

Sample Size Determination (Capability Indices)

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Summary

This procedure determines a suitable sample size for estimating three capability indices:

- **C_p** – This two-sided capability index compares the distance between the specification limits to 6-sigma:

$$C_p = \frac{USL - LSL}{6\sigma} \quad (1)$$

- **C_{pk}** – The smaller of two one-sided indices:

$$C_{pk} = \min\left(\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\right) \quad (2)$$

- **C_{pm}** – A modified version of *C_p* measuring variation around the target or nominal value *T* rather than the estimated process mean:

$$C_{pm} = \frac{C_p}{\sqrt{1 + \frac{(\mu - T)^2}{\sigma^2}}} \quad (3)$$

where LSL and USL are the lower and upper specification limits, respectively, μ is the process mean, and σ is the process sigma. The estimates of the indices, which will be labeled \hat{C}_p , \hat{C}_{pk} , and \hat{C}_{pm} are calculated by replacing the mean and sigma with estimated values.

Sample StatFolio: *capsize.sgp*

Sample Data:

None.

Data Input

The first dialog box displayed by this procedure is used to specify the problem of interest to the analyst.

The dialog box is titled "Sample Size Determination (Capability Indices)". It contains the following fields and controls:

- Index:** Three radio buttons are present: Cp (unselected), Cpk (selected), and Cpm (unselected).
- Buttons:** OK, Cancel, and Help buttons are located on the right side.
- Estimated index:** A text box containing the value 1.33.
- Mean minus target:** A text box containing the value 1.0, followed by the label "sigma".
- Relative error:** A text box containing the value 10.0, followed by a percent sign (%).
- Confidence Level:** A text box containing the value 95.0, followed by a percent sign (%).

- **Index:** the capability index to be estimated. It is assumed that a random sample of size n will be used to estimate the indicated index. The procedure will determine a suitable value for n .
- **Relative error:** the maximum desired difference between the estimate of the capability index and its lower confidence bound.
- **Estimated index:** the anticipated value of the capability index (required for Cpk only). This is the value at which the *relative error* will be fixed. For Cp and Cpm, the relative error is the same for all values of the index.
- **Confidence level:** the level of confidence for the lower confidence bound.
- **Mean minus target:** the anticipated difference between the estimated mean and the target value (required for Cpm only). This is the value at which the *relative error* will be fixed.

For example, the above dialog box indicates that the lower 95% confidence bound for Cpk should be no more than 10% below the estimated value when \hat{C}_{PK} equals 1.33.

Analysis Summary

The *Analysis Summary* displays the required sample size:

Sample Size Determination (Capability Indices)

Capability index: Cpk
 Estimate: 1.33
 Relative error: 10.0%
 Confidence level: 95.0%

The required sample size is 154.

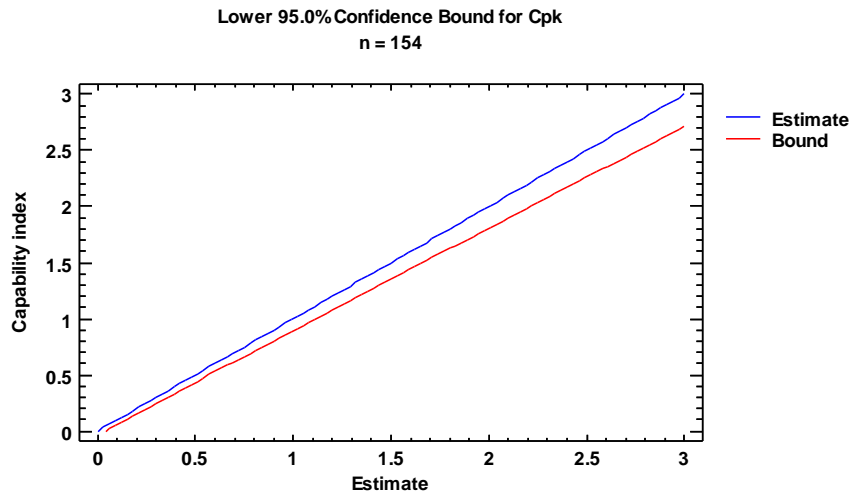
The StatAdvisor

To be 95.0% confident that the true value of Cpk is no less than 10.0% below the estimated value, the required sample size is 154 if the estimate equals 1.33.

In the current example, a sample of $n = 154$ observations is required to achieve the desired lower bound.

Confidence Bounds

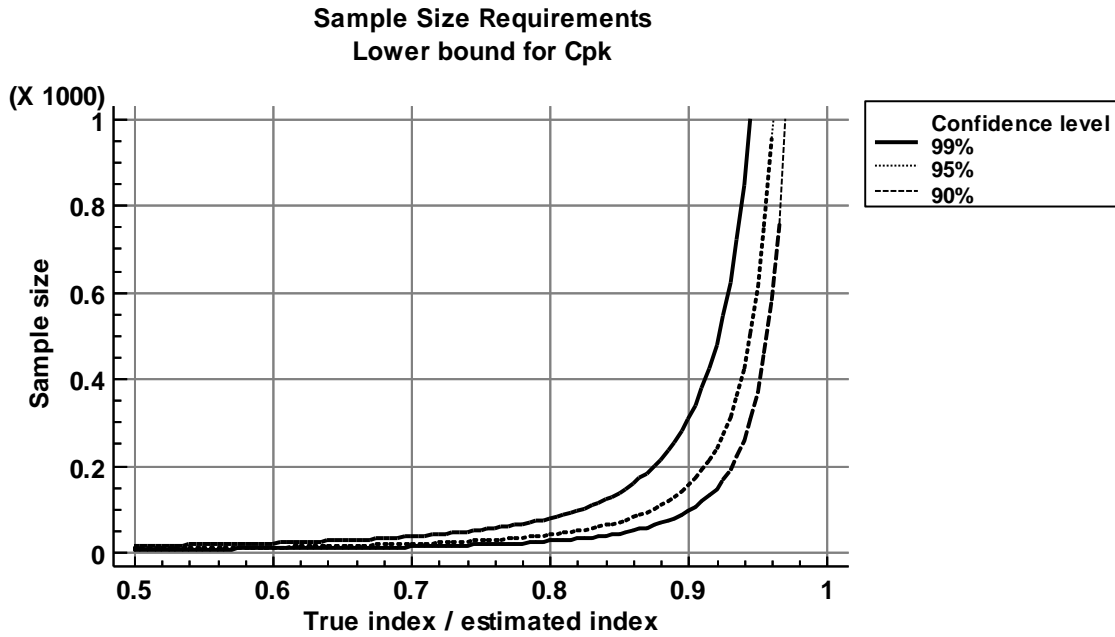
The *Confidence Bounds* plot shows the lower confidence bound as a function of the estimated capability index:



For a sample of $n = 154$, the bound is 10% below the estimate when \hat{C}_{PK} equals 1.33. For Cpk, the relative error increases as the estimate increases.

Sample Size Curves

The *Sample Size Curves* plot the smallest sample sizes required such that the ratio of the true capability index to its estimated value equals or exceeds the value plotted on the horizontal axis:



For example, the middle of the 3 curves crosses $X = 0.9$ at $n = 154$, which demonstrates that 154 observations are required such that the true value of Cpk is not less than 0.9 times its estimated value.

Calculations

The program finds the smallest n such that the lower confidence bound for the ratio of the true capability index to its estimated value satisfies the condition specified in the *relative error* field of the dialog box.

Let α equal 1 minus the desired confidence level. Then the equations for the lower confidence bound of that ratio are shown below:

- **Cp**

$$\sqrt{\frac{X_{\alpha,n-1}^2}{n-1}} \tag{4}$$

where $X_{\alpha,n-1}^2$ is the value at which the cumulative chi-square distribution with $n-1$ degrees of freedom equals α .

- **Cpk**

$$1 - z_{\alpha} \sqrt{\frac{1}{9n} + \frac{\hat{C}_{PK}^2}{2(n-1)}} / \hat{C}_{PK} \tag{5}$$

where z_{α} is the value at which the cumulative standard normal distribution equals $1-\alpha$ and \hat{C}_{PK} is the value entered in the *estimated index* field on the dialog box.

- **Cpm**

$$\sqrt{\frac{nX_{\alpha,f}^2}{(n-1)f}} \tag{6}$$

where

$$f = \frac{(n + \lambda)^2}{n + 2\lambda} \tag{7}$$

and

$$\lambda = n\delta^2 \tag{8}$$

where δ is the value entered in the *mean minus target* field on the dialog box.

Reference: “Sample Size Determination for the Estimate of Process Capability Indices” by Chin-Chuan Wu and Hsin-Lin Kuo, Information and Management Sciences, Vol. 15, No. 1, pp. 1-12.