# Thrust Calculation

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The thrust required to move a mass a given distance within a given time may be calculated by summing all of the forces that act on the mass. These forces generally fall within the following four categories:

- **Gravity** is important when something is being raised or lowered in a system. Lifting a mass vertically is one example, as is sliding something on an incline.
- **Friction forces** exist in almost all systems and must be considered.
- **Applied forces** come from springs, other actuators, magnets, etc., and are the forces that act on the mass other than friction, gravity, and the actuator's thrust. The spring shown in the figure below is an example of an Applied force.
- **Actuator thrust** is the required actuator force, and is what we need to determine.
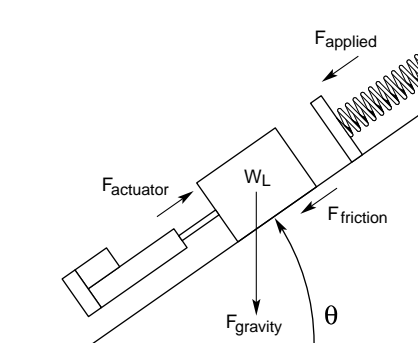
The figure above shows a general case where the force required by an actuator must be determined. All of the above forces are included, and it is important to note that all of these forces can change over time, so the thrust must be calculated for each section of the move profile. The worst case thrust and speed required should be used to pick the appropriate actuator. All of these forces added up (Σ) must be equal to mass $\times$ acceleration, or:

$$\Sigma F = m \times a \quad \text{, or,} \tag{1}$$

$$F_{actuator} - F_{applied} - F_{friction} - F_{gravity} = ma = \left(\frac{W_t}{g}\right) a \tag{2}$$

$$F_{actuator} = \left(\frac{W_t}{g}\right) a + F_{applied} + F_{friction} + F_{gravity} \tag{3}$$

where $\quad W_t = W_{load} + W_{actuator}$ \qquad (4)

$\qquad F_{friction} = \mu W_L \cos\theta, \quad$ and

$\qquad F_{gravity} = W_L \sin\theta$

$W_{actuator}$ becomes important when the acceleration force, $(W_t/g)a$, is a significant part of the thrust calculation. For simplicity, start by neglecting this weight, and calculate the required thrust without it. After selecting an actuator, add its mass to the mass of the load and recalculate. To make these equations clear, lets begin with an example.

Example 1

We would like to move a 200 lb weight a distance of 10 inches in 2 seconds. The mass slides up and incline with a friction coefficient of 0.1 at an angle of 45°. There is a spring that will be in contact with the mass during the last 0.5 inch of travel and has a spring rate of 100 lb/in. What is the maximum thrust and velocity?

Solution

We need to look at the thrust requirement during each part of the move, and find the points of maximum thrust and maximum speed. Choosing a trapezoidal profile we calculate that v_{max} is 7.5 in/sec and the peak acceleration is 11.25 in/sec² (see Move Profile Section).

Acceleration Section:

Ma	$= 200 \text{ lb}/386 \text{ in/sec}^2 \times 11.25 \text{ in/sec}^2$
	$= 5.83 \text{ lb}$
$F_{applied}$	$= 0 \text{ lb}$
$F_{friction}$	$= [200 \text{ lb} \times \cos(45)] \times 0.1 = 14.14 \text{ lb}$
$F_{gravity}$	$= 200 \text{ lb} \times \sin(45) = 141.4 \text{ lb}$
F_{total}	$= 161 \text{ lb}$

Slew Section:

Ma	$= 0 \text{ lb (since } a=0)$
$F_{applied}$	$= 0 \text{ lb}$
$F_{friction}$	$= [200 \text{ lb} \times \cos(45)] \times 0.1 = 14.14 \text{ lb}$
$F_{gravity}$	$= 200 \text{ lb} \times \sin(45) = 141.4 \text{ lb}$
F_{total}	$= 156 \text{ lb}$

Deceleration Section:

Ma	$= 200 \text{ lb}/386 \text{ in/sec}^2 \times -11.25 \text{ in/sec}^2$
	$= -5.83 \text{ lb}$
$F_{applied}$	$= K \times x = 0.5 \text{ in} \times 100 \text{ lb/in} = 50 \text{ lb}$ (worst case)
$F_{friction}$	$= [200 \text{ lb} \times \cos(45)] \times 0.1 = 14.14 \text{ lb}$
$F_{gravity}$	$= 200 \text{ lb} \times \sin(45) = 141.4 \text{ lb}$
F_{total}	$= 200 \text{ lb}$

So the worst case required thrust is <u>200 lb.</u> And the worst case velocity is <u>7.5 in/sec.</u>

Thrust Calculation

Actuator Mass

In applications where the **acceleration force**, $(W_t/g)a$, is a significant part of the required thrust, the actuator mass must be considered in the thrust calculation. After an actuator is chosen, the actuator weight (linear inertia), $W_{actuator}$, is added to the weight of the load. $W_{actuator}$ can be determined using the tables and equation in the actuator data section. To illustrate, we will use the previous example.

1. The first step is to pick an actuator with the above thrust and speed capability. One such actuator is an EC3-B32-20-16B-12. This is an EC3 actuator with a B32 motor, a 2:1 gear reduction, a 16mm lead ballscrew, and a 12 inch stroke.

2. The next step is to look up the effective **Actuator Linear Inertia** in the tables located in the particular actuator section (do not include the "load" term in the equation). An entry from this table can be seen in the table below. The B32 motor inertia is 1.00×10^{-3} in-lb-sec^2. The effective actuator weight, calculated from the table is 247 lb.

3. The final step is to add this weight to the weight of the load, W_L, and recalculate the peak thrust required for each section of the move profile **(do not add this weight to the gravity or friction terms)**:

Acceleration Section:

$$Ma = 447 \text{ lb}/386 \text{ in/sec}^2 \times 11.25 \text{ in/sec}^2$$

$$= \textbf{13.03 lb}$$

$$F_{total} = \textbf{169 lb}$$

Slew Section:

$$Ma = \textbf{0 lb} \text{ (since a=0)}$$

Deceleration Section:

$$Ma = 447 \text{ lb}/386 \text{ in/sec}^2 \times -11.25 \text{ in/sec}^2)$$

$$= \textbf{-13.03 lb}$$

$$F_{total} = \textbf{193 lb}$$

We can see from this calculation that the addition of this extra "acceleration weight" increases the thrust required during acceleration, but reduces the peak thrust required during deceleration. The EC3-B32-20-16B-12 will work in the application.
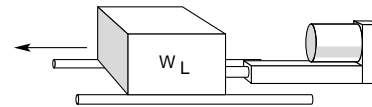
Vertical and Horizontal Cases

In a vertical system, θ is 90°, sin90 = 1, and $F_{gravity}$ is equal to W_L. Since cos90 = 0, $F_{friction}$ = 0.
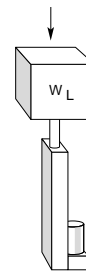
$$F_{actuator} = (W_t/g)a + F_{applied} + F_{gravity}$$
$$\boxed{F_{actuator} = (W_t/g)a + F_{applied} + W_L}$$

In a horizontal system, $\sin\theta = 0$, so gravity would play no part ($F_{gravity} = 0$), and $\cos\theta = 1$, so $F_{friction}$ would be equal to μW_L, or 50 lb.

$$F_{actuator} = (W_t/g)a + F_{applied} + F_{friction}$$
$$\boxed{F_{actuator} = (W_t/g)a + F_{applied} + \mu W_L}$$

RMS Thrust

For all <u>Servo Motor</u> applications, the RMS Thrust needs to be calculated. This thrust must fall within the continuous duty region of the actuator. Use the following equation when calculating RMS Thrust:
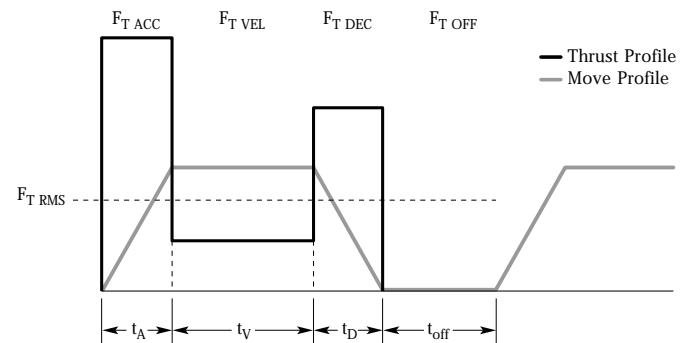
$$F_{T \, RMS} = \sqrt{\frac{(F_{T \, ACC})^2 t_a + (F_{T \, VEL})^2 t_v + (F_{T \, DEC})^2 t_d + (F_{T \, OFF})^2 t_{off}}{t_a + t_v + t_d + t_{off}}}$$

EC3 Cylinder	Actuator Linear Inertia = [A + B*(stroke, in) + D]/C			
Leadscrew	**Ratio**	**A (lb-in-s²)**	**B (lb-in-s²/in)**	**C (lb-in-s²/lb)**
16x16	1:1	1.1909E-03	1.1760E-05	2.6039E-05
	1.5:1	7.4495E-04	5.2280E-06	1.1573E-05
	2:1	4.7866E-04	2.7650E-06	6.1212E-06

Motor Inertia (in-lb-sec²)	
	D (lb-in-s²)
B32	1.00 E-03