

Sample Size Determination (Two or More Samples)

This procedure determines a suitable sample size for estimating or testing hypotheses concerning any of the following:

1. the difference between the **means** μ_1 and μ_2 of two normal distributions.
2. the ratio of the **standard deviations** σ_1 and σ_2 of two normal distributions.
3. the difference between the **proportions** θ_1 and θ_2 of two binomial distributions.
4. the difference between the **rates** λ_1 and λ_2 of two Poisson distributions.
5. the **pairwise differences** between the means of more than two normal distributions.

It finds a sample size that achieves either of two goals:

1. generates a *confidence interval* for the difference or ratio of specified width.
2. yields the desired *power* in a test of hypotheses concerning the difference or ratio.

Sample StatFolio: *samsize2.sgp*

Sample Data:

None.

Data Input

The first dialog displayed by this procedure is used to specify the problem of interest to the analyst.

- **Compare:** the problem of interest. It is assumed that random samples of size n_j will be taken from j populations that follow the specified distribution and used to estimate or test the value of the indicated parameters. The procedure will determine suitable values for n_j .
- **Hypothesized Difference or Ratio:** the anticipated value of the difference or ratio. If performing a hypothesis test, this value forms the null hypothesis (usually 0). If constructing a confidence interval, this value is only used if the desired width of the interval is specified in relative (percentage) terms.
- **Hypothesized Within-Group Sigmas:** the anticipated value of the standard deviation σ within each of the j populations sampled, assumed to be the same for all populations. When comparing 2 or more means, this value is a critical part of the calculation and should either be known exactly or be a reliable estimate from previous data.
- **Hypothesized Means:** an approximate value for the means μ_j . This value is not used in the calculations.
- **Hypothesized Proportions:** an approximate value θ_0 for the binomial proportions. This value is used to determine the likely standard error of the difference between the two proportions.
- **Hypothesized Rates:** an approximate value λ_0 for the Poisson rates. This value is used to determine the likely standard error of the difference between the two rates.
- **Number of Means:** the number of samples k when comparing more than 2 means.
- **Percent of Data in First Sample:** when comparing two samples, the percent of data in the first sample:

$$\rho = 100 \frac{n_1}{n_1 + n_2} \% \quad (1)$$

Except in rare cases, the percentage should be set to 50%.

For example, the above dialog box indicates a desire to compare the means of 2 normal distributions, thought to be around $\mu = 10$ with standard deviations of $\sigma = 3$. The null hypothesis is that the difference between the means ($\mu_1 - \mu_2$) equals 0. Equal sample sizes for the 2 samples are desired.

The second dialog box elicits information about the goal of the analysis:

Sample-Size Determination Options

Control

Absolute Error
+ 3.

Relative Error
+ 10. %

Power
90. %
Difference to Detect:
3.

Sample Size
30

Confidence Level: 95. %

Alternative Hypothesis

Not Equal
 Less Than
 Greater Than

Sigma

To Be Estimated
 Known

OK
Cancel
Help

- **Control:** specifies the goal from among the following choices:
 1. **Absolute error:** a confidence interval for the difference or ratio is to be constructed. That interval should not deviate from the point estimate of the difference or ratio in either direction by more than the absolute distance W indicated. Note: when comparing more than 2 means, the interval used is based on Tukey's multiple comparison method.
 2. **Relative error:** a confidence interval for the difference or ratio is to be constructed. That interval should not deviate from the point estimate of the difference or ratio in either direction by more than the relative percentage P indicated. This is identical to the absolute error case with W set equal to P times the specified difference or ratio.
 3. **Power:** a hypothesis test is to be performed. The power of the test $100(1-\beta) \%$ should equal the percentage specified when the true value of the difference or ratio deviates from the null hypothesis by the indicated $\Delta = \text{Difference to Detect}$. Power is defined as the probability of rejecting the null hypothesis when it is false. If a two-sided test is to be performed, then that power must be achieved both above and below the value specified by the null hypothesis. Note: when comparing more than 2 means, power refers to the F test for between group differences in the ANOVA table and Δ refers to the largest difference between any 2 means.
 4. **Sample Size:** the predetermined sample size n , assumed to be the same for all samples. This option is used to plot the power curve for a sample size that was not calculated by this procedure.

- **Confidence Level:** the level of confidence $100(1-\alpha)\%$ used when constructing confidence intervals. The value α is also used as the level of Type I error when testing hypotheses. A Type I error occurs when the null hypothesis is falsely rejected.
- **Alternative Hypothesis:** select *Not Equal* for a two-sided hypothesis test, *Less Than* if the alternative hypothesis is that the parameter is less than the value specified by the null hypothesis, or *Greater Than* if the alternative hypothesis is that the parameter is greater than the value specified by the null hypothesis.
- **Sigma:** when comparing or testing normal means, whether the standard deviation is assumed to be known (z test) or if it will be estimated from the data (t test).

For example, the dialog box above indicates that the following test is to be performed:

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
 Alt. hypothesis $H_A: \mu_1 - \mu_2 \neq 0$

The probability of a Type I error (rejecting a true null hypothesis) is set to $\alpha = 5\%$, while the probability of a Type II error (not rejecting a false null hypothesis) is set to $\beta = 10\%$ when the true absolute difference between the means equals 3.

The following table may be helpful in remembering how to set the error probabilities.

	Do Not Reject H_0	Reject H_0
H_0 is true	Correct decision	Type I error risk = α
H_0 is false	Type II error risk = β	Correct decision power = $1-\beta$

Analysis Summary

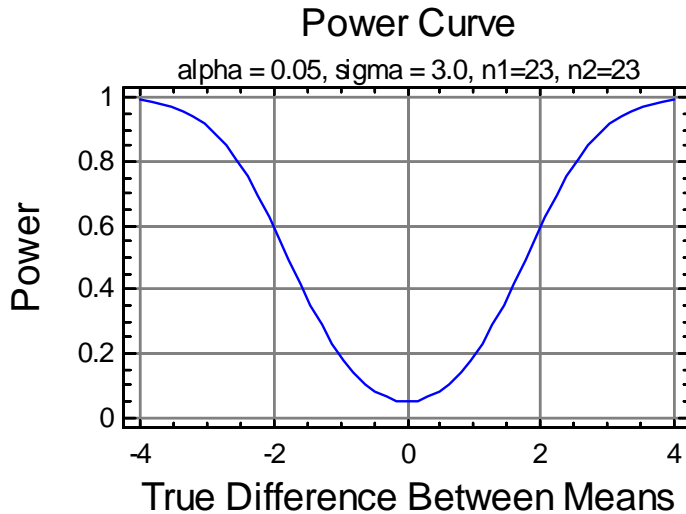
The *Analysis Summary* displays the desired goal and the sample sizes that will achieve it:

<u>Sample-Size Determination</u>
Parameter to be estimated: difference between two normal means Desired power: 90.0% for difference = 0.0 versus difference = 3.0 Type of alternative: not equal Alpha risk: 5.0% Sigma: 3.0 (to be estimated)
The required sample size is 23 observations from sample 1 and 23 observations from sample 2.

In the current example, samples of $n = 23$ observations from each population are required to achieve the power requested.

Power Curve

The *Power Curve* shows the power of the specified test of hypotheses for the derived sample sizes.



It can be seen that the power (probability of rejecting the null) is only around α when the true difference is close to zero, by it rises to $1-\beta$ when the difference varies in either direction by the specified *Difference to Detect*.

Calculations

- **Normal Mean – Confidence Interval**

If σ is assumed to be known, find the smallest n_1 and n_2 such that

$$z_{\alpha/2} \sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq W \quad (2)$$

If σ will be estimated from the data, find the smallest n_1 and n_2 such that

$$t_{\alpha/2, n_1+n_2-2} \sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq W \quad (3)$$

- **Normal Mean – Hypothesis Test**

If σ is assumed to be known, find the smallest n_1 and n_2 such that

$$\left[\frac{(z_{\alpha/2} + z_{\beta}) \sigma}{\delta} \right]^2 \leq \frac{n_1 n_2}{n_1 + n_2} \quad (4)$$

If σ is to be estimated from the data, find the smallest n_1 and n_2 such that

$$\left[\frac{(t_{\alpha/2, n_1+n_2-2} + t_{\beta, n_1+n_2-2})\sigma}{\delta} \right]^2 \leq \frac{n_1 n_2}{n_1 + n_2} \quad (5)$$

• **Normal Sigma – Confidence Interval**

Find the smallest n_1 and n_2 such that

$$\Delta \left(1 - \frac{1}{\sqrt{F_{\alpha/2, n_1-1, n_2-1}}} \right) \leq W \quad (6)$$

and

$$\Delta \left(\sqrt{F_{\alpha/2, n_2-1, n_1-1}} - 1 \right) \leq W \quad (7)$$

• **Normal Sigma – Hypothesis Test**

Find the smallest n_1 or n_2 such that

$$n_2 = 1 + \left(\frac{n_1/n_2 + 1}{2n_1/n_2} \right) \left[\frac{z_{\alpha/2} + z_{\beta}}{\ln(1 + \Delta)} \right]^2 \quad \text{if } \Delta > 0 \quad (8)$$

or

$$n_1 = 1 + \left(\frac{n_2/n_1 + 1}{2n_2/n_1} \right) \left[\frac{z_{\alpha/2} + z_{\beta}}{\ln\left(\frac{1}{1 + \Delta}\right)} \right]^2 \quad \text{if } \Delta < 0 \quad (9)$$

• **Binomial Proportion – Confidence Intervals**

Find the smallest n_1 and n_2 such that

$$z_{\alpha/2} \sqrt{\theta_0(1 - \theta_0) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} \leq W \quad (10)$$

• **Binomial Proportion – Hypothesis Tests**

Find the smallest n_1 and n_2 such that

$$\left[\frac{(z_{\alpha/2} + z_{\beta})}{2 \left(\sin^{-1} \sqrt{\theta_0 + \frac{\Delta}{2}} - \sin^{-1} \sqrt{\theta_0 - \frac{\Delta}{2}} \right)} \right]^2 \leq \frac{n_1 n_2}{n_1 + n_2} \quad (11)$$

• **Poisson Rate – Confidence Intervals**

Find the smallest n_1 and n_2 such that

$$z_{\alpha/2} \sqrt{\lambda_0 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} \leq W \quad (12)$$

• **Poisson Rate – Hypothesis Tests**

Find the smallest n_1 and n_2 such that

$$\left[\frac{(z_{\alpha/2} + z_{\beta})}{\Delta} \right]^2 \sqrt{(\lambda_0 + \Delta)(\lambda_0 - \Delta)} \leq \frac{n_1 n_2}{n_1 + n_2} \quad (13)$$

• **More Than 2 Normal Means – Confidence Intervals**

Using Tukey’s T, find the smallest common sample size n such that:

$$T_{\alpha,k,k(n-1)} \sigma \sqrt{\frac{2}{n}} \leq W \quad (14)$$

• **More Than 2 Normal Means – Hypothesis Test**

Find the smallest common sample size n for which the power of the between group F-test in the analysis of variance table equals or exceeds that specified when the largest difference between any two means equals Δ , based on a non-central F distribution with non-centrality parameter

$$\phi = \frac{|\Delta|}{\sigma} \sqrt{\frac{n}{2k}} \quad (15)$$

Note: for all one-sided tests, replace $\alpha/2$ by α in the equations for the hypothesis tests.