

Statistical Tolerance Limits (Observations)

Statistical tolerance limits give a range of values for X such that one may be $100(1-\alpha)\%$ confident that P percent of the population from which a data sample comes falls within that range. STATGRAPHICS calculates the following types of tolerance limits:

1. Parametric tolerance limits: these limits assume that the data are a random sample from a normal, lognormal or Weibull distribution.
2. Nonparametric limits: these limits do not assume normality. However, they are not as precise or as flexible as those based on the assumption of a specific distribution.

If you have a column containing n observations, you can use the *Statistical Tolerance Limits (Observations)* procedure described in this document to calculate tolerance limits. If you have only summary statistics (mean and standard deviation), then use the *Statistical Tolerance Limits (Mean and Sigma)* procedure.

Sample StatFolio: *tollims.sgp*

Sample Data:

The file *bottles.sgd* contains the measured bursting strength of $n = 100$ glass bottles, similar to a dataset contained in Montgomery (2005). Each row consists of a sample tested at 10 minute intervals. The table below shows a partial list of the data from that file:

<i>strength</i>	<i>time</i>
255	0:10
232	0:20
282	0:30
260	0:40
255	0:50
233	1:00
240	1:10
255	1:20
254	1:30
259	1:40
235	1:50
262	2:00

Bottles are required to have a bursting strength between 200 psi and 300 psi.

Data Input

The data input dialog for this procedure is shown below:

- **Data:** a column of numeric data containing measurements made on samples taken from a single population. A minimum of $n = 5$ observations is required.
- **Date/Time/Labels:** optional column containing row identifiers, used to scale the X axis in the *Run Chart*. If this field is left blank, then row numbers are used to identify observations.
- **LSL, Nominal, USL:** optional values for the lower specification limit, the nominal or target value, and the upper specification limit. If entered, these values will be added to the graphics output for comparison with the estimated tolerance limits
- **Select:** subset selection.

Analysis Options

A statistical tolerance limit allows you to make statements about the population from which your data was sampled. A two-sided interval makes a statement of the form:

“I am _____% confident that _____% percent of the population from which my data come lies between _____ and _____.”

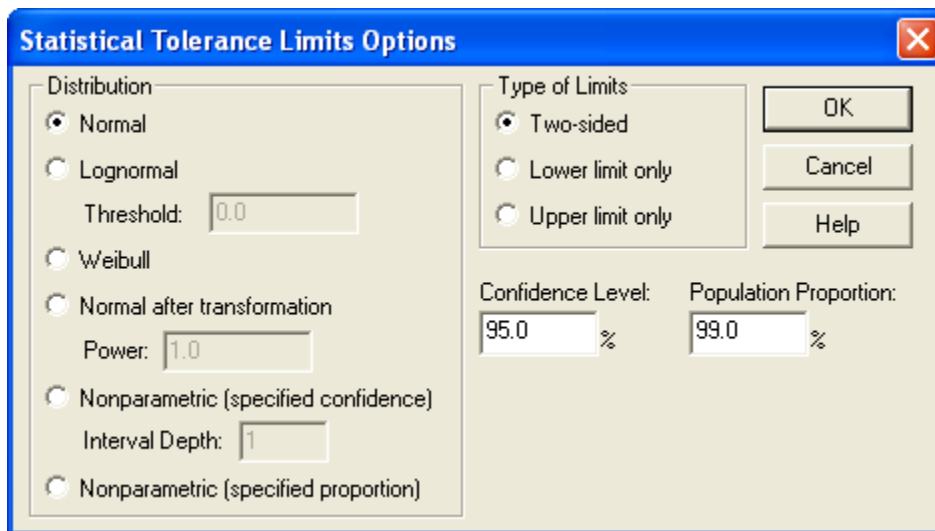
A one-sided lower tolerance bound makes a statement of the form:

“I am _____% confident that _____% percent of the population from which my data come lies at or above _____.”

A one-sided upper tolerance bound makes a statement of the form:

“I am _____% confident that _____% percent of the population from which my data come lies at or below _____.”

The *Analysis Options* dialog box is used to select the type of tolerance limits desired:



- **Distribution:** the assumed distribution for the observations. There are various choices:
 1. *Normal* – the observations are assumed to come from a normal distribution.
 2. *Lognormal* - the observations are assumed to come from a lognormal distribution. The two-parameter lognormal distribution usually requires that all $X > 0$. However, you can change the minimum or *threshold* value if desired.
 3. *Weibull* - the observations are assumed to come from a Weibull distribution. The two-parameter Weibull distribution usually requires that all $X \geq 0$. However, you can change the minimum or *threshold* value if desired.

4. *Normal after transformation* – the observations are assumed to follow a normal distribution after transforming them according to

$$X' = (X + T)^p \quad (1)$$

where p is the specified *power* and T is the specified *threshold*. For example, setting $p = 0.5$ indicates that the square roots of the data follow a normal distribution.

5. *Nonparametric (specified confidence)* – no distribution is assumed. In this case, the confidence level may be specified but not the population proportion. Nonparametric tolerance limits use the d -th smallest and the d -th largest observation to set the limits, where d is called the *interval depth*.
6. *Nonparametric (specified proportion)* – no distribution is assumed. In this case, the population proportion may be specified but not the confidence level.
- **Type of limits** – either a two-sided tolerance interval or a one-sided tolerance bound may be requested.
 - **Confidence level** – the confidence level at which the limits will be calculated.
 - **Population proportion** – the proportion of the population bound by the tolerance limits.

Analysis Summary

Given a level of confidence $(1-\alpha)$ and a population percentage P , the *Analysis Summary* calculates and displays the tolerance limits:

<u>Statistical Tolerance Limits - strength</u>
Data variable: strength
Distribution: Normal
sample size = 100
mean = 254.64
std. dev. = 10.6823
95.0% tolerance interval for 99.0% of the population
Xbar +/- 2.93431 sigma
Upper: 285.985
Lower: 223.295

If a normal distribution is assumed, two-sided tolerance limits may be calculated by taking the sample mean plus and minus a multiple of the standard deviation, according to

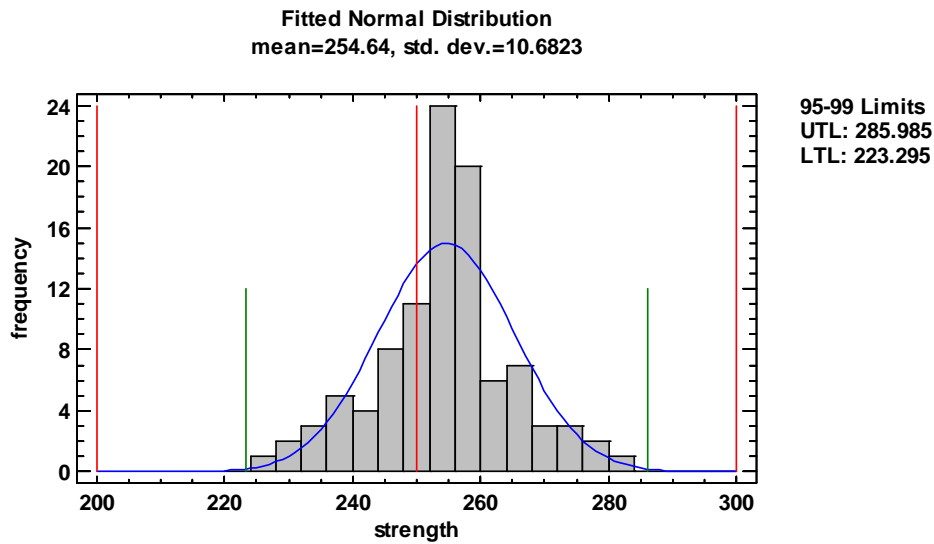
$$\bar{x} \pm Ks \quad (2)$$

The factor K depends upon the sample size n , the level of confidence $(1-\alpha)$, and the specified percentage P .

For example, the above table states that one may be 95% confident that 99% of the population from which the data came lies between 223.295 and 285.985.

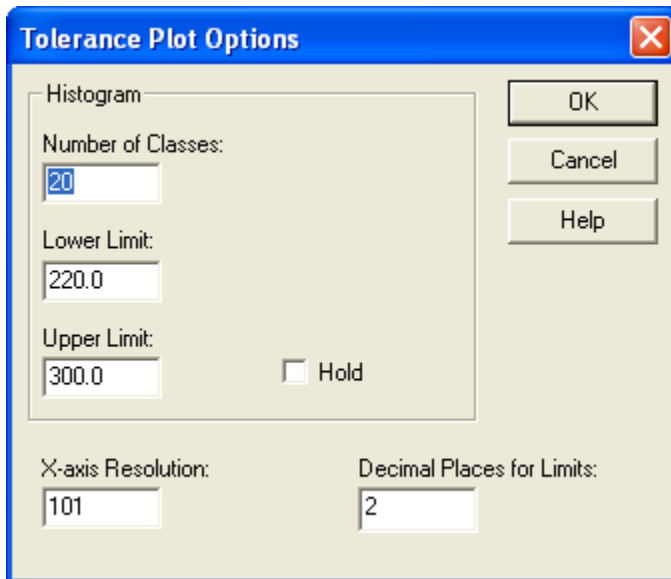
Tolerance Plot

This plot shows a histogram of the data together with the fitted distribution (unless nonparametric limits have been requested):



The shorter vertical lines indicate the location of the tolerance limits. The taller vertical lines show the specification limits (if any).

Pane Options



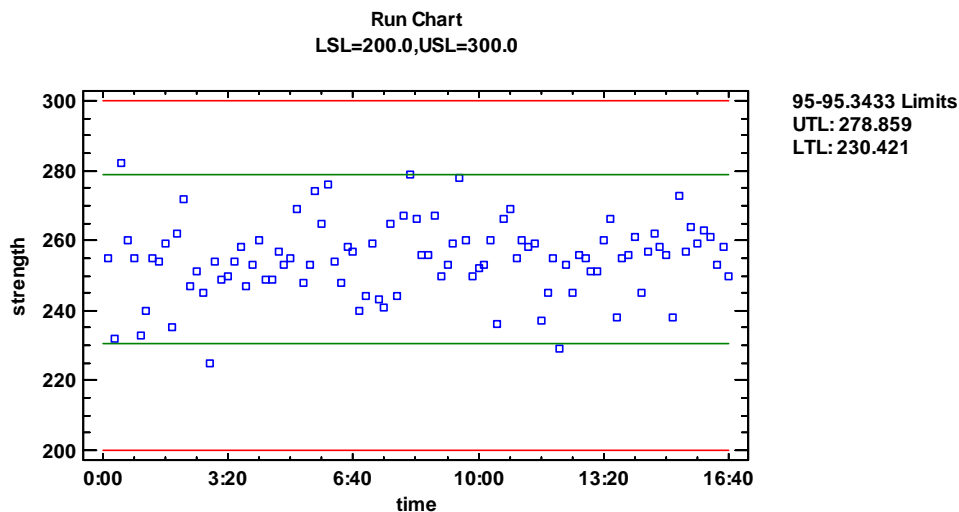
- **Number of classes:** the number of intervals into which the data will be divided. Intervals are adjacent to each other and of equal width. The number of intervals into which the data is

grouped by default is set by the rule specified on the *EDA* tab of the *Preferences* dialog box on the *Edit* menu.

- **Lower Limit:** lower limit of the first interval.
- **Upper Limit:** upper limit of the last interval.
- **Hold:** maintains the selected number of intervals and limits even if the source data changes. By default, the number of classes and the limits are recalculated whenever the data changes. This is necessary so that all observations are displayed even if some of the updated data fall beyond the original limits.
- **Decimal Places for Limits:** number of decimal places to use when displaying the estimated limits.

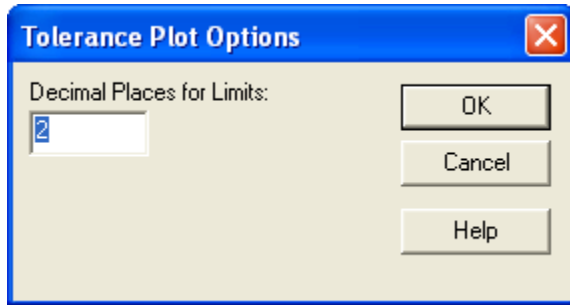
Run Chart

This plot shows a sequential plot of the observations together with the estimated tolerance limits and the specification limits (if any):



If an entry was made in the *Date/Time/Labels* field on the initial data input dialog box, those values are used to scale the horizontal axis. Otherwise, the data are plotted versus row number.

Pane Options



- **Decimal Places for Limits:** number of decimal places to use when displaying the estimated limits.

Tests for Normality

The estimated tolerance limits are strongly dependent on the assumed distribution of the observations. By default, it is usually assumed that the data follow a normal distribution. The *Tests for Normality* pane performs one or more tests to determine whether or not this is a reasonable assumption. For each test, the hypotheses of interest are:

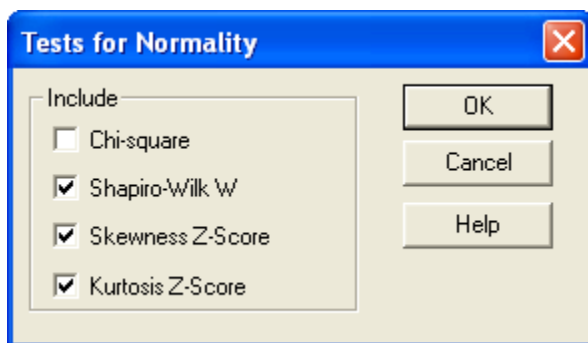
- Null hypothesis: data are independent samples from a normal distribution
- Alt. hypothesis: data are not independent samples from a normal distribution

Tests for Normality for strength		
Test	Statistic	P-Value
Shapiro-Wilk W	0.97781	0.416181
Skewness Z-score	0.399024	0.689872
Kurtosis Z-score	1.18663	0.235371

The tests to be run are selected using *Pane Options*. Each test is displayed with its associated test statistic and *P-Value*. Small P-values (below 0.05 if operating at the 5% significance level) lead to a rejection of the null hypothesis and thus to a rejection of the normal distribution. In the above table, the P-Values are all well above 0.05, so there is not statistically significant non-normality in the data.

For a detailed description of the tests, see the documentation for *Distribution Fitting (Uncensored Data)*.

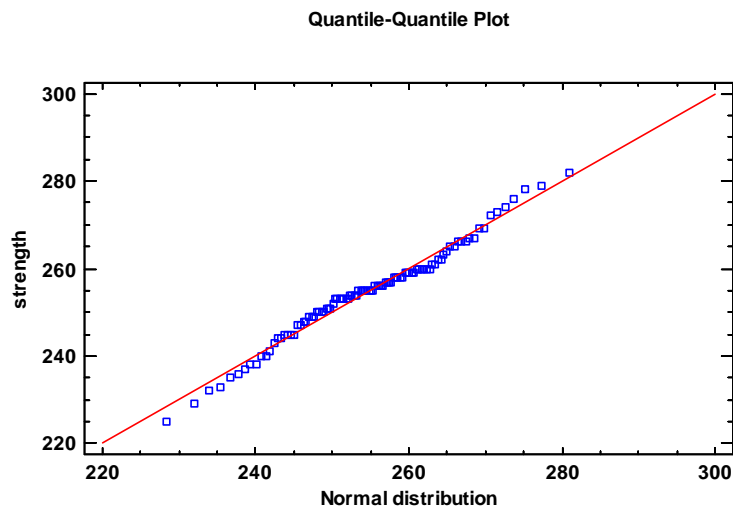
Pane Options



Include – select the tests to include in the output. The default tests are defined on the *Dist. Fit* tab of the *Preferences* dialog box accessed from the *Edit* menu.

Quantile-Quantile Plot

The *Quantile-Quantile Plot* shows the percentiles of the observed data plotted versus the equivalent percentiles of the fitted distribution:



If the assumed distribution fits the data well, the points should lie close to the line.

Goodness-of-Fit Tests

If a distribution other than the normal is fit to the data, its adequacy as a model for the data can be tested using the *Goodness-of-Fit Tests*. Up to 7 different tests may be performed. For all tests, the hypotheses of interest are:

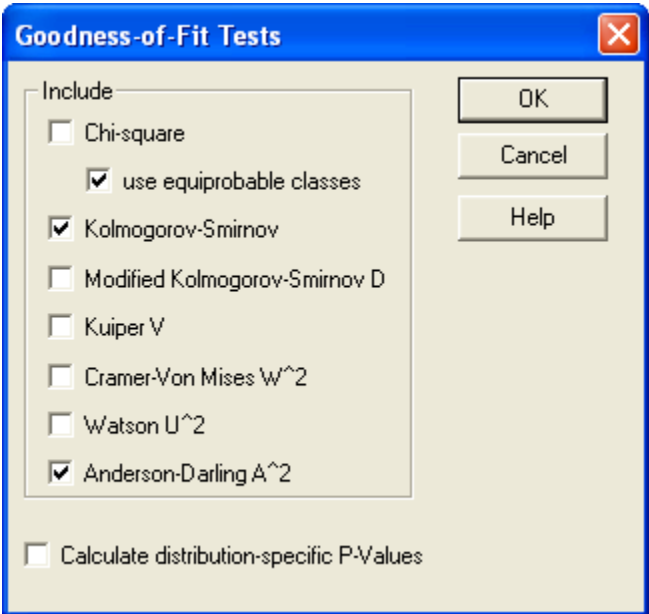
- Null hypothesis: data are independent samples from the specified distribution
- Alt. hypothesis: data are not independent samples from the specified distribution

Goodness-of-Fit Tests for strength			
Kolmogorov-Smirnov Test			
	<i>Lognormal Distribution</i>		
DPLUS	0.0833684		
DMINUS	0.107325		
DN	0.107325		
P-value	0.199867		
<i>EDF Statistic</i>	<i>Value</i>	<i>Modified Form</i>	<i>P-Value</i>
Anderson-Darling A ²	0.88292	0.88292	>=0.10

*Indicates that the P-Value has been compared to tables of critical values specially constructed for fitting the currently selected distribution. Other P-values are based on general tables and may be very conservative.

The tests to be run are selected using *Pane Options*. Two common tests are shown above. In each case, if the P-Value is large (greater or equal to 0.05 if operating at the 5% significance level), then the distribution is a reasonable model for the data. For a detailed description of the tests, see the documentation for *Distribution Fitting (Uncensored Data)*.

Pane Options



- **Include:** select one or more tests to perform. For the chi-squared test, select *use equiprobable classes* to group data into classes with equal expected frequencies. If this option is not checked, classes will be created that match the *Capability Plot*.
- **Calculate distribution-specific P-Values:** if checked, the P-Values will be based on tables or formulas specifically developed for the distribution being tested. Otherwise, the P-Values will be based on a general table or formula that applies to all distributions. The general approach is more conservative (will not reject a distribution as easily) but may be preferred when comparing P-Values amongst different distributions.

Nonparametric Limits

The d -th smallest and d -th largest values in a data sample may be used to construct a tolerance interval for the population from which the data come without assuming any specific distribution. The resulting tolerance limits give a range of values for X such that one may be $100(1-\alpha)\%$ confident that **at least** P percent of the population from which a data sample comes falls within that range. The interval can be quite conservative, with the actual percentage being much larger than that stated.

Statistical Tolerance Limits - strength

Data variable: strength

Nonparametric Tolerance Limits

sample size = 100

depth: 1

95.0% tolerance interval for 95.3433% of the population

Upper: 282.0

Lower: 225.0

The output above shows that one can be 95% confident that at least 95.3433% of the population lies between 225 (the smallest observation) and 282 (the largest observation).

Calculations**Normal Tolerance Limits**

2-sided limits:

$$\bar{x} \pm Ks \quad (3)$$

1-sided bounds:

$$\bar{x} - Ks \quad \text{or} \quad \bar{x} + Ks \quad (4)$$

Lognormal Tolerance Limits

The limits are calculated by taking the logarithms of the data, calculating normal tolerance limits in the log scale, and then inverting the limits.

Weibull Tolerance Limits

$(1-\alpha)\%$ one-sided limits are calculated using one end of the $(1-2\alpha)\%$ confidence limits for the p -th percentile of the Weibull distribution, using the method described in the document titled *Weibull Analysis*. Conservative two-sided limits are calculated using the $(1-\alpha)\%$ confidence limits for the $p/2$ -th and the $(1-p/2)$ -th percentiles.

Normal Tolerance Limits (after transformation)

The limits are calculated by transforming the data, calculating normal tolerance limits in the transformed metric, and then inverting the limits.

Nonparametric Tolerance Limits

Let $x_{(d)}$ be the d -th smallest value in the sample, and let $x_{(n-d+1)}$ be the d -th largest value. These values form a tolerance interval that contains at least $100P\%$ of the population with $100(1-\alpha)\%$ confidence where

$$P = (q-1)/(q+1) \quad (5)$$

$$q = \frac{4(n-d+.5)}{X_{\alpha,4d}^2} \quad (6)$$