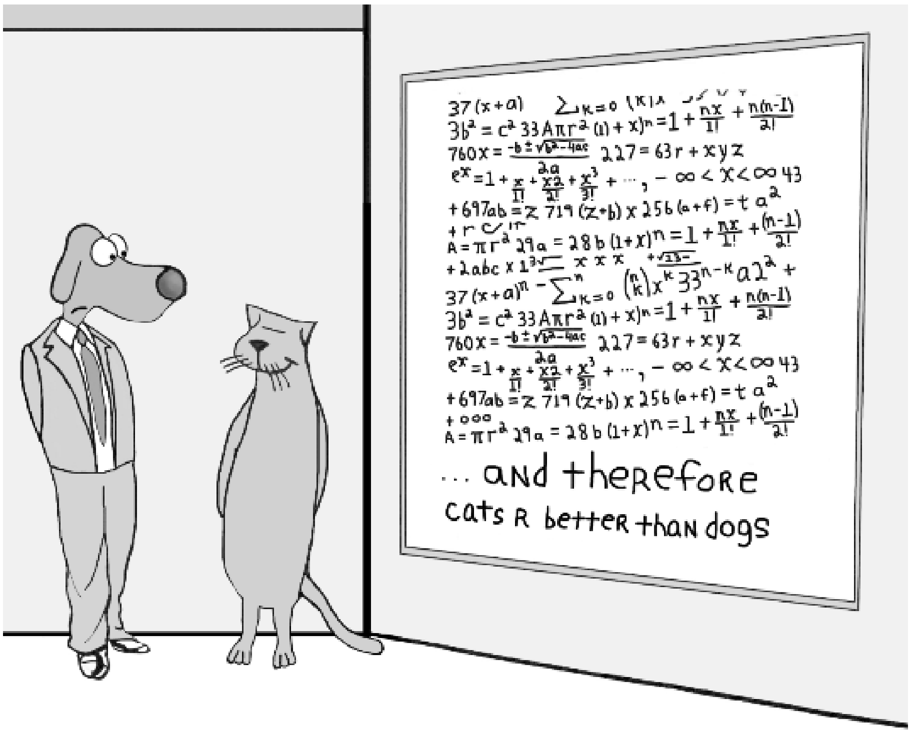
Year 12 Extension Maths, Vectors Worksheet

8 questions on Vectors from the Specialist Mathematics/Extension Mathematics national curriculum for Year 12.



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Questions

1. A scalar entity has magnitude (size) but no direction.
   A vector entity has both magnitude and direction.

   Are the following scalar, vector or neither?
   (i) length
   (ii) brand of car
   (iii) change in temperature
   (iv) volume
   (v) salary
   (vi) acceleration.

   a) (i) scalar; (ii) neither; (iii) vector;
      (iv) scalar; (v) scalar; (vi) vector.
   b) (i) scalar;(ii) neither; (iii) vector;
      (iv) vector; (v) scalar; (vi) vector.
   c) (i) scalar; (ii) neither; (iii) vector;
      (iv) scalar; (v) neither; (vi) vector.
   d) (i) scalar; (ii) vector; (iii) vector;
      (iv) scalar; (v) neither; (vi) vector.

Answer: ____________________

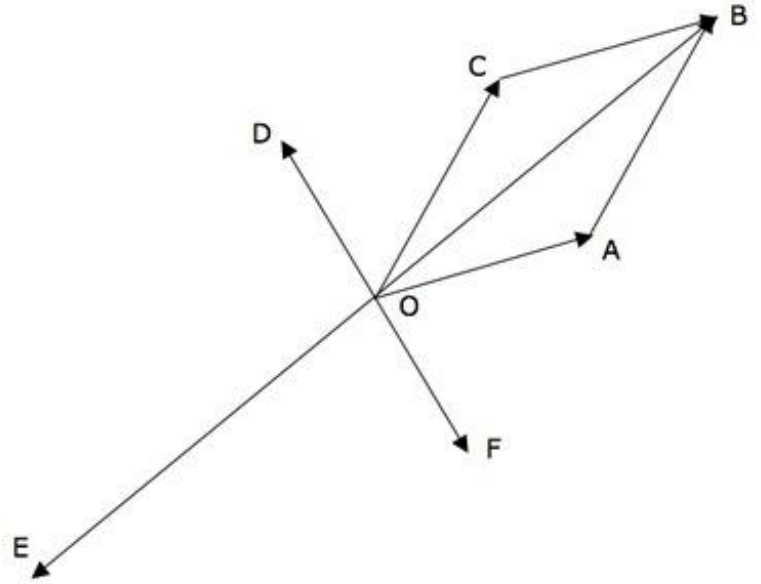
2. A vector in three dimensions **cannot** be represented by a directed line segment.

   a) True

   b) False

Answer: ____________________

3. The length of OB = length of OE. FD is parallel to AC.
   The length of OD = length of OF = distance from A to C.



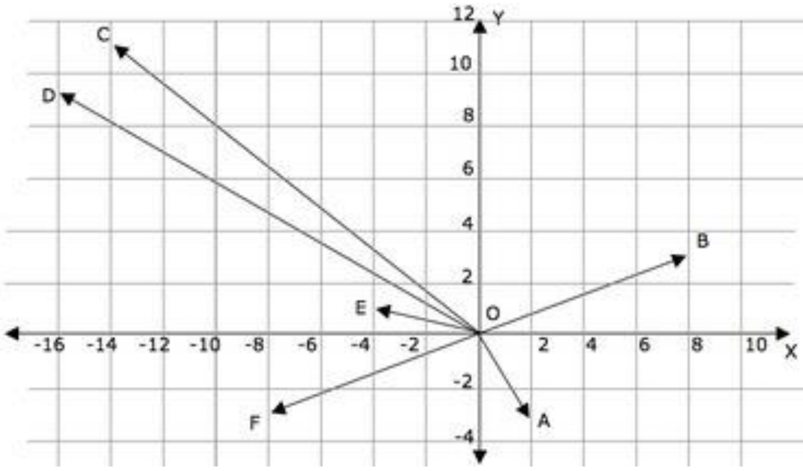
Which vector is equal to OA – OC?

a) OB
b) OF
c) OD
d) OE

Answer: ____________________

4. Read the equation and answer the question below:

$\underline{OA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, $\underline{OE} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$.



$v = 3\ \underline{OE} - 2\ \underline{OA}$.

(i) Which directed line segment is equal to *v* ?

(ii) What is the magnitude of $\underline{OE}$?

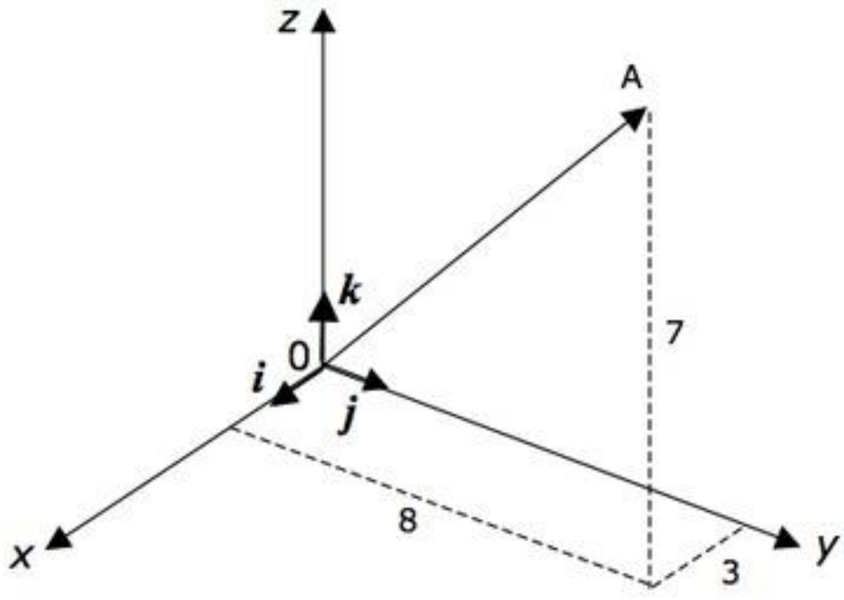
(iii) What is the magnitude of $\underline{OA}$?

(iv) What is the magnitude of *v*?

a) (i) $\underline{OD}$; (ii) √17;
   (iii) √13; (iv) √337.
b) (i) $\underline{OC}$; (ii) √13;
   (iii) √17; (iv) √317.
c) (i) $\underline{OF}$; (ii) √17;
   (iii) √13; (iv) – √317.
d) (i) $\underline{OB}$; (ii) √17;
   (iii) √13; (iv) √73.

Answer: ____________________

5. A unit vector is a vector with magnitude equal to one.
   In three dimensions ***i***, ***j*** and ***k*** are unit vectors parallel to the *x*-axis, the *y*-axis and the *z*-axis respectively.



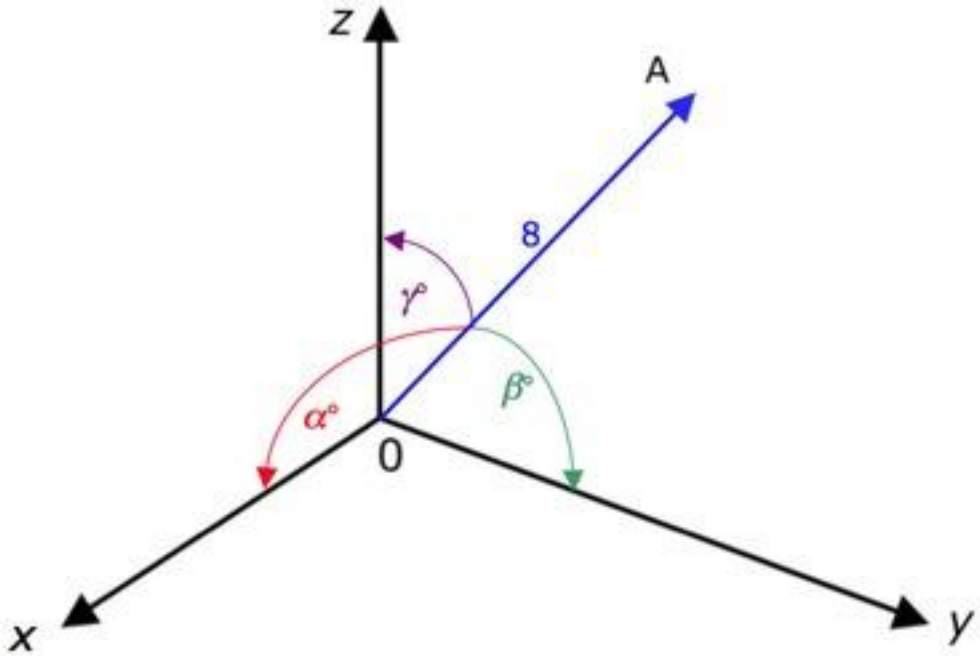
The directed line segment OA can be written as the column matrix $\begin{pmatrix} 3 \\ 8 \\ 7 \end{pmatrix}$.

How else can OA be written?

a) $i + j + k$
b) $3i + 8j$
c) $3i + 8\boldsymbol{j} + 7k$
d) $8i + 3j + 7k$

Answer: ____________________

6. The vector OA has magnitude 8 and makes angles of $\alpha°$, $\beta°$ and $\gamma°$ with the positive directions of the x-axis, the y-axis and the z-axis respectively.



Which of the following is the resolution of OA into three mutually perpendicular directions?

(i) $(8 \cos \alpha°)\,\boldsymbol{i} + (8 \cos \beta°)\,\boldsymbol{j} + (8 \cos \gamma°)\,\boldsymbol{k}$

(ii) $(8 \sin \alpha°)\,\boldsymbol{i} + (8 \sin \beta°)\,\boldsymbol{j} + (8 \sin \gamma°)\,\boldsymbol{k}$

(iii) $(2 \sin \alpha°)\,\boldsymbol{i} + (2 \sin \beta°)\,\boldsymbol{j} + (2 \sin \gamma°)\,\boldsymbol{k}$

(iv) $(2 \cos \alpha°)\,\boldsymbol{i} + (2 \cos \beta°)\,\boldsymbol{j} + (2 \sin \gamma°)\,\boldsymbol{k}$

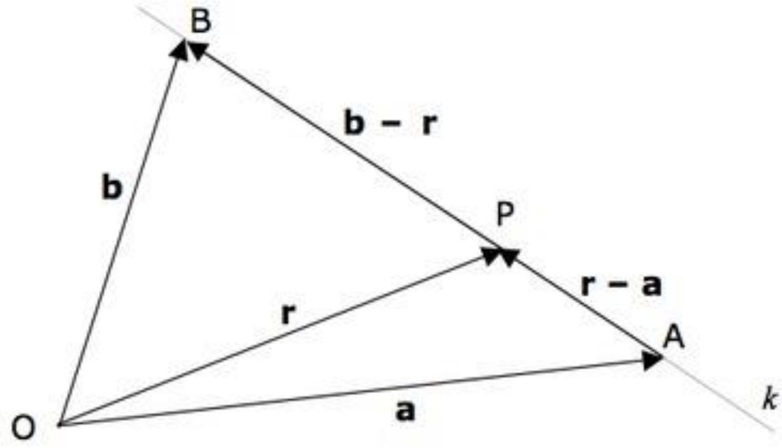
Answer: ______________________

7. A and B are fixed points on a line *k*.
   P is a point lying on the line *k* such that AP : PB = *m* : *n*.

   a is the position vector <u>OA</u>.

   b is the position vector <u>OB</u>.

   r is the position vector <u>OP</u>.



From the Triangle Rule for addition of vectors, <u>AP</u> = ***r*** – ***a*** and<u>PB</u> = ***b*** – ***r***.

What does ***r*** equal when written in terms of ***a***, ***b***, *m* and *n*?

(i) $\dfrac{m\mathbf{a} - n\mathbf{b}}{m - n}$ (ii) $\dfrac{m\mathbf{a} + n\mathbf{b}}{m + n}$

(iii) $\dfrac{n\mathbf{a} + m\mathbf{b}}{m + n}$ (iv) $\dfrac{n\mathbf{a} - m\mathbf{b}}{m + n}$.

Enter your answer as (i), (ii), (iii) or (iv).

Answer: ____________________

8.

If $\underline{OA} = \mathbf{a} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$, $\underline{OB} = \mathbf{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

and $\underline{OC} = \mathbf{c} = \mathbf{a} + \mathbf{b} = \begin{pmatrix} -2+3 \\ 4+5 \end{pmatrix} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$,

which statements are true?

(i) $\underline{OC}$ is perpendicular to $\underline{AB}$.

(ii) OACB is a parallelogram.

(iii) The angle between $\underline{OA}$ and $\underline{OB}$ is 58° to the nearest degree.

(iv) The magnitude of $\underline{OC}$ is equal to the sum of the magnitudes of $\underline{OA}$ and $\underline{OB}$.

(v) $\underline{AB}$ is parallel to $3\boldsymbol{b} - 2\boldsymbol{a}$.

a) (ii) and (iii)

b) (iii) and (iv)

c) (i), (ii) and (iv)

d) (i), (iv) and (v)

Answer: ______________________

The Answers.

Hey! No peeking until you've finished…

Question 1

Answer: A

(i) Length has magnitude but does not have a direction. For example, a length of 10km could be taken in any number of directions: North, South, East, altitude etc. However, the length on its own gives us no indication of direction, so it is a **scalar** quantity.
(ii) The brand of a car has **neither** magnitude nor direction.
(iii) A change in temperature has magnitude and direction because it can be up or down. A change in temperature is found by ΔT = final temperature - initial temperature. If the final temperature is less than the initial temperature, then ΔT will be negative. It is this negative that indicates the temperature has decreased. Change in temperature is a **vector** quantity.
(iv) Volume has magnitude but does not have a direction. This is a **scalar**.
(v) A salary (wage) has magnitude but does not require direction in order to have meaning. This is a **scalar** quantity.
(vi) Acceleration has magnitude and direction. A negative acceleration indicates that an object is slowing down, while a positive acceleration indicates the object's velocity is increasing. This is a **vector**.

Question 2

Answer: FALSE

By definition, a directed line segment has direction as well as magnitude and can represent a vector.

Directed line segments and vectors are similar, but not quite the same. Both have magnitude and direction, but the directed line segment is fixed between two locations in space. As such, it is possible to represent a vector with an infinite number of directed line segments. The directed line segments must have only the correct length and direction and then, no matter which two points A and B they lie between, they represent the same vector.

The statement is false.

Question 3

Answer: B

OA – OC = OA + CO = CO + OA.

To find the magnitude and direction of OA – OC, start from C, go to O and then to A.

Therefore OA – OC = CA, which is parallel to and has the same magnitude and direction as OF. It is important that your vector is CA = OF, rather than AC, which has the wrong direction to be the sum of OA - OC.

The correct answer is the vecotr **OF**.

Question 4

Answer: A

(i) The *x*-coordinate for the end-point of ***v*** is 3 x (-4) – 2 x 2 = –16.

The *y*-coordinate for the end-point of ***v*** is 3 x 1 – 2 x (-3) = 9.

The coordinates of the end-point of the vector ***v*** are (–16, 9).

v is <u>OD</u>.

(ii) The magnitude of <u>OE</u> = $\sqrt{[(-4)^2 + 1^2]} = \sqrt{17}$.

(iii) The magnitude of <u>OA</u> = $\sqrt{[2^2 + (-3)^2]} = \sqrt{13}$.

(iii) The magnitude of ***v*** = $\sqrt{[(-16)^2 + 9^2]} = \sqrt{337}$.

Question 5

Answer: C

The side, in the O*x* direction of the rectangle on the *xy*-plane has length 3 units.

The side, in the Oy direction of the rectangle on the *xy*-plane has length 8 units.

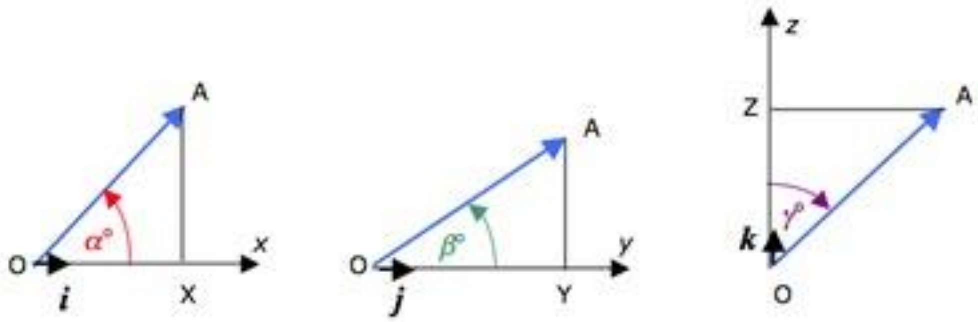
The diagonal of the rectangle on the *xy*-plane is the vector 3***i*** + 8***j***.

<u>OA</u> is the vector sum of the diagonal 3***i*** + 8***j*** and the vertical vector 7***k***.

Therefore <u>OA</u> = 3***i*** + 8***j*** + 7***k***.

Question 6

Answer: A



OX = OA cos α°, OY = OA cos β°, OZ = OA cos γ°.

But OA = 8,

<u>OX</u> =8 cos α° ***i***, <u>OY</u> = 8 cos β° ***j***, <u>OZ</u> = 8 cos γ° ***k***.

<u>OA</u> = <u>OX</u> + <u>OY</u> + <u>OZ</u> = (8 cos *α*°) ***i*** + (8 cos *β*°) ***j*** + (8 cos *γ*°) ***k***.

<u>OA</u> has been resolved into its components in three mutually perpendicular directions.

Question 7

Answer: i

Since P divides a line interval AB in the ratio $m : n$ then $n\underline{AP} = m\underline{PB}$.

But $\underline{AP} = \boldsymbol{r} - \boldsymbol{a}$ and $\underline{PB} = \boldsymbol{b} - \boldsymbol{r}$.

Then $n\boldsymbol{r} - n\boldsymbol{a} = m\boldsymbol{b} - m\boldsymbol{r}$ and $(m + n)\boldsymbol{r} = m\boldsymbol{b} + n\boldsymbol{a}$.

Question 8

Answer: D

(i) $\underline{OC} = 1\boldsymbol{i} + 9\boldsymbol{j}$, $|\underline{OC}| = \sqrt{(1^2 + 9^2)} = \sqrt{82}$.

$\underline{AB} = \underline{OB} - \underline{OA} = 3\boldsymbol{i} + 5\boldsymbol{j} - (-2\boldsymbol{i} + 4\boldsymbol{j}) = 5\boldsymbol{i} + 1\boldsymbol{j}$

$\underline{OC} . \underline{AB} = (1\boldsymbol{i} + 9\boldsymbol{j}) . (5\boldsymbol{i} + 1\boldsymbol{j})$

$= 1\boldsymbol{i} . 5\boldsymbol{i} + 1\boldsymbol{i} . 1\boldsymbol{j} + 9\boldsymbol{j} . 5\boldsymbol{i} + 9\boldsymbol{j} . 1\boldsymbol{j}$

$= 5 + 0 + 0 + 9 = 14 \neq 0$ and $\underline{OC}$ is **not** perpendicular to $\underline{AB}$

(ii) OACB is a parallelogram from the definition of vector addition.

(iii) $\underline{OA} . \underline{OB} = |\underline{OA}| \times |\underline{OB}| \times \cos \vartheta°$, where $\vartheta°$ is the angle between them.

$\underline{OA} . \underline{OB} = (-2\boldsymbol{i} + 4\boldsymbol{j}) . (3\boldsymbol{i} + 5\boldsymbol{j}) = -6 + 20 = 14$

$|\underline{OA}| \times |\underline{OB}| \times \cos \vartheta° = \sqrt{([-2]^2 + 4^2)} \times \sqrt{(3^2 + 5^2)} \times \cos\vartheta°$

$= \sqrt{20} \times \sqrt{34} \times \cos \vartheta°$

$\sqrt{20} \times \sqrt{34} \times \cos \vartheta° = 14$ and $\cos \vartheta° = 14 \div (\sqrt{20} \times \sqrt{34})$.

Therefore $\vartheta° = 57.52880771 ...° = 58°$ to the nearest degree.

(iv) $|\underline{OC}| = \sqrt{82}$, $|\underline{OA}| = \sqrt{20}$ and $|\underline{OB}| = \sqrt{34}$

The magnitude of $\underline{OC}$ is **not** equal to the sum of the magnitudes of $\underline{OA}$ and $\underline{OB}$.

(v) $\underline{AB} = 5\boldsymbol{i} + 1\boldsymbol{j}$, $|\underline{AB}| = \sqrt{26}$

$3\boldsymbol{b} - 2\boldsymbol{a} = 13\boldsymbol{i} + 7\boldsymbol{j}$, $|3\boldsymbol{b} - 2\boldsymbol{a}| = \sqrt{(169 + 49)} = \sqrt{218}$

$\underline{AB} . (3\boldsymbol{b} - 2\boldsymbol{a}) = (5\boldsymbol{i} + 1\boldsymbol{j}) . (13\boldsymbol{i} + 7\boldsymbol{j}) = 65 + 7$

$= 72 = \sqrt{26} \times \sqrt{218} \times \cos \theta°$, and $\cos \theta° \neq 1$.

Therefore $\underline{AB}$ is **not** parallel to $3\boldsymbol{b} - 2\boldsymbol{a}$.