Year 11 Mathematics Worksheet

10 questions on Matrices from the Maths B (General Maths) national curriculum for Year 11.

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Questions

1. What does the number in the second row and the third column tell you?

The 3 x 5 matrix shows the numbers of sandwiches sold over five days at the school canteen.

	Mon	Tue	Wed	Thu	Fri
Cheese	50	45	52	47	39
Salad	61	44	48	38	51
Chicken	53	46	39	42	52

a) 48 salad sandwiches were sold on Wednesday.

b) 61 salad sandwiches were sold on Monday.

c) 46 chicken sandwiches were sold on Thursday.

d) 45 cheese sandwiches were sold on Tuesday.

Answer: ______________________

2.

$$\text{If } A = \begin{pmatrix} 5 & 7 & 3 & 1 \\ 8 & 0 & 2 & 9 \\ 0 & 4 & 6 & 7 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 1 & 4 & 0 \\ 5 & 3 & 2 & 1 \\ 6 & 0 & 1 & 2 \end{pmatrix}$$

which matrix is equal to 2A – B?

a) $\begin{pmatrix} 12 & 15 & 10 & 1 \\ 21 & 6 & 6 & 19 \\ 6 & 8 & 13 & 16 \end{pmatrix}$

b) $\begin{pmatrix} 12 & 15 & 10 & 1 \\ 21 & 6 & 6 & 19 \\ 6 & 8 & 13 & 16 \end{pmatrix}$

c) $\begin{pmatrix} 12 & 15 & 10 & 1 \\ 21 & 6 & 6 & 19 \\ 6 & 8 & 13 & 16 \end{pmatrix}$

d) $\begin{pmatrix} 12 & 15 & 10 & 1 \\ 21 & 6 & 6 & 19 \\ 6 & 8 & 13 & 16 \end{pmatrix}$

Answer: ____________________

3. This first matrix, **A**, shows the cost price and marked selling price for shirts and shorts.

PRICES (dollars)	Shirt Price	Shorts Price
Cost	25	30
Marked	40	50

The second matrix, **B**, shows the numbers of shirts and shorts sold in two successive weeks.

NUMBERS	Week 1	Week 2
Shirts sold	34	28
Shorts sold	29	32

The matrix product **AB** =

$$\begin{pmatrix} 25 \times 34 + 30 \times 29 = 1720 & 25 \times 28 + 30 \times 32 = 1660 \\ 40 \times 34 + 50 \times 29 = 2810 & 40 \times 28 + 50 \times 32 = 2720 \end{pmatrix}$$

What does the entry in the first row, second column of **AB** represent?

a) Total marked price of shirts and shorts sold in Week 2.

b) Total cost price of shirts and shorts sold in Week 2.

c) Total marked price of shirts and shorts sold in Week 1.

d) Total cost price of shirts and shorts sold in Week 1.

Answer: ________________________

4.

A = $\begin{pmatrix} 2 & 8 \\ 7 & 6 \end{pmatrix}$, **B** = $\begin{pmatrix} 1 & 6 \\ 3 & 7 \end{pmatrix}$, **C** = $\begin{pmatrix} 1 & 6 \end{pmatrix}$, **D** = $\begin{pmatrix} 2 \\ 7 \end{pmatrix}$.

Which matrix product is equal to $\begin{pmatrix} 2 & 12 \\ 7 & 42 \end{pmatrix}$?

a) CD

b) BD

c) AB

d) DC

Answer: ______________________________

5.

$$\begin{array}{r} \\ \textbf{Cheese} \\ \textbf{Salad} \\ \textbf{Chicken} \end{array} \begin{array}{c} \begin{array}{ccccc} \textbf{Mon} & \textbf{Tue} & \textbf{Wed} & \textbf{Thu} & \textbf{Fri} \end{array} \\ \begin{pmatrix} 50 & 45 & 52 & 47 & 39 \\ 61 & 44 & 48 & 38 & 51 \\ 53 & 46 & 39 & 42 & 52 \end{pmatrix} \end{array}$$

The 3 x 5 matrix shows the number of sandwiches bought on different days in the school canteen.

The price of a cheese sandwich is $1.20, the price of a salad sandwich is $1.50 and the price of a chicken sandwich is $1.85.

If **S** = $\begin{pmatrix} 50 & 45 & 52 & 47 & 39 \\ 61 & 44 & 48 & 38 & 51 \\ 53 & 46 & 39 & 42 & 52 \end{pmatrix}$ and **P** = (1.2 1.5 1.85)

which matrix shows the amount of money spent on sandwiches on each of the five days?

a) SP

b) S + P

c) PS

d) P + S

Answer: ___________________________

6. **A** and **B** are both 2 x 2 square matrices.

AB = ***I*** = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Which statement is false?

a) If A = – I then A = B.

b) AB = BA

c) AB ≠ BA

d) B = A^{-1}

Answer: _____________________

7. A = $\begin{pmatrix} 4 & 2 \\ 3 & 9 \end{pmatrix}$.

What does the determinant of A equal?

a) 42

b) 30

c) $\begin{pmatrix} 9 & -2 \\ -3 & 4 \end{pmatrix}$

d) $\begin{pmatrix} 0.3 & -1/15 \\ -0.1 & 2/15 \end{pmatrix}$

Answer: ______________________

8. Which of the matrices A, B, C and D have inverses?

A = $\begin{pmatrix} 4 & 2 \\ 3 & 9 \end{pmatrix}$, B = $\begin{pmatrix} 4 & 6 \\ 6 & 9 \end{pmatrix}$, C = $\begin{pmatrix} 6 & 6 \\ 8 & 8 \end{pmatrix}$, D = $\begin{pmatrix} 0 & 1 \\ 3 & 7 \end{pmatrix}$

a) B and C

b) B and D

c) A and C

d) A and D

Answer: _________________

9. Bob has an encoding matrix C that he uses when writing messages in code to Allen who knows the decoding matrix C^{-1}.

Bob first changes the letters in his message to numbers (a = 1 to z = 26), groups the numbers in threes and then puts the numbers in columns of 3 x 3 matrices A and B.
Bob encodes A and B by pre-multiplying them by C before sending the message to Allen in matrices E and F.

CA = **E** and **CB** = **F** with

$$\mathbf{C} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} 18 & 45 & 22 \\ 10 & 40 & 21 \\ 18 & 35 & 41 \end{pmatrix} \text{ and } \mathbf{F} = \begin{pmatrix} 22 & 30 & 17 \\ 11 & 27 & 17 \\ 27 & 27 & 4 \end{pmatrix}$$

What was Bob's original message?

a) See you later at home Allen.

b) Meet you at school Bob.

e) Allen you’ll bat third.

d) The answer is twenty.

Answer: ______________________

10. A take-away coffee bar charges \$4.20 for a cappuccino coffee and \$3.80 for a plain coffee.

On one day the coffee bar makes total sales worth \$1854.00 and uses 458 cardboard coffee mugs.

This information is shown in the matrix equation:

$$\begin{pmatrix} 4.2 & 3.8 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 1854 \\ 458 \end{pmatrix}$$

where *m* and *n* are the numbers of cappuccinos and regular coffees sold. What is the next step in solving this matrix equation to find the number of each kind of coffee sold?

Note that: $\begin{pmatrix} 4.2 & 3.8 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2.5 & -9.5 \\ -2.5 & 10.5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

a) $\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 2.5 \\ -2.5 \end{pmatrix} \begin{pmatrix} -9.5 & 1854 \\ 10.5 & 458 \end{pmatrix}$

b) $\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 2.5 & -2.5 \\ -9.5 & 10.5 \end{pmatrix} \begin{pmatrix} 1854 \\ 458 \end{pmatrix}$

$$\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 2.5 & -9.5 \\ -2.5 & 10.5 \end{pmatrix} \begin{pmatrix} 1854 \\ 458 \end{pmatrix}$$

c)

$$\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 1854 \\ 458 \end{pmatrix} \begin{pmatrix} 2.5 & -9.5 \\ -2.5 & 10.5 \end{pmatrix}$$

d)

Answer: ______________________

The Answers.

Hey! No peeking until you've finished…

Question 1

Answer: a) **48 salad sandwiches** were sold on Wednesday.

The second row is for salad sandwiches.
The third column is for Wednesday.
The number in the second row and the third column tells us that **48 salad sandwiches** were sold on Wednesday.

Question 2

Answer: A) $\begin{pmatrix} 8 & 13 & 2 & 2 \\ 11 & -3 & 2 & 17 \\ -6 & 8 & 11 & 12 \end{pmatrix}$

The elements in the first row of 2A – B are:
2 x 5 – 2, 2 x 7 – 1, 2 x 3 – 4, 2 x 1 – 0.
The elements in the second row of 2A – B are:
2 x 8 – 5, 2 x 0 – 3, 2 x 2 – 2, 2 x 9 – 1.
The elements in the third row of 2A – B are:
2 x 0 – 6, 2 x 4 – 0, 2 x 6 – 1, 2 x 7 – 2.

$$2A - B = \begin{pmatrix} 8 & 13 & 2 & 2 \\ 11 & -3 & 2 & 17 \\ -6 & 8 & 11 & 12 \end{pmatrix}$$

Question 3

Answer: b) The total cost price for both kinds of garments sold in Week 2

The elements in the first row of **A** are the cost prices of shirts and shorts.
The elements in the second column of **B** are the numbers of each garment sold in Week 2.
The element in the first row and second column of **AB** is the cost price of shirts times the number sold plus the cost price of shorts times the number sold in

Week 2.
The total cost price for both kinds of garments sold in Week 2 equals $1660.

Question 4

Answer: D) DC

The only products which will have 2 rows and 2 columns are $\mathbf{A}^2$, $\mathbf{B}^2$, **AB**, **BA** and **DC**.
Eliminate $\mathbf{A}^2$, $\mathbf{B}^2$, **AB**, **BA** by checking that the element in the first row and column of these products is not 2.
Complete the multiplication for **DC.**

Question 5

Answer: C) PS

The product **SP** is not possible.

The product **PS** is of size 1 x 5. It has one row and five columns.

Each element in **PS** is the sum of the amounts of money from selling each kind of sandwich on a particular day.
For example: The element in the third column of **PS** will show that on Wednesday the 52 cheese sandwiches at $1.20 each, 48 salad sandwiches at $1.50 each and the 39 chicken sandwiches at $1.85 each would have sold for a total of $(52 x 1.2 + 48 x 1.5 + 39 x 1.85)

Question 6

Answer: C) AB ≠ BA.

By definition, if $\mathbf{AB} = \boldsymbol{I}$ then **A** and **B** are inverses and $\mathbf{B} = \mathbf{A}^{-1}$.
In a special case if $\mathbf{A} = -\boldsymbol{I}$ then $-\boldsymbol{I}\mathbf{B} = \boldsymbol{I}$ and $-\mathbf{B} = \boldsymbol{I}$. Therefore $\mathbf{B} = -\boldsymbol{I}$ and $\mathbf{A} = \mathbf{B}$.
So these two statements above can be eliminated.
That leaves **AB** = **BA** and **AB** ≠ **BA** which cannot both be true.

If **A** = $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ then $\mathbf{A}^{-1}$ = $\frac{1}{ps - rq}\begin{pmatrix} s & -q \\ -r & p \end{pmatrix}$ = **B**.

BA = $\frac{1}{ps - rq}\begin{pmatrix} s & -q \\ -r & p \end{pmatrix}\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ = ***I*** = **AB**.

Therefore the false statement is **AB** ≠ **BA**.

Question 7

Answer: B) 30

From the definition, the "determinant of A" = |A| = 4 x 9 – 3 x 2 = **30**.

Question 8

Answer: D) A and D

A matrix with determinant zero does not have an inverse because in the set of real numbers division by zero is not defined.

|A| = 36 – 6 ≠ 0. Matrix A has an inverse.
|B| = 36 – 36 = 0. Matrix B does not have an inverse.
|C| = 48 – 48 = 0. Matrix C does not have an inverse.
|D| = 0 – 3 ≠ 0. Matrix D does has an inverse.
Therefore, **Matrix A** and **Matrix D** both have inverses.

Question 9

Answer: B) Meet you at school Bob.

Let $C^{-1} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}$

Then

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} a+c & a+b & b+c \\ d+f & d+e & e+f \\ g+k & g+h & h+k \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Equating corresponding elements:
a + c = 1, a + b = 0, b + c = 0, from which a = c = 1/2 and b = – 1/2.
d + f = 0, d + e = 1, e + f = 0, from which d = e = 1/2 and f = – 1/2.
g + k = 0, g + h = 0, h + k = 1, from which h = k = 1/2 and g = – 1/2.
To complete the solution find A and B from $A = C^{-1}E$ and $B = C^{-1}F$ and replace the numbers in A and B with letters of the alphabet.
Read down the columns.

Question 10

Answer: C) $\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 2.5 & -9.5 \\ -2.5 & 10.5 \end{pmatrix}\begin{pmatrix} 1854 \\ 458 \end{pmatrix}$

You were only asked for the next step but the full solution is shown.

Since $\begin{pmatrix} 4.2 & 3.8 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2.5 & -9.5 \\ -2.5 & 10.5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 2.5 & -9.5 \\ -2.5 & 10.5 \end{pmatrix}\begin{pmatrix} 4.2 & 3.8 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 2.5 & -9.5 \\ -2.5 & 10.5 \end{pmatrix}\begin{pmatrix} 1854 \\ 458 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 284 \\ 174 \end{pmatrix}$$

$$\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 284 \\ 174 \end{pmatrix}$$

The coffee bar sold 284 cappuccinos and 174 plain coffees.