

U Chart

Summary

The **U Chart** procedure creates a control chart for data that describes the number of nonconformities per unit recorded as the result of inspecting m samples. The samples could represent a group of n items, a fixed period of time, a specified square meters of fabric, or some other sampling unit of fixed size. Unlike the data plotted using a p chart or np chart, the inspection of an item within the sample does not result in simply an acceptable or unacceptable rating. Rather, it results in a count of the total number of defects or problems observed with that item. The chart may be constructed in either *Initial Study* (Phase 1) mode, where the current data determine the control limits, or in *Control to Standard* (Phase 2) mode, where the limits come from either a known standard or from prior data.

The data for this analysis consist of m samples from a population detailing:

n_j = size of sample j . This could be expressed as the number of items inspected or the amount of product or time that the sample represents.

c_j = number of nonconformities observed in sample j

Many of the options for the U chart are the same as those of the P chart. This document highlights the differences between the 2 charts. For information on options such as runs test and ARL curves, consult the *P Chart* documentation.

Sample StatFolio: *uchart.sgp*

Sample Data:

The file *cloth.sgd* contains data on $m = 10$ samples of dyed cloth, presented by Montgomery (2005). A sample of cloth from each of 10 rolls was taken and the number of defects was counted. The table below shows the data in that file:

Sample	Square Meters	Nonconformities
1	500	14
2	400	12
3	650	20
4	500	11
5	475	7
6	500	10
7	600	21
8	525	16
9	600	19
10	625	23

Note that the size of the samples varies from roll to roll.

Data Input

The data required by this procedure is a numeric column containing the number of defects or nonconformities items in each of m samples, together with the sample sizes.

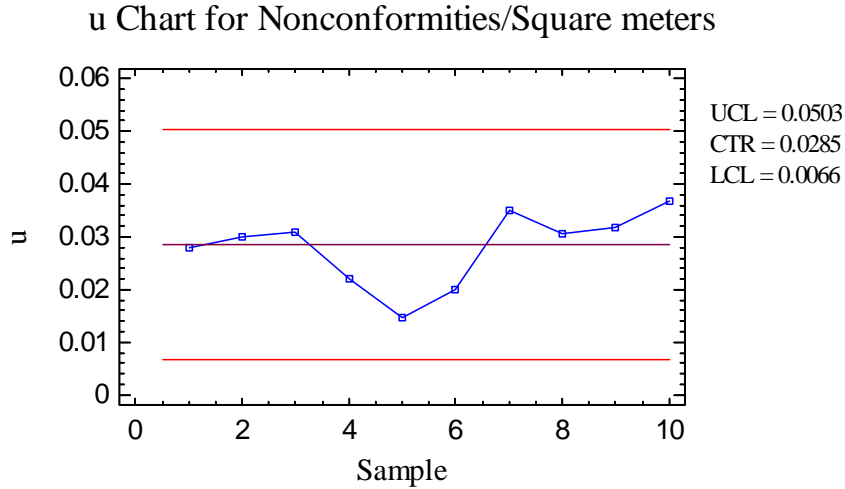
- **Counts:** a column containing the number of nonconformities in each sample.
- **Sample Sizes:** the sizes of the samples. If all sample sizes are equal, you can enter a single value such as “500”. Otherwise, enter a column with the sizes corresponding to each count.
- **Sample Labels:** optional labels for each sample.
- **LSL, Nominal, USL:** optional lower specification limit, nominal (target) value, and upper specification limit.
- **Select:** subset selection.

In entering the data, the above dialog box calculates the rate of detects per square meter “on-the-fly” by dividing the counts by the sample sizes:

$$u_j = c_j / n_j \quad (1)$$

U Chart

The *U Chart* plots the rates of nonconformities per unit within each sample.



In Phase 1 (*Initial Studies*) mode, the centerline and control limits are determined from the data. The centerline is located at:

$$\bar{u} = \frac{\sum_{j=1}^m n_j u_j}{\sum_{j=1}^m n_j} \quad (2)$$

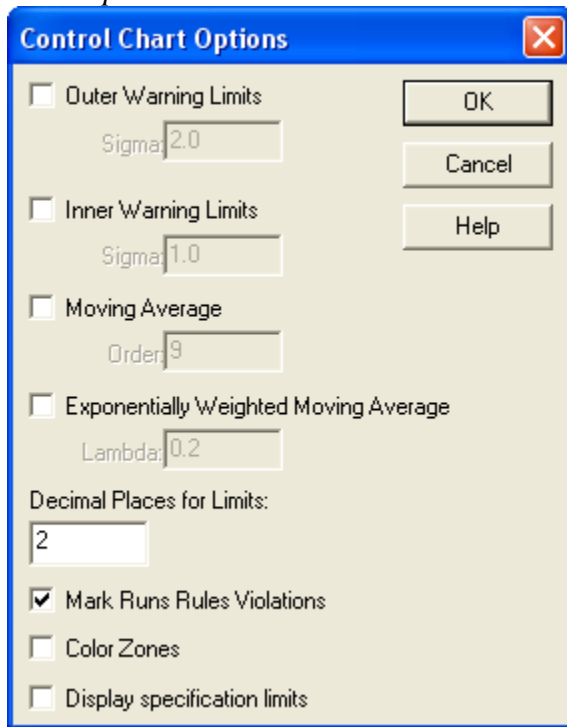
The control limits are placed above and below the centerline at:

$$\bar{u} \pm 3\sqrt{\bar{u}/n} \quad (3)$$

where k is the sigma multiple specified on the *Control Charts* tab of the *Preferences* dialog box ($k = 3$ except in rare cases) and n is the sample size. If the sample sizes are not equal, then depending on *Analysis Options*, n is replaced by either:

- (1) \bar{n} , the average sample size. In this case, the control limits are the same for all samples.
- (2) n_j , the individual sample sizes. In this case, the control limits are step functions.

Any points beyond the control limits will be flagged using a special point symbol. Any point excluded from the analysis, usually by clicking on a point on the chart and pressing the *Exclude/Include* button, will be indicated by an X. If so indicated on the *Pane Options* dialog box, unusual sequences of points may also be flagged. In the current chart, all samples are well within the control limits.

Pane Options

- **Outer Warning Limits:** check this box to add warning limits at the specified multiple of sigma, usually at 2 sigma.
- **Inner Warning Limits:** check this box to add warning limits at the specified multiple of sigma, usually at 1 sigma.
- **Moving Average:** check this box to add a moving average smoother to the chart. In addition to the sample data, the average of the most recent q points will also be displayed, where q is the *order* of the moving average. The default value $q = 9$ since the 1-sigma inner warning limits for the original data are equivalent to the 3-sigma control limits for that order moving average.
- **Exponentially Weighted Moving Average:** check this box to add an EWMA smoother to the chart. In addition to the sample data, an exponentially weighted moving average of the sample data will also be displayed, where λ is the smoothing parameter of the EWMA. The default value $\lambda = 0.2$ since the 1-sigma inner warning limits for the original rates are equivalent to the 3-sigma control limits for that EWMA.
- **Decimal Places for Limits:** the number of decimal places used to display the control limits.
- **Mark Runs Rules Violations:** flags with a special point symbol any unusual sequences or runs. The runs rules applied by default are specified on the *Runs Tests* tab of the *Preferences* dialog box.
- **Color Zones:** check this box to display green, yellow and red zones.

- **Display Specification Limits:** whether to add horizontal lines to the chart displaying the location of the specification limits (if any).

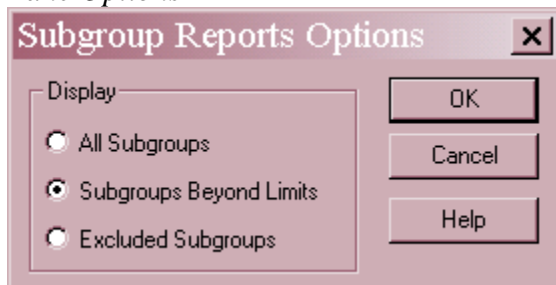
U Chart Report

This pane tabulates the values plotted on the control chart:

u Chart Report		
All Samples		
X = Excluded * = Beyond Limits		
<i>Sample</i>	<i>Size</i>	<i>u</i>
1	500.0	0.028
2	400.0	0.03
3	650.0	0.0307692
4	500.0	0.022
5	475.0	0.0147368
6	500.0	0.02
7	600.0	0.035
8	525.0	0.0304762
9	600.0	0.0316667
10	625.0	0.0368

Out-of-control points are indicated by an asterisk. Points excluded from the calculations are indicated by an X.

Pane Options



- **Display:** specify the samples (subgroups) to display in the report.

Analysis Summary

The *Analysis Summary* summarizes the data and the control chart.

<u>u Chart - Nonconformities/Square meters</u>	
Number of samples = 10	
Average sample size = 537.5	
0 samples excluded	
u Chart	
Period	#1-10
UCL: +3.0 sigma	0.0502969
Centerline	0.0284651
LCL: -3.0 sigma	0.00663337
0 beyond limits	
Estimates	
Period	#1-10
Mean u	0.0284651
Sigma	0.00727725

Included in the table are:

- **Sample Information:** the number of samples m and the average sample size

$$\bar{n} = \frac{\sum_{j=1}^m n_j}{m} \tag{4}$$

If any samples have been excluded from the calculations, that number is also displayed.

- **u Chart:** a summary of the centerline and control limits for the u chart. Using *Analysis Options*, separate limits may be calculated for different periods (sets of samples).
- **Estimates:** estimates of the process mean rate

$$\hat{\mu} = \bar{u} \tag{5}$$

and the process sigma

$$\hat{\sigma} = \sqrt{\bar{u} / \bar{n}} \tag{6}$$

Analysis Options

- **Type of Study:** determines how the control limits are set. For an *Initial Study* (Phase 1) chart, the limits are estimated from the current data. For a *Control to Standard* (Phase 2) chart, the control limits are determined from the information in the *Control to Standard* section of the dialog box.
- **Normalize:** if selected, all statistics will be normalized by calculating Z-scores and the Z-scores plotted on the charts. The centerline on such a chart is always located at 0, and the control limits are always located at $\pm k$.
- **Avg. Subgroup Size:** if checked, the control limits will be horizontal lines based on the average sample size. If not checked, the individual sample sizes will be used, resulting in step function control limits if the sample sizes are not all equal.
- **Use Zone Format:** if checked, the statistics will be plotted using a zone chart rather than the usual format. See the example below.
- **Recalculate at:** the control limits of the chart may be recalculated at up to 4 locations along the X axis by specifying the sample numbers at which new estimates are to be initiated. Separate estimates of the mean number of nonconforming items per unit will be obtained using the data in the different sections. In such cases, the control limits will be adjusted at the start of each new period.
- **U Chart Control Limits:** specify the multiple k to use in determining the upper and lower control limits on the u chart. To suppress a limit completely, enter 0.
- **Control to Standard:** to perform a Phase 2 analysis, select *Control to Standard* for the *Type of Study* and then enter either of the following:

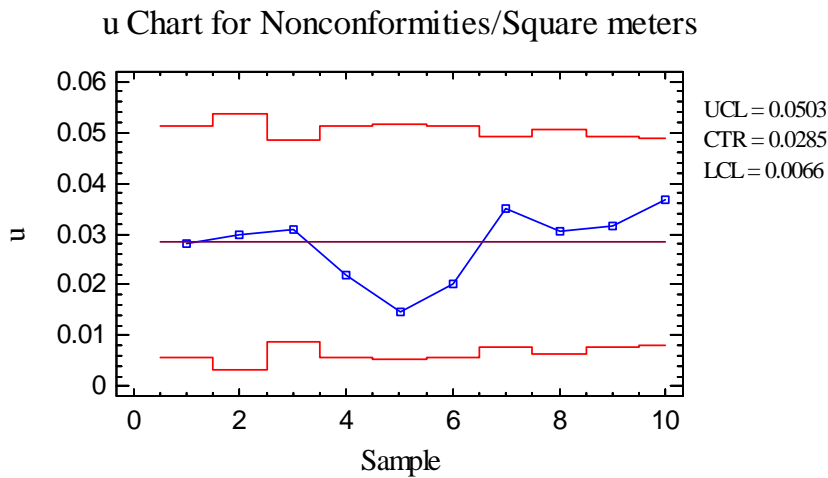
Specify Parameters: specify the established standard mean number of nonconforming items per unit.

Specify Control Limits: specify the location of the centerline and control limits exactly where you wish them to be placed.

- **Exclude:** use this button to exclude specific samples from the calculations.

Example: Using Individual Sample Sizes

The above u chart used the average sample size to position the control limits. When the sample sizes are not equal, a more exact analysis lets the control limits vary according to the actual size of each sample. In STATGRAPHICS, this is done by unchecking *Avg. Subgroup Size* on the *Analysis Options* dialog box. The resulting control limits are step functions:



The control limits are tighter for the smaller samples, since the number of nonconforming items in smaller samples is less variable than it is in larger samples.

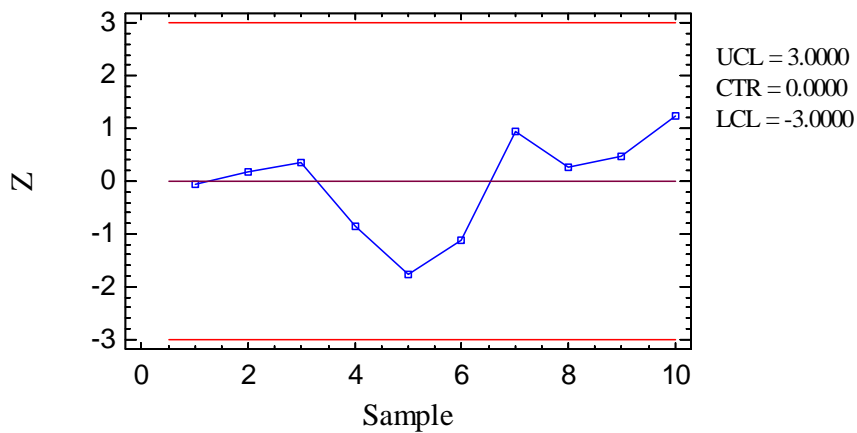
Example: Using a Normalized Chart

Unequal sample sizes can also be dealt with by normalizing all values on the chart. This is done by checking the *Normalize* field on the *Analysis Options* dialog box. If *Avg. Sample Size* is left unchecked, then Z scores are calculated for each sample according to:

$$Z_j = \frac{u_j - \bar{u}}{\sqrt{\bar{u} / n_j}} \tag{7}$$

The Z-scores are plotted on the chart:

u Chart for Nonconformities/Square meters



On all normalized charts, the centerline is located at 0 and the control limits at $\pm k$. The plotted values represent the number of standard deviations that each rate is above or below its expected value.

Save Results

The following results can be saved to the datasheet:

1. *us* – the sample rates.
2. *Sizes* – the sample sizes.
3. *Labels* – the sample labels.
4. *Mean u* – the estimated mean rate of nonconformities per unit.
5. *Sigma* – the estimated standard deviation of the rates for a sample of average size.