T Chart

Summary

The **T Chart** procedure creates a control chart based on the intervals of time between the occurrence of rare events. It uses the Weibull distribution, which assumes that events occur at locations recorded in a continuous time scale. This type of chart is used frequently in the health sciences to monitor the occurrence of events such as post-surgical infections.

Note: If events can only occur at discrete points in time (or are recorded at discrete locations such as a day), then a G Chart should be used in place of a T Chart.

The chart may be constructed in either *Initial Study* (Phase 1) mode, where the current data determine the control limits, or in *Control to Standard* (Phase 2) mode, where the limits come from either a known standard or from prior data.

The data for this analysis may be in either of 2 formats:

1. *Time of occurrence*. In this format, the data describe the time at which each event occurred:

   t_i = time of occurrence of event *i*.

2. *Time between occurrences*. In this format, the data represent the amount of time between each pair of consecutive occurrences.

   x_j = time between the occurrence of event *j*-1 and occurrence of event *j*.

If the times of occurrence are given, the times between occurrence of events are calculated according to:

$$x_j = t_j - t_{j-1} \tag{1}$$

The number of interevent times will be represented by *m*.

Sample StatFolio: *tchart.sgp*

Sample Data

The file *infection times.sgd* contains the date and time at which infections occurred in a hospital at the beginning of 2012. Since there were 100 infections, this results in a total of $m = 99$ measurements of the time between infections. The first several rows of that file are shown below:

Date of infection
1/1/12 8:11
1/2/12 5:42
1/3/12 21:55
1/5/12 5:24
1/7/12 10:14
1/9/12 11:11

The chart below shows the times between the occurrence of infections, plotted versus the date and time of infection:
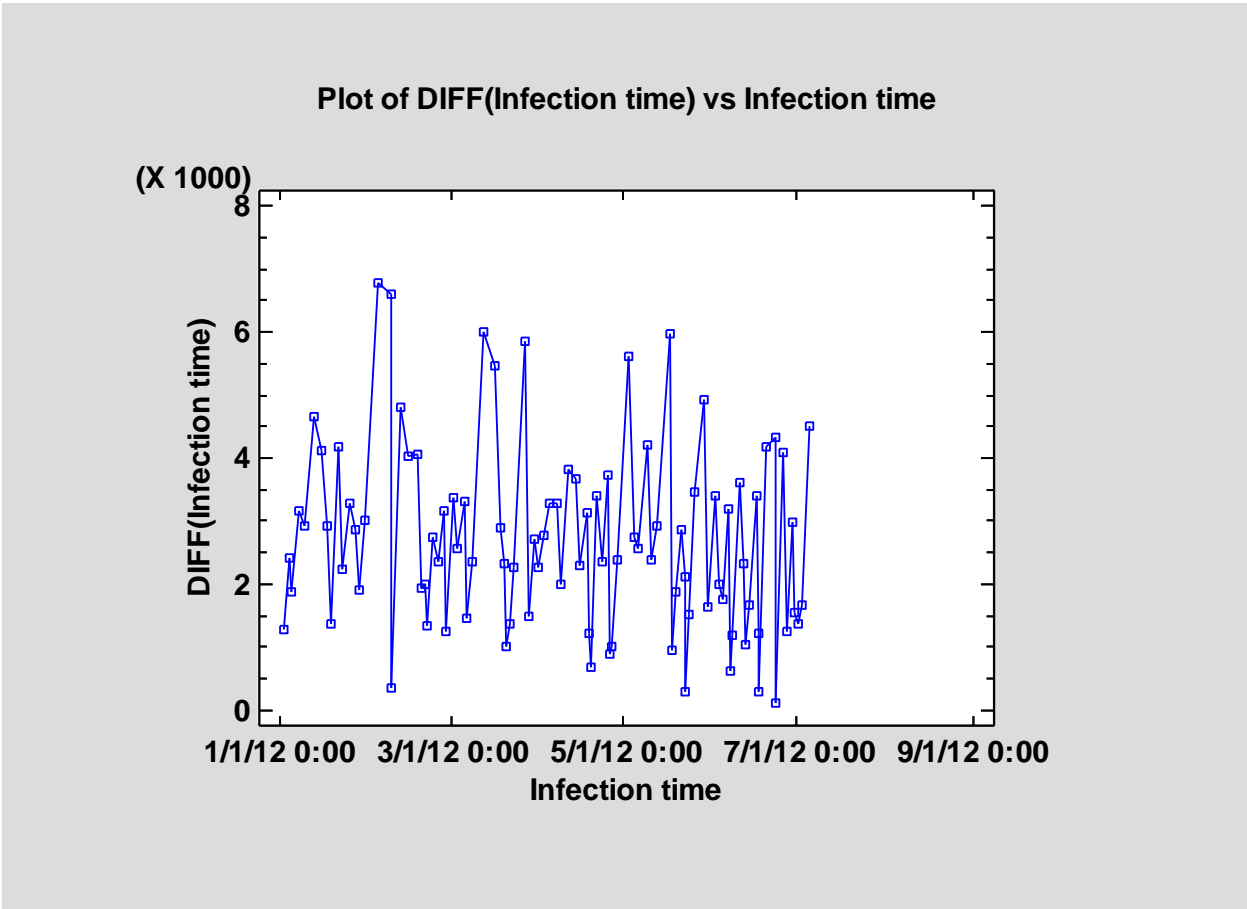

Time is measured in minutes.

Data Input

The data input dialog box for the t chart is shown below:

- **Data:** the data to be analyzed in the format specified by the *Type of data* radio buttons.

- **Type of data**: an indication of the format in which the data have been entered. If *Time of occurrence* is selected, then the data specify the time at which each event occurred. If *Time between occurrences* is selected, then the data represent the time between each event. Note that the t chart assumes that events occur over a continuous interval of time. Events occurring at discrete points (such as days) should be plotted using a *g chart*.

- **Labels:** optional labels for each data value.

- **LSL, Nominal, USL**: optional lower specification limit, nominal (target) value, and upper specification limit for the time between events.

- **Select:** subset selection.

T Chart

The *T Chart* plots the time between events *x*:
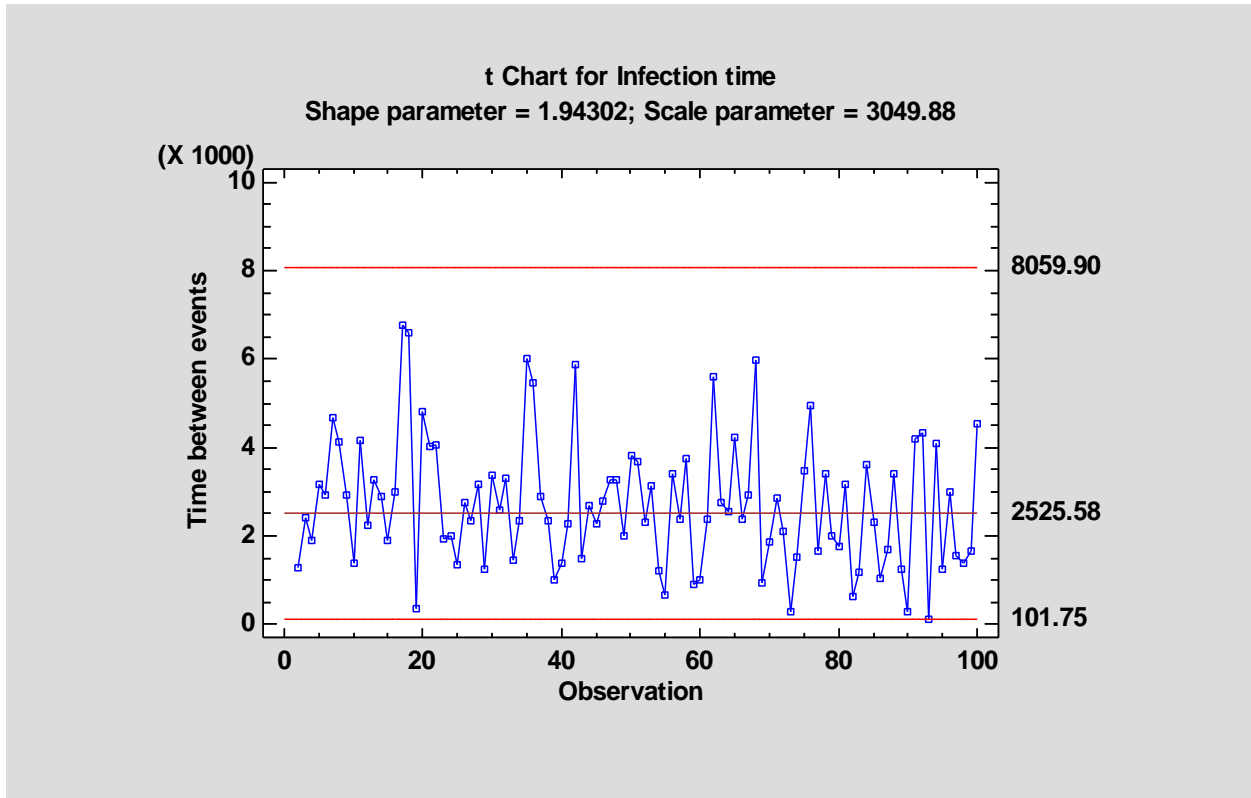
It is assumed that the time between events follows a Weibull distribution which has a probability density function given by:

$$f(x) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha} \tag{2}$$

The parameter α is called the shape parameter, while β is called the scale parameter. The mean time between events is equal to

$$\mu = \frac{\beta}{\alpha} \Gamma\left(\frac{1}{\alpha}\right) \tag{3}$$

and the standard deviation is equal to

$$\sigma = \left(\beta/\sqrt{\alpha}\right)\left[2\Gamma\left(\frac{2}{\alpha}\right) - \frac{1}{\alpha}\Gamma\left(\frac{1}{\alpha}\right)^2\right]^{1/2} \tag{4}$$

In Phase 1 (*Initial Studies*) mode, the centerline and control limits are determined from the data. To calculate the limits, the maximum likelihood estimators for the parameters are first calculated. The centerline is then placed at the 50th percentile of the estimated Weibull

distribution. The lower and upper control lines are placed at the 0.135th percentile and the 99.865th percentile (assuming that 3-sigma limits are requested).

In Phase 2 (*Control to Standard*) mode, the user specifies the assumed values of the shape and scale parameters on the *Analysis Options* dialog box, as shown below:
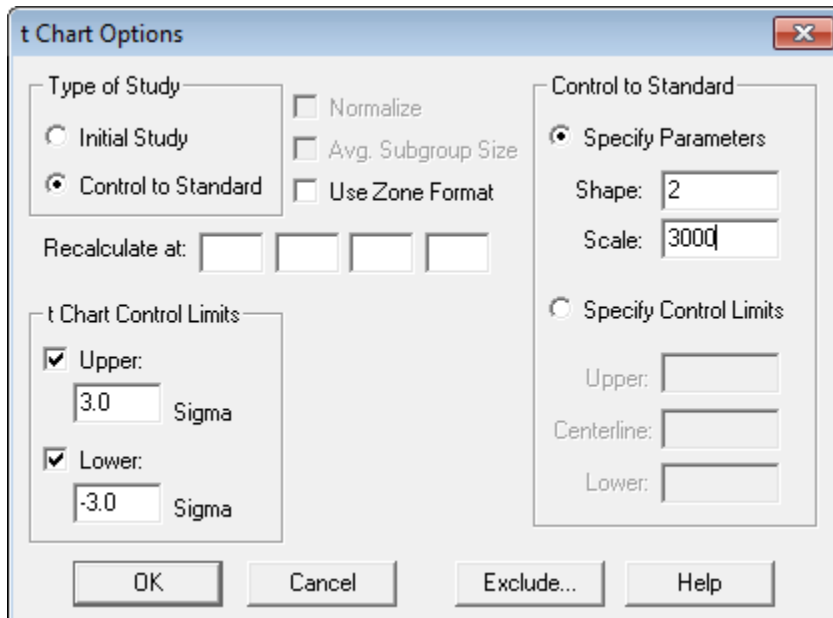
Any points beyond the control limits will be flagged using a special point symbol. Any point excluded from the analysis, usually by clicking on a point on the chart and pressing the *Exclude/Include* button, will be indicated by an X. If so indicated on the *Pane Options* dialog box, unusual sequences of points may also be flagged. In the current chart, all samples are within the control limits.

Pane Options
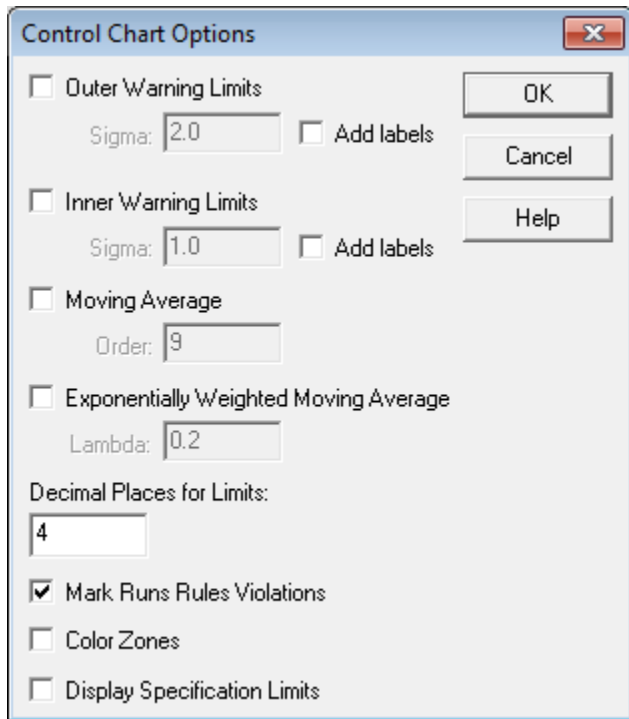
- **Outer Warning Limits**: check this box to add warning limits at the specified multiple of sigma, usually at 2 sigma.

- **Inner Warning Limits**: check this box to add warning limits at the specified multiple of sigma, usually at 1 sigma.

- **Moving Average:** check this box to add a moving average smoother to the chart. In addition to the sample data, the average of the most recent q points will also be displayed, where q is the *order* of the moving average.

- **Exponentially Weighted Moving Average:** check this box to add an EWMA smoother to the chart. In addition to the sample data, an exponentially weighted moving average of the sample data will also be displayed, where λ is the smoothing parameter of the EWMA.

- **Decimal Places for Limits:** the number of decimal places used to display the control limits.

- **Mark Runs Rules Violations:** flags with a special point symbol any unusual sequences or runs. The runs rules applied by default are specified on the *Runs Tests* tab of the *Preferences* dialog box.

- **Color Zones:** check this box to display green, yellow and red zones.

- **Display Specification Limits**: whether to add horizontal lines to the chart displaying the location of the specification limits (if any).

Note: the warning limits are placed relative to the control limits. By default, they are located at one-third and two-thirds of the distance to the "3-sigma" limits.
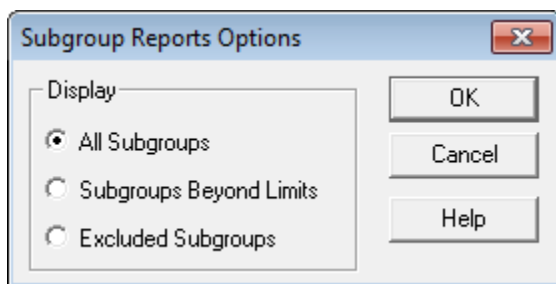
T Chart Report

This pane tabulates the values plotted on the control chart:

T Chart Report
All Observations
X = Excluded   * = Beyond Limits

Observation	Infection time	Time between events
1	1/1/12 8:11	
2	1/2/12 5:42	1291.0
3	1/3/12 21:55	2413.0
4	1/5/12 5:24	1889.0
5	1/7/12 10:14	3170.0
6	1/9/12 11:11	2937.0
7	1/12/12 16:56	4665.0
8	1/15/12 13:39	4123.0
9	1/17/12 14:26	2927.0
10	1/18/12 13:26	1380.0
11	1/21/12 10:53	4167.0
12	1/22/12 23:58	2225.0
13	1/25/12 6:34	3276.0
14	1/27/12 6:31	2877.0
15	1/28/12 14:18	1907.0
16	1/30/12 16:20	3002.0
17	2/4/12 9:15	6775.0
18	2/8/12 23:19	6604.0
19	2/9/12 5:15	356.0
20	2/12/12 13:34	4819.0

Out-of-control points are indicated by an asterisk. Points excluded from the calculations are indicated by an X.

Pane Options

- **Display:** specify the observations to display in the report.

Analysis Summary

The *Analysis Summary* summarizes the data and the control chart:

T Chart - Infection time
Number of observations = 99
0 observations excluded

t Chart

Period	#2-100
UCL: +3.0 sigma	8059.9
Centerline	2525.58
LCL: -3.0 sigma	101.748

0 beyond limits

Parameter Estimates

Period	#2-100
Shape	1.94302
Scale	3049.88

Included in the table are:

- **Sample Information**: the number of interevent times *m* and the number of observations (if any) that have been excluded from the calculations.

- **t Chart:** a summary of the centerline and control limits for the t chart. Using *Analysis Options*, separate limits may be calculated for different periods (sets of samples).

- **Estimates:** estimates of the shape parameter $\hat{\alpha}$ and the scale parameter $\hat{\beta}$.

Analysis Options

- **Type of Study:** determines how the control limits are set. For an *Initial Study* (Phase 1) chart, the limits are estimated from the current data. For a *Control to Standard* (Phase 2) chart, the control limits are determined from the information in the *Control to Standard* section of the dialog box.

- **Use Zone Format:** if checked, the statistics will be plotted using a zone chart rather than the usual format.

- **Recalculate at:** the control limits of the chart may be recalculated at up to 4 locations along the X axis by specifying the sample numbers at which new estimates are to be initiated. Separate estimates of the shape and scale parameters will be obtained using the data in the different sections. In such cases, the control limits will be adjusted at the start of each new period.

- **t Chart Control Limits:** specify the multiple *k* to use in determining the upper and lower control limits on the t chart. To suppress a limit completely, enter 0.

- **Control to Standard:** to perform a Phase 2 analysis, select *Control to Standard* for the *Type of Study* and then enter either of the following:

  Specify Parameters: specify the established standard shape and scale parameters.

  Specify Control Limits: specify the location of the centerline and control limits exactly where you wish them to be placed.

- **Exclude**: use this button to exclude specific samples from the calculations.
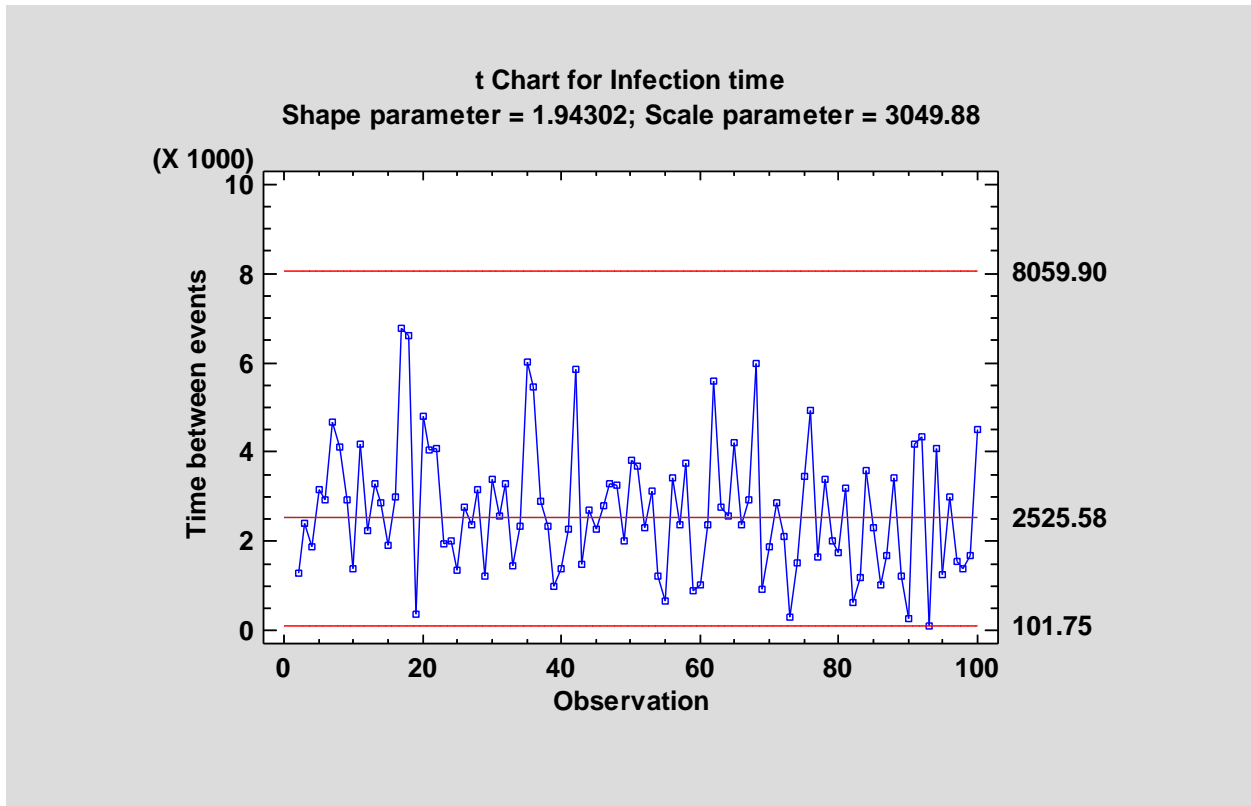
Runs Tests

The *Runs Tests* pane displays the results of standard tests applied to the t chart to look for unusual sequences of points.

Runs Tests
Rules
(A) runs above or below centerline of length 8 or greater.
(B) runs up or down of length 8 or greater.
(G) sets of 8 observations alternating up and down

Violations

Observation	t Chart
26	G
27	G
28	G
29	G
30	G
31	G
32	G
33	G
34	G

Depending on the default settings on the *Runs Tests* tab of the *Preferences* dialog box, STATGRAPHICS will look for up to 7 different types of patterns:

    A: A group of 8 or more points, all above or all below the centerline.

    B: A group of 8 or more points, all increasing or all decreasing.

    C: A group of 5 points in which at least 4 are more than 1-sigma away from the centerline, on the same side of the centerline.

    D: A group of 3 points in which at least 2 are more than 2-sigma away from the centerline, on the same side of the centerline.

    E: A group of 15 or more points, all within 1-sigma.

    F: A group of 8 or more points, all beyond 2-sigma, but not necessarily on the same side of the centerline.

    G: A group of 8 or more points, all following an alternating up and down pattern.

Any such runs will be indicated in the above table and also on the control charts (unless suppressed). The output above shows that an unusual sequence of alternating ups and downs is first signaled at observation #26. This may be seen in the chart below:

t Chart for Infection time
Shape parameter = 1.94302; Scale parameter = 3049.88

Pane Options

Select the runs tests to be applied and the parameters that define those tests.

Notes:

1. The consecutive zeroes run test is not relevant for this type of control chart.
2. For run tests C through F, the "1-sigma" and "2-sigma" limits are placed one-third and two-thirds of the distance to the control limits, respectively.

Save Results

The following results can be saved to the datasheet:

1. *Times* – the interevent times.
2. *Labels* – the sample labels.
3. *Shape parameter* – the estimated or specified shape parameter.
4. *Scale parameter* – the estimated or specified scale parameter.

References

Cleary, Barbara A. (2010) *Charting Rare Events Effectively*. QualityDigest.com