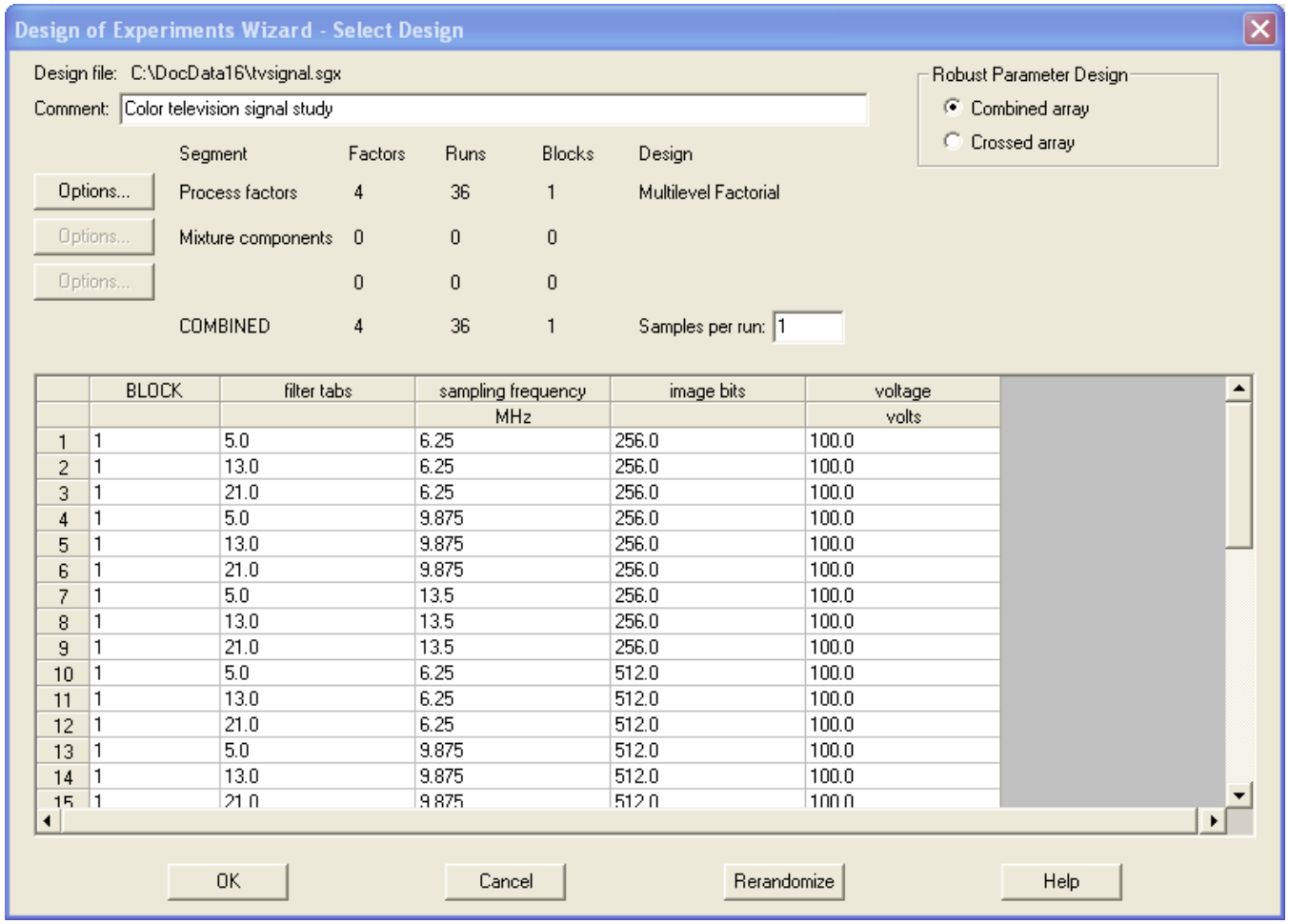
Select a single design for both the controllable and noise factors.



Step 3: Select design (cont.)

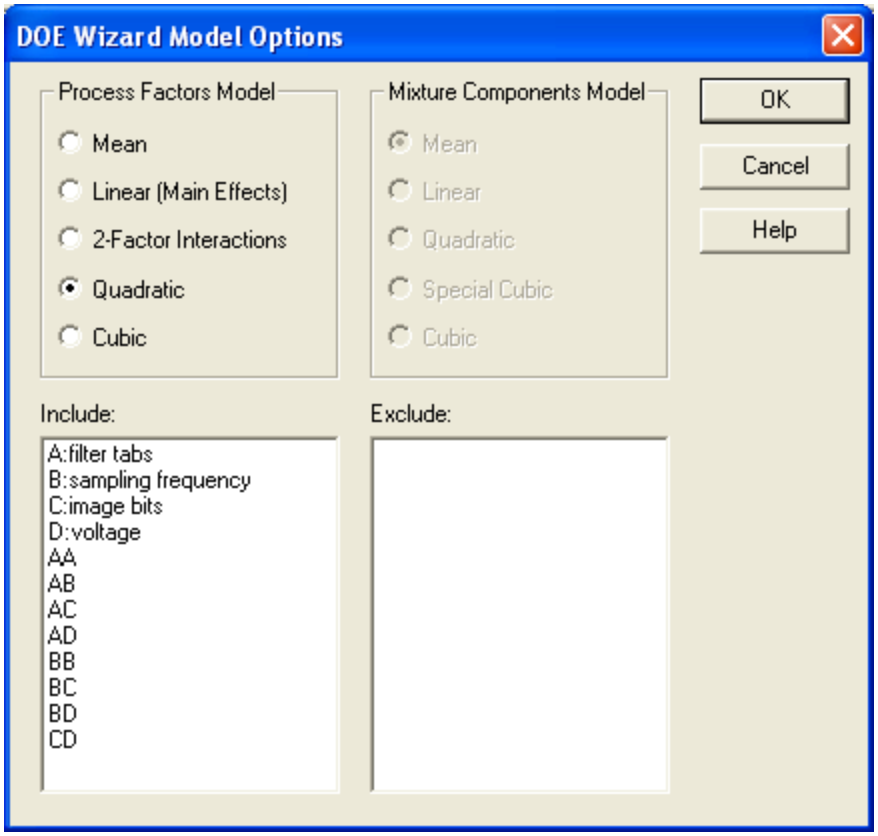


Step 3: Select design (cont.)

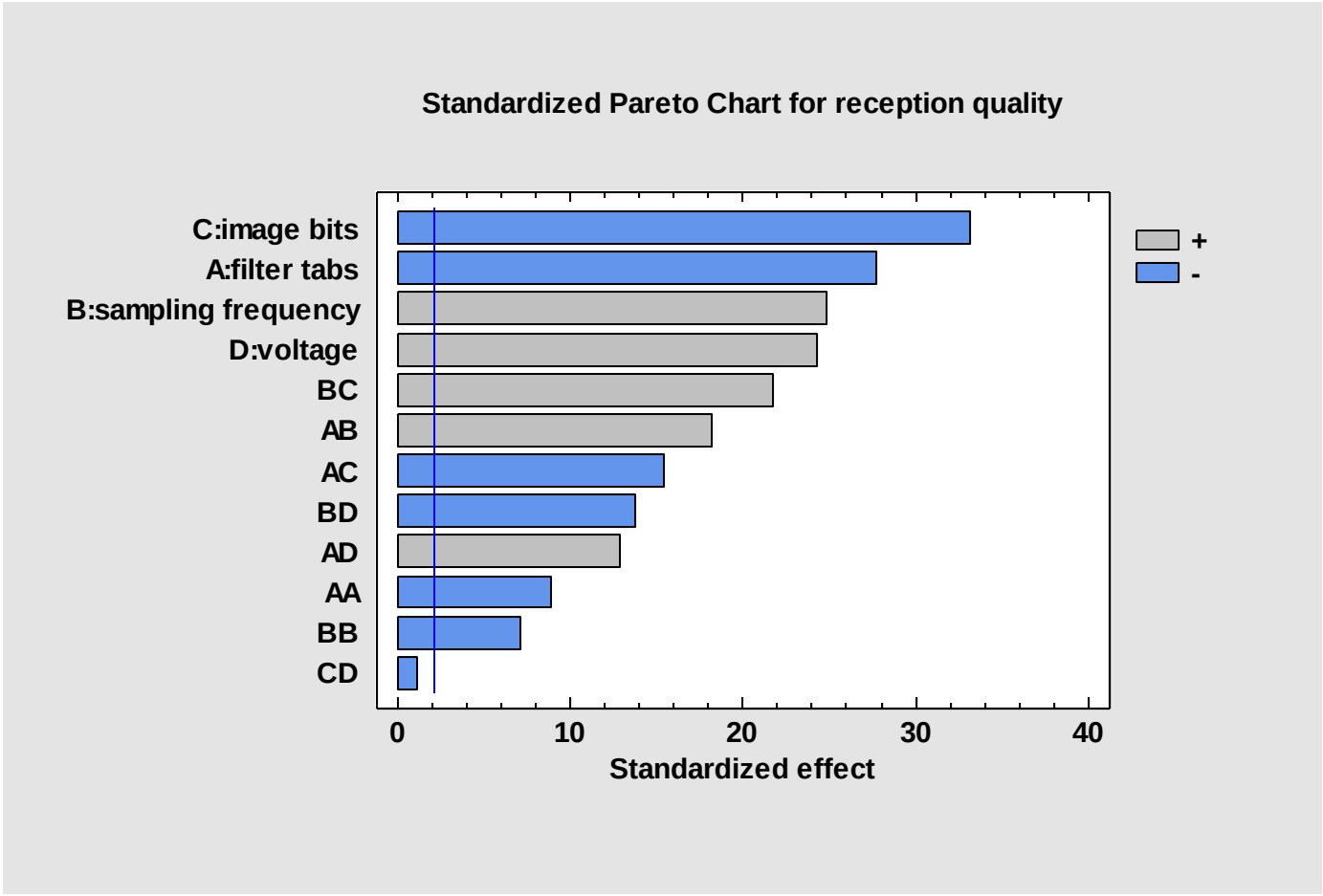


Step 4: Select model

The default model has two-factor interactions and quadratic effects for the 3-level factors.

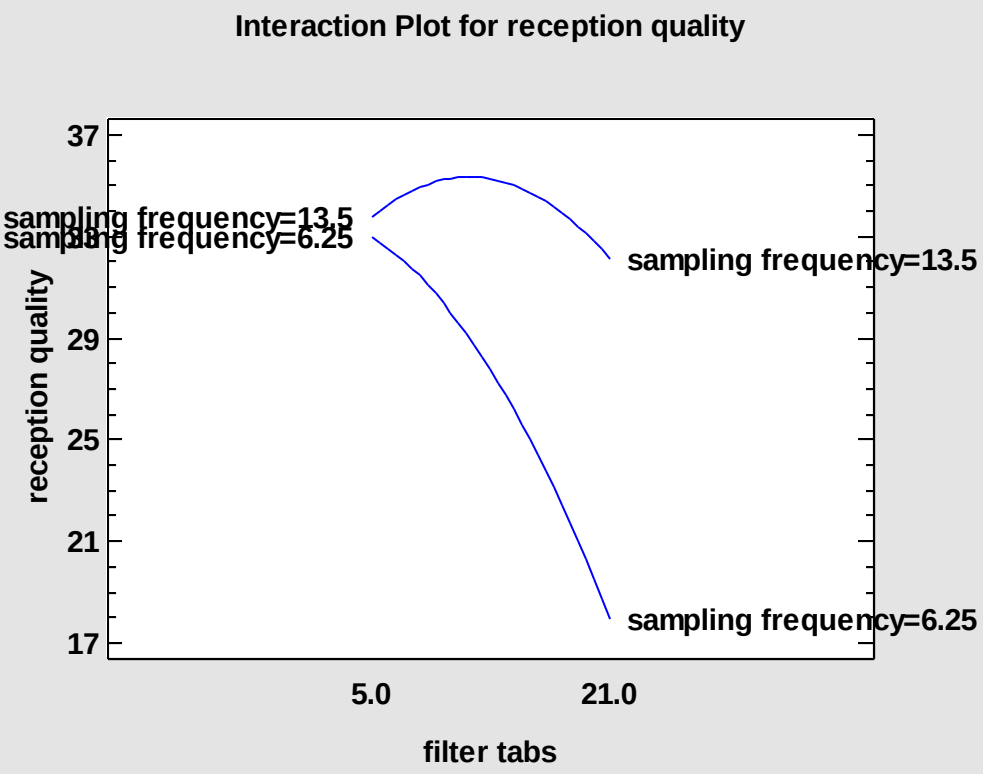


Step 8: Analyze data



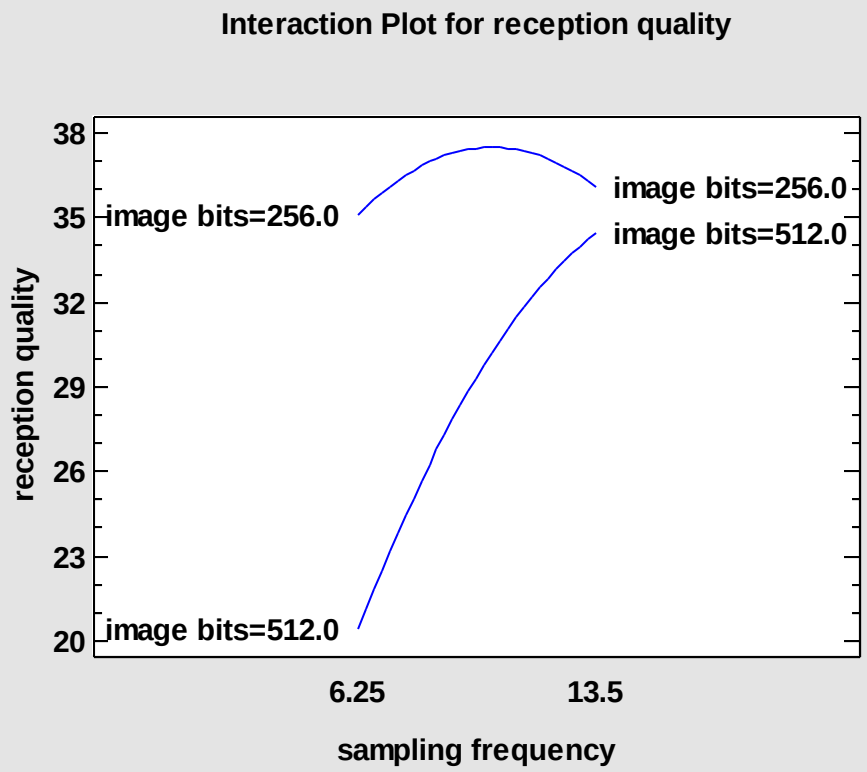
Filename: tvsignal.sgx

Step 8: Analyze data (cont.)



Reception quality is good at high sampling frequency, or at low sampling frequency and low filter tabs.

Step 8: Analyze data (cont.)



Reception quality is less affected by changes in image bits at high sampling frequency.

Step 8: Analyze data (cont.)

Reception quality is less affected by changes in image bits at low filter tabs.

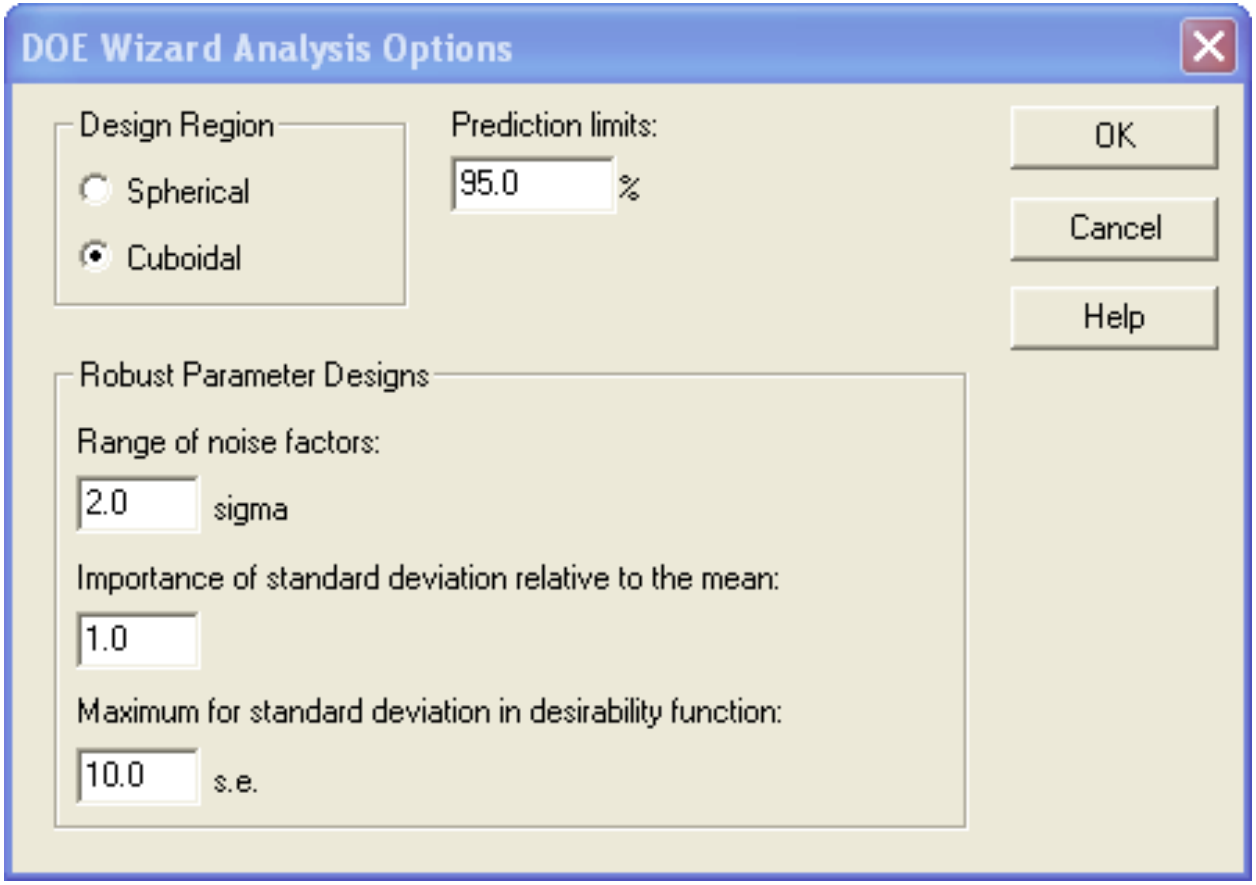
Step 9: Optimize response

"Desirability "– on a scale of 0 to 1, how desirable are the values of the response variables at a selected combination of the controllable factors.

In this case, desirability is a combination of the mean response and the standard deviation of the response at that combination. The transmission of error formula is used to estimate the std. deviation:

$$se = \sqrt{\sum_{i=1}^{r}\left[\frac{\partial y(x,z)}{\partial z_i}\sigma_z\right]^2 + \sigma^2}$$

Analysis Options



Step 9: Optimize response (cont.)

Step 9: Optimize the responses

Response Values at Optimum

Response	*Optimized*	*Prediction*	*Lower 95.0% Limit*	*Upper 95.0% Limit*
reception quality	yes	35.3544	34.5018	36.207

Standard Deviation	*Desirability*
0.738624	0.883626

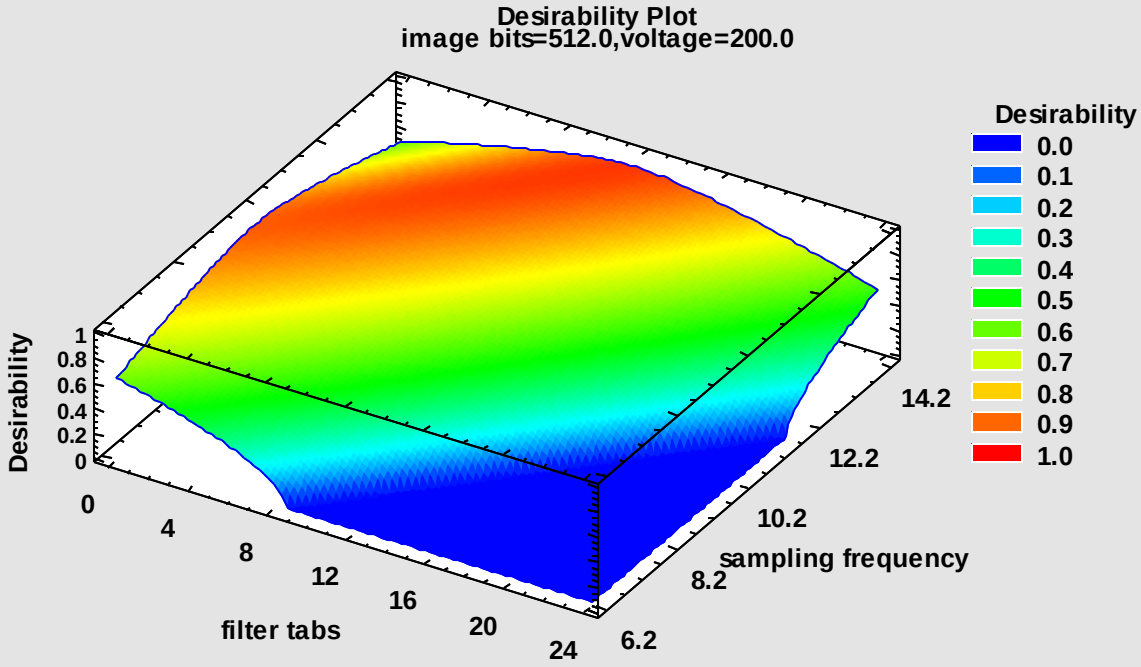
Factor Settings at Optimum

Factor	*Setting*
filter tabs	9.83853
sampling frequency	13.5

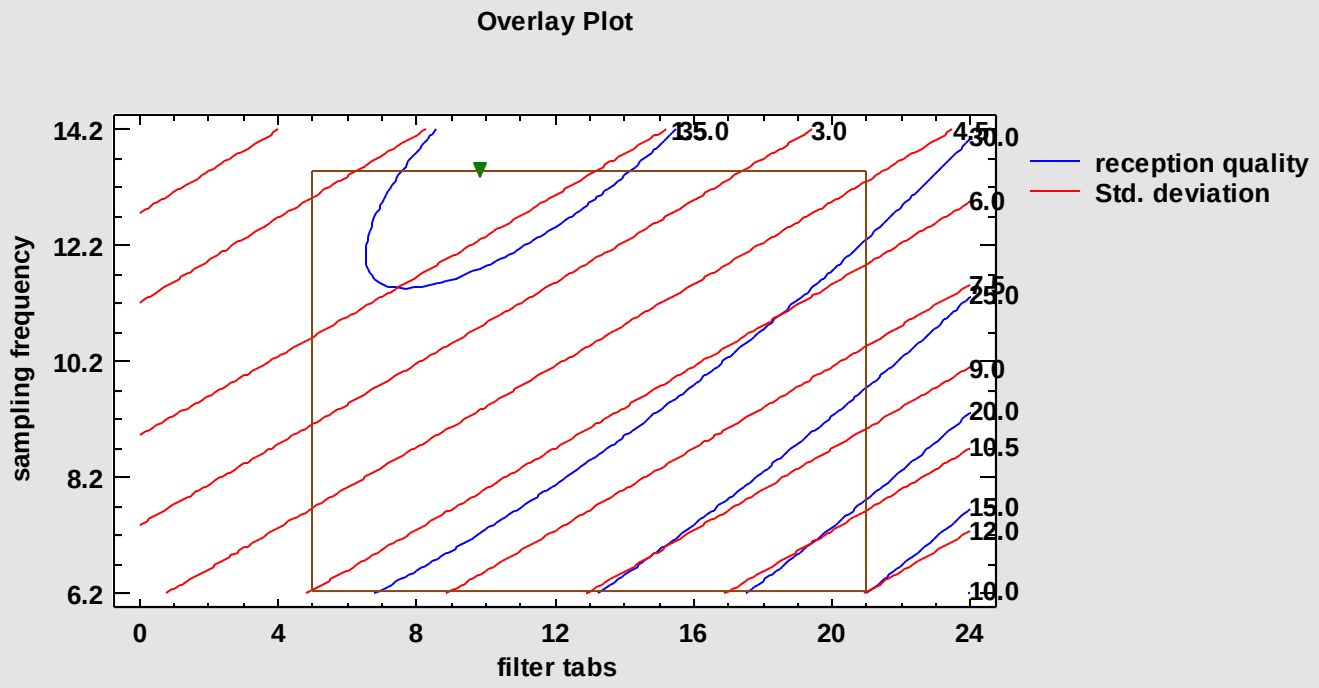
Transmission of error formula used to estimate the std. deviation of the response at the optimal conditions.

$$se = \sqrt{\sum_{i=1}^{r}\left[\frac{\partial y(x,z)}{\partial z_i}\sigma_z\right]^2 + \sigma^2}$$

Step 9: Optimize response (cont.)



Step 9: Optimize response (cont.)



More Information

(1) Myers, R. H., Montgomery, D. C. and Anderson-Cook, C. M. (2009). Response Surface Methodology: Process and Product Optimization Using Designed Experiments, 3rd edition. New York: John Wiley and Sons.

(2) Statgraphics Centurion PDF file: Doe Wizard – Inner-Outer Arrays

(3) Statgraphics Centurion PDF file: Doe Wizard – Robust Parameter Designs

(4) www.statgraphics.com