Year 10 Numeracy Worksheet

10 questions on Sine and Cosine Rules for Year 10 students.

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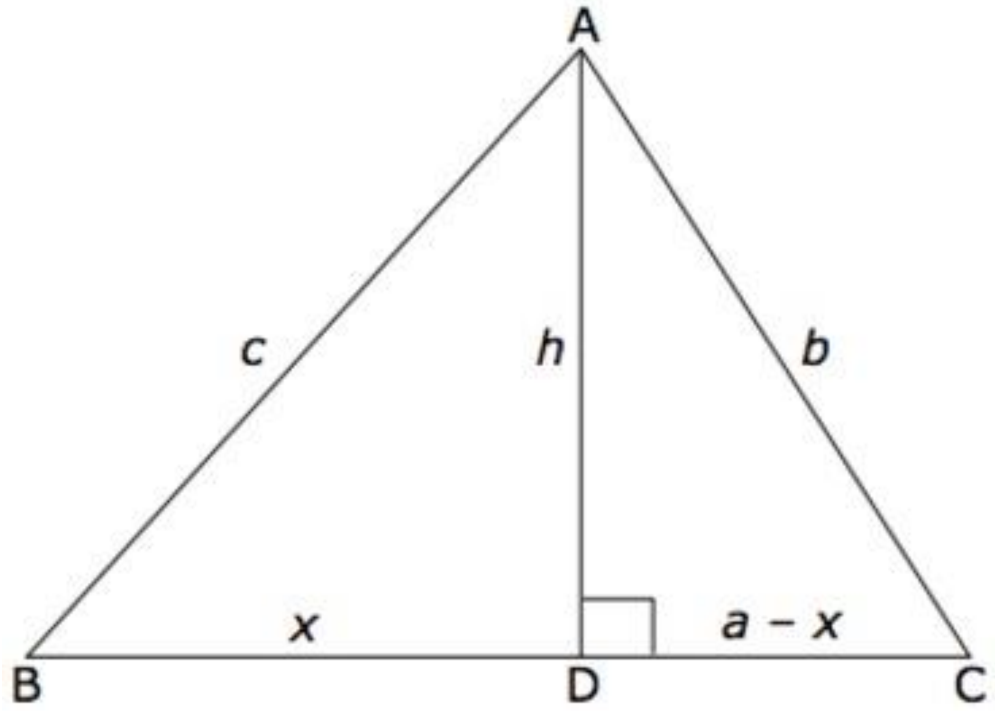
Questions

1. Mel is proving the Cosine Rule.



Mel draws the triangle and writes:

In the right-angled ΔADB, $x/c = \cos \angle B$.

$\therefore x = c \cos \angle B$.

By Pythagoras' theorem, $h^2 = c^2 - x^2$.

In the right-angled ΔADC, by Pythagoras' theorem, $h^2 = b^2 - (a - x)^2$

$\therefore c^2 - x^2 = b^2 - (a - x)^2$.

$\therefore c^2 - x^2 = b^2 - a^2 + 2ax - x^2$

...., line missing (i).

$\therefore c^2 = b^2 - a^2 + 2ac \cos \angle B$.

...., line missing (ii).

What are the two missing lines?

a. (i) $c^2 = b^2 - a^2 + 2ax$,

(ii) $b^2 = a^2 + c^2 - 2ac \cos \angle B$.

b. (i) $c^2 = b^2 - a^2 + 2ax$,

(ii) $b^2 = a^2 + c^2 + 2ac \cos \angle B$.

c. (i) $c^2 = b^2 - a^2 + 2ax \cos \angle B$,

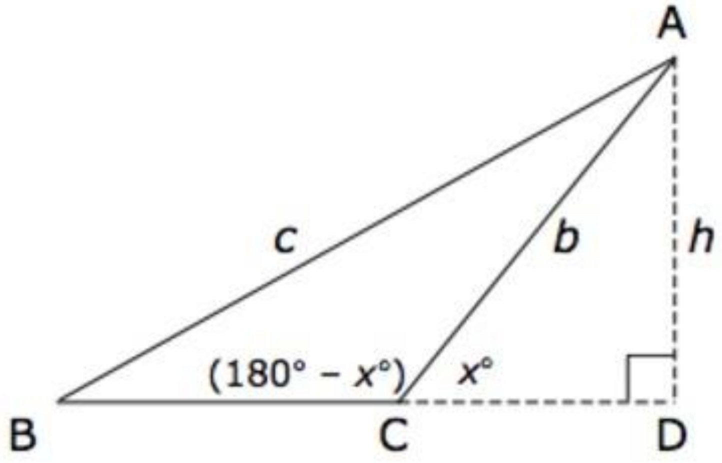
(ii) $b^2 = a^2 - c^2 + 2ac \cos \angle B$.

d. (i) $c^2 + 2x^2 = b^2 - a^2 + 2ax$,

(ii) $b^2 = a^2 + c^2 + 2ac \cos \angle B$.

Answer: ____________________

2. Jan is proving the Sine Rule in an obtuse-angled triangle.



Jan draws the triangle and writes:

sin $\angle$ACD = sin $x°$ = sin $(180 – x)°$ = sin $\angle$ACB.

$\therefore$ sin $x°$ = sin $\angle$ACB.

In the right-angled ΔADC, h/b = sin $x°$.

$\therefore$ $h = b$ sin $x°$.

...., line missing (i).

In the right-angled Δ ADB, h/c = sin $\angle$B.

$\therefore$ $h = c$ sin $\angle$B.

In the obtuse-angled ΔABC,

...., line missing (ii).

...., line missing (iii).

What are the three missing lines?

a. (i) $\therefore h = b \sin \angle C$.
(ii) $\therefore b \sin \angle B = c \sin \angle C$.
(iii) $\dfrac{b}{\sin \angle B} = \dfrac{c}{\sin \angle C}$

b. (i) $\therefore h = b \sin \angle C$.
(ii) $\therefore b \sin \angle C = c \sin \angle B$.
(iii) $\dfrac{b}{\sin \angle C} = \dfrac{c}{\sin \angle B}$

c. (i) $\therefore h = b \sin \angle C$.
(ii) $\therefore b \sin \angle B = c \sin \angle C$.
(iii) $\dfrac{b}{\sin \angle C} = \dfrac{c}{\sin \angle B}$

d. (i) $\therefore h = b \sin \angle C$.
(ii) $\therefore b \sin \angle C = c \sin \angle B$.
(iii) $\dfrac{b}{\sin \angle B} = \dfrac{c}{\sin \angle C}$

Answer: ____________________

3. Which Rule would you use to solve a non-right-angled triangle if you were given

(i) two angles and one side

(ii) three sides

(iii) two sides and an included angle

(iv) two sides and a non-included angle?

a. (i) sine rule, (ii) sine rule,

   (iii) cosine rule, (iv) cosine rule.

b. (i) cosine rule, (ii) cosine rule,

   (iii) cosine rule, (iv) sine rule.

c. (i) sine rule, (ii) cosine rule,

   (iii) cosine rule, (iv) sine rule.

d. (i) sine rule, (ii) cosine rule,

   (iii) sine rule, (iv) cosine rule.

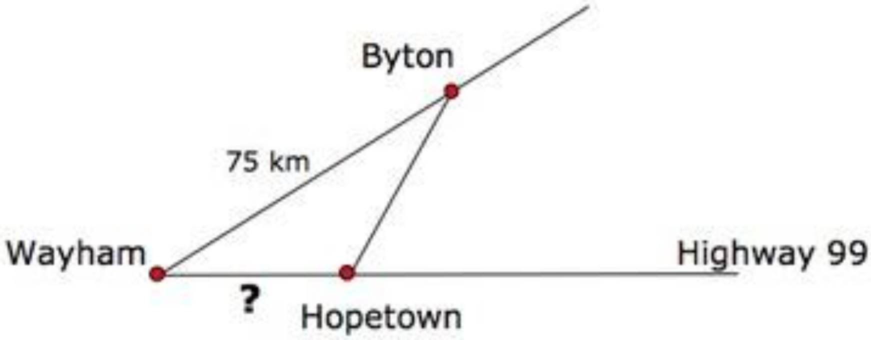
   Answer: ____________________

4. The bearing of Wayham from Byton is 238°.

The bearing of Hopetown from Byton 195°.

Hopetown is due east of Wayham along Highway 99.

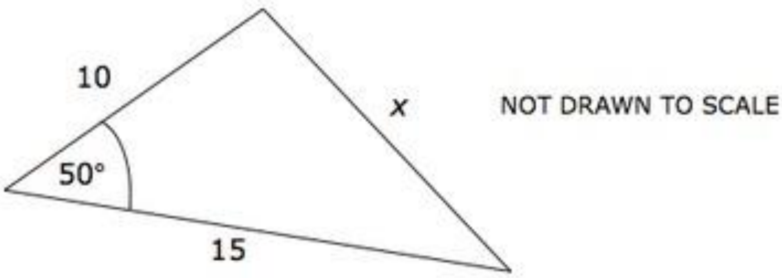
It is 75 km from Byton directly to Wayham.



How far is it from Wayham to Hopetown, to the nearest km?

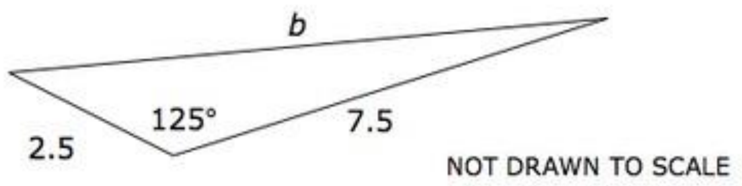
Answer: ____________________

5. Use the cosine rule to calculate the value of *x* in the triangle, (correct to 1 decimal place).



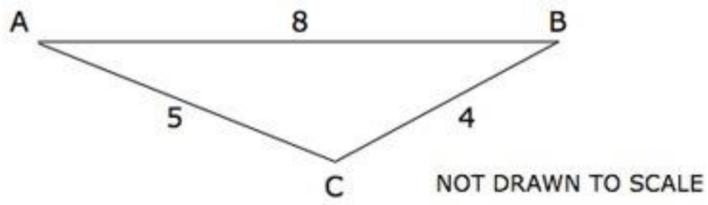
Answer: ____________________

6. Use the cosine rule to calculate the value of *b* in the obtuse-angled triangle, (correct to 2 decimal places).



Answer: ______________________

7. To the nearest minute, find the size of all the angles in the triangle ABC.



a. Angle A = 24° 9', Angle B = 31° 45',

   Angle C = 125°6'

b. Angle A = 125° 6', Angle B = 24° 9',

   Angle C = 92°

c. Angle A = 31° 45', Angle B = 24° 9',

   Angle C = 125°6'

d. Angle A = 67° 6', Angle B = 21° 9',

   Angle C = 94°

Answer: ______________________

8. In a triangle ABC, sin A = 0.45, sin B = 0.64 and b =12.

Find sin C and the length of AB, both correct to 2 decimal places.

a. sin C = 0.92

   AB = 17.19

b. sin C = 66.92

   AB = 17.2

c. sin C = 0.92

   AB = 17.20

d. sin C = 66.92

   AB = 17.19

Answer: ______________________

9. Find both unknown angles in ΔPQR in which q = 10, r = 9 and ∠R = 55°.

Give the answers correct to the nearest minute.

a. ∠P = 55° 28',

   ∠Q = 65°31'.

b. ∠P = 59.28°,

   ∠Q = 66.32°.

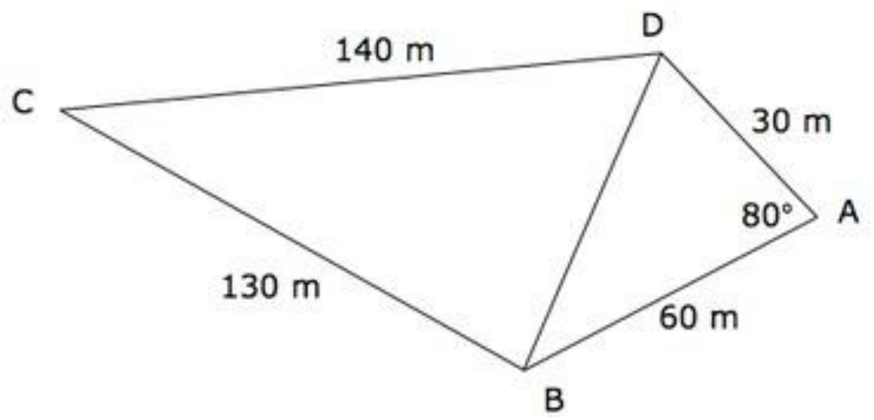
c. $\angle P = 59° 28'$,

$\angle Q = 66°32'$.

d. $\angle P = 65.53°$,

$\angle Q = 59.47°$.

Answer: ____________________



10.

Use the measurements on the diagram to find the size of $\angle BCD$. Give your answer in degrees and minutes, correct to the nearest minute. Do not round off until all the calculator work is completed.

Answer: ___________________

The Answers.

Hey! No peeking until you've finished…

Question 1

Answer: a. **(i)** $\boldsymbol{c^2 = b^2 - a^2 + 2ax,}$

(ii) $\boldsymbol{b^2 = a^2 + c^2 - 2ac}$ **cos** $\boldsymbol{\angle}$**B.**

(i) From the line above:

Left hand side = $c^2 - x^2 + x^2 = c^2$

Right hand side = $b^2 - a^2 + 2ax - x^2 + x^2 = b^2 - a^2 + 2ax$

The missing line is $\boldsymbol{c^2 = b^2 - a^2 + 2ax}$

(ii) From the line above: $c^2 = b^2 - a^2 + 2ac \cos \angle B$.

Make b^2 the subject by adding $a^2 - 2ac \cos \angle B$ to both sides.

Right hand side = $b^2 - a^2 + 2ac \cos \angle B + a^2 - 2ac \cos \angle B = b^2$.

Left hand side = $c^2 + a^2 - 2ac \cos \angle B$.

Therefore the second missing line is $\boldsymbol{b^2 = c^2 + a^2 - 2ac}$ **cos cos** $\boldsymbol{\angle}$**B,**

which is one way of writing the cosine rule.

Question 2

Answer: a.

(i) $\therefore h = b \sin \angle C$.

(ii) $\therefore b \sin \angle C = c \sin \angle B$.

(iii) $$\frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

(i) $h = b \sin x°$ and $\sin x° = \sin (180 - x)°$.

Then $h = b \sin (180 - x)° = \boldsymbol{b \sin \angle C}$.

(ii) Equating the values of h:

***b* sin ∠C = *c* sin ∠B**.

(iii) **Divide both sides of line (ii) by sin ∠B and sin ∠C**.

Question 3

Answer: c. (i) sine rule, (ii) cosine rule,

(iii) cosine rule, (iv) sine rule.

Choose between a) the cosine rule which requires two sides and an included angle or three sides and b) the sine rule which requires a side and the angle opposite to it as well as either another side or another angle.

(i) and (iv) If you know two angles in a triangle you can find the third angle. With two angles and one side, or two sides and a non-included angle you will always know a side and the opposite angle. Therefore, you can use the **sine rule**.

You need at least two sides to use the cosine rule, but the angle must have the two known sides as its arms.

(ii) and (iii) The six different ways of writing the cosine rule are:

$a^2 = b^2 + c^2 - 2bc \cos A$ and $\cos A = (b^2 + c^2 - a^2) \div 2bc$.

$b^2 = a^2 + c^2 - 2ac \cos B$ and $\cos B = (a^2 + c^2 - b^2) \div 2ac$.

$c^2 = a^2 + b^2 - 2ab \cos C$ and $\cos C = (a^2 + b^2 - c^2) \div 2ab$.

If you know a, b and c you can find any angle. If you know two sides and the included angle you can find the third side and then any other angle. Therefore, you can use the **cosine rule**.

Question 4

Answer: The distance from Wayham to Hopetown = 53 km, to the nearest km.

Draw a labelled sketch showing the triangle with vertices Byton, Wayham and Hopetown.

Mark the known angles and distances.

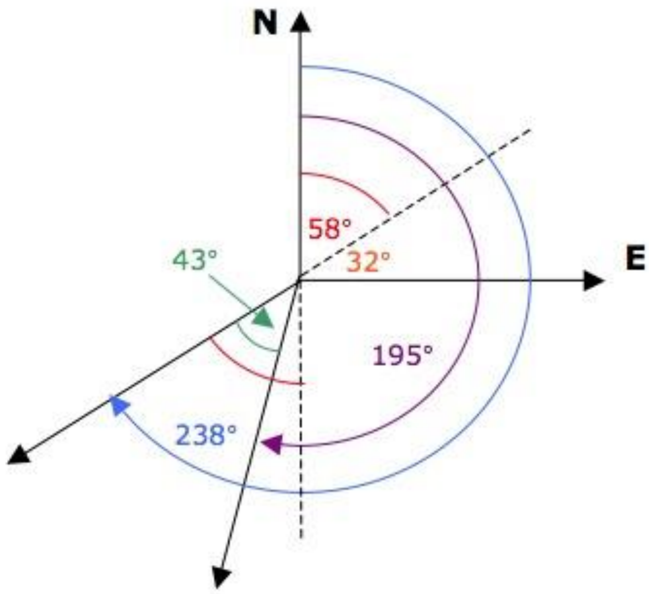
Decide which rule to use, write the formula and substitute the known values.

Bearings are measured clockwise from north.

From the differences in the bearings, 238° – 195° = 43°, the angle between the roads from Byton to Wayham and from Byton to Hopetown is 43°.

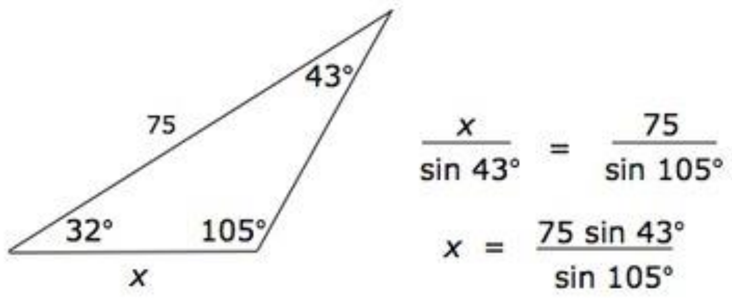
The bearing of Byton from Wayham is 58°.

Hopetown is due east of Wayham so the angle between the roads from Wayham to Byton and Wayham to Hopetown is 32°.



The third angle in the triangle = 180° – 43° – 32° = 105°

Using the sine rule:



By calculator, x = 52.95424929...

The distance from Wayham to Hopetown = **53 km**, to the nearest km.

Question 5

Answer: x = 11.5 correct to 1 decimal place.

NOTE that • can be used for a multiplication sign.

Identify the rule to use: The cosine rule: $C^2 = A^2 + B^2 - 2 \bullet A \bullet B \bullet \cos c$

Substitute in the values given in the diagram: $x^2 = 10^2 + 15^2 – 2 \bullet 10 \bullet 15 \bullet \cos 50°$

$x^2 = 100 + 225 – 300 \bullet \cos 50°$

$x = \sqrt{(325 – 300 \bullet \cos 50°)}$

$x = 11.49624796...$

x = **11.5** correct to 1 decimal place.

Question 6

Answer: *b* = 9.17, correct to 2 decimal places.

Identify the rule to use: The Cosine Law: $c^2 = a^2 + b^2 - 2ab\cos C$

Substitute in the given values:

$b^2 = 2.5^2 + 7.5^2 – 2 \times 2.5 \times 7.5 \times \cos 125°$

$b = \sqrt{(2.5^2 + 7.5^2 – 2 \times 2.5 \times 7.5 \times \cos 125°)}$

$b = 9.165648715$

$b =$ **9.17**, correct to 2 decimal places.

Note that the angle C is always opposite the side c. In this Cosine Law, the side and angles are interchangeable (i.e. swapping c with b to find b is alright).

Question 7

Answer: a. Angle A = 24° 9', Angle B = 31° 45',

Angle C = 125°6'

The cosine law is: $c^2 = a^2 + b^2 - 2ab \cos(C)$

Note that C is always the angle opposite the side c.

$$\cos \angle A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 8^2 - 4^2}{2 \times 5 \times 8}$$

$$\angle A = \text{invcos} \frac{5^2 + 8^2 - 4^2}{2 \times 5 \times 8}$$

$= 24.146848 \ldots = 24°\ 8'\ 48.65'' = 24°\ 9'$ to nearest minute.

$$\angle B = \text{invcos} \frac{4^2 + 8^2 - 5^2}{2 \times 4 \times 8}$$

$= 30.75351981 \ldots = 30°\ 45'\ 12.67'' = 30°\ 45'$ to nearest minute.

$$\angle C = \text{invcos} \frac{4^2 + 5^2 - 8^2}{2 \times 4 \times 5}$$

$= 125.0996322 \ldots = 125°\ 5'\ 58.68'' = 125°\ 6'$ to nearest minute.

Question 8

Answer: c. sin C = 0.92

AB = 17.20

The angle sum of a triangle is 180°.

$\angle C = 180 - (\text{invsin } 0.45 + \text{invsin } 0.64) = 113.4644966 ...°$

Then, $\sin C = 0.917306984$ ... or **0.92** to 2 decimal places.

The sine of two angles and a side opposite one of the angles are given.

Write down the sine rule and substitute the given values.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{0.45} = \frac{12}{0.64} = \frac{c}{\sin C}$$

$$c = \frac{12 \sin C}{0.64}$$

$c = 17.19950595$ ...

AB = **17.20** correct to 2 decimal places.

Question 9

Answer: c. $\angle$**P = 59° 28',**

$\angle$**Q = 66°32'.**

Method: Draw a diagram showing the given data.

Decide which rule you can use.

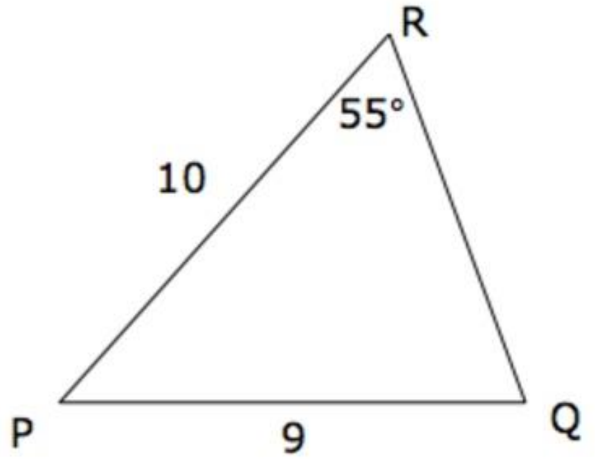
Write the sine rule with the sines of the unknown angles in the numerators

and the known lengths in the denominators.

Make sure that your calculator is in degrees mode.

There are 60 seconds in one minute. Round up to the minute above when there are 30 or more seconds.

Draw a diagram showing the given data:



When given two sides and a non-included angle, write down the sine rule.

$$\frac{\sin P}{p} = \frac{\sin Q}{q} = \frac{\sin R}{r}$$

$$\frac{\sin P}{p} = \frac{\sin Q}{10} = \frac{\sin 55°}{9}$$

$\angle Q = \text{invsin}\left(\frac{10 \sin 55°}{9}\right)$,

$\angle Q = 65.52870797\ ...° = 65°\ 31'\ 43.35'' = 66°\ 32'$, to the nearest minute.

$\angle P = 59°\ 28'$, angle sum of a triangle.

Question 10

Answer: $\angle C = 26°\ 19'$, to the nearest minute.

In ΔABD two sides and the included angle are known.

Use the cosine rule.

$DB^2 = 30^2 + 60^2 - 2 \times 30 \times 60 \cos 80° = 3874.86656...$

In ΔBCD, three sides are now known.

Use the cosine rule.

$$\cos C = \frac{140^2 + 130^2 - 3874.86656}{2 \times 140 \times 130} = 0.8962912088 \ldots$$

$C = \text{invcos } 0.8962912088$

$\angle C = 26.3247602198 \ldots°$

$\angle$**C = 26° 19'**, to the nearest minute.

NOTE: The calculation could have been done in one calculator run.

$$C = \text{invcos } \frac{140^2 + 130^2 - 30^2 - 60^2 + 2 \times 30 \times 60 \cos 80°}{2 \times 140 \times 130}$$