Year 12 Mathematics Worksheet

10 questions on Algebra, Functions and Graphs for Year 12 students.

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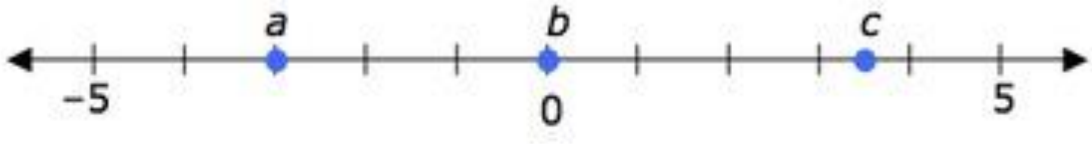
Questions

1. The absolute value of a real number, $|x|$, is the distance of x from the origin on a number line.



The numbers *a*, *b* and *c* are plotted on the number line above.

Which of the following are true?

(i) $|a| = a$
(ii) $|b| = 0$
(iii) $|-c| = c$
(iv) $|c - a| = a + c$
(v) $|a + c| = |a| + |c|$
(vi) $|a| + a = b$

a) (iv), (v), (vi)

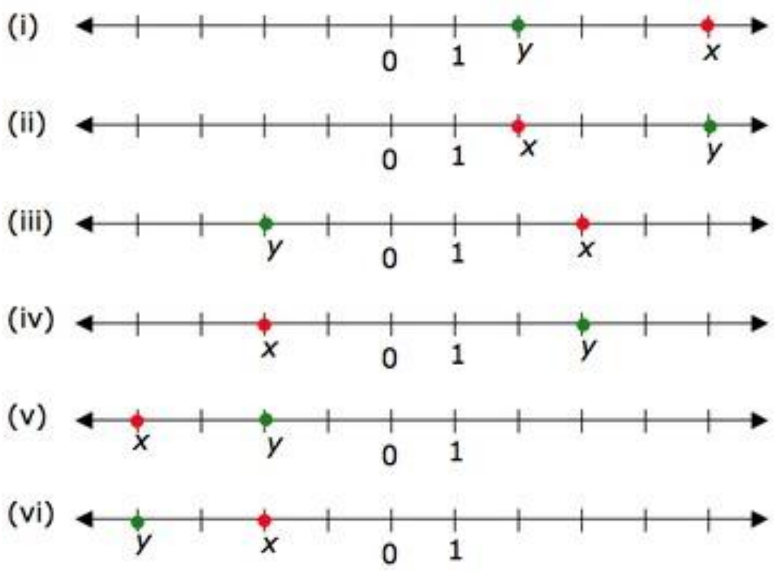
b) (i), (iii), (v)

c) (ii), (iii), (vi)

d) (i), (ii), (iii)

Answer: ____________________

2. In which of the following is the distance between x and y on the number line equal to $|x - y|$?



a) Only (ii), (iv), (v)

b) Only (i), (iii), (vi)

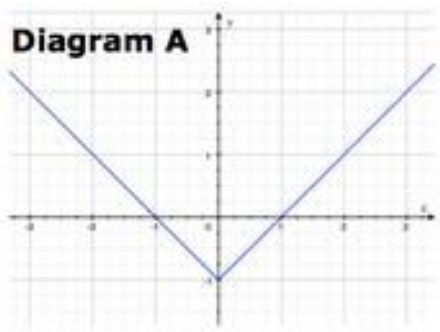
c) All of them.

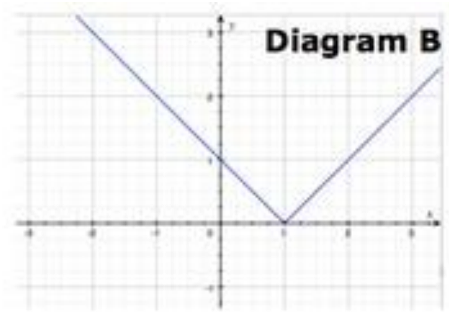
d) None of them

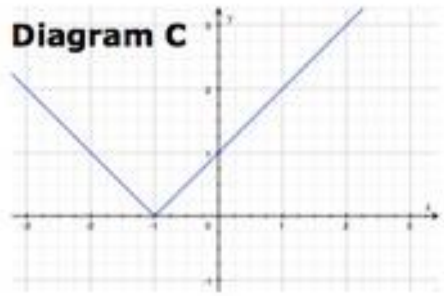
Answer_______________________

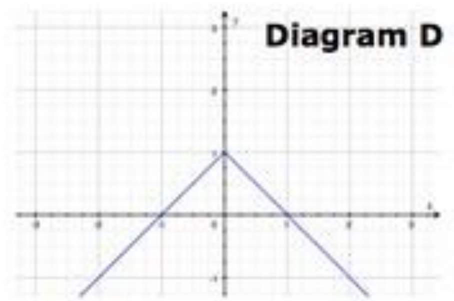
3. Match the diagrams A, B, C and D to the equations:

(i) $y = 1 - |x|$, (ii) $y = |x| - 1$, (iii) $y = |x + 1|$ and (iv) $y = |x - 1|$.









a)
A: $y = |x - 1|$
B: $y = 1 - |x|$
C: $y = |x| - 1$
D: $y = |x + 1|$

b)
A: $y = |x - 1|$
B: $y = |x + 1|$
C: $y = |x| - 1$
D: $y = 1 - |x|$

c)
A: $y = |x| - 1$
B: $y = |x - 1|$
C: $y = |x + 1|$
D: $y = 1 - |x|$

d)
A: $y = |x| - 1$
B: $y = |x + 1|$
C: $y = |x - 1|$
D: $y = 1 - |x|$

Answer___________________________

4. Perry is looking for another way to write the equation $y = |2x - 6|$.

Which should he choose?

$$y \left.\begin{aligned} &= 2x + 6, && \text{if } x < 3 \\ &= 0, && \text{if } x = 3 \\ &= 2x - 6 && \text{if } x > 3 \end{aligned}\right\}$$

a)

$$y \left.\begin{aligned} &= 2x - 6, && \text{if } x < 3 \\ &= 0, && \text{if } x = 3 \\ &= -2x - 6 && \text{if } x > 3 \end{aligned}\right\}$$

b)

$$y \left.\begin{aligned} &= -2x + 6, && \text{if } x < 3 \\ &= 0, && \text{if } x = 3 \\ &= 2x - 6 && \text{if } x > 3 \end{aligned}\right\}$$

c)

$$y \left.\begin{aligned} &= -2x - 6, && \text{if } x < 3 \\ &= 0, && \text{if } x = 3 \\ &= -2x + 6 && \text{if } x > 3 \end{aligned}\right\}$$

d)

Answer____________________

5. By drawing the graphs of $y = |x - 2|$ and $y = |3x + 6|$ on the same grid, Sean solves the equation $|x - 2| = |3x + 6|$ graphically.

What answer should Sean get?

a) 3 and 6

b) 4 and 1

c) –3 and –6

d) –4 and –1

Answer_____________

6. Solve $|x + 2| - |3x - 6| \geq 0$ algebraically.

a) $-4 < x < 1$

b) $4 \leq x \leq -1$

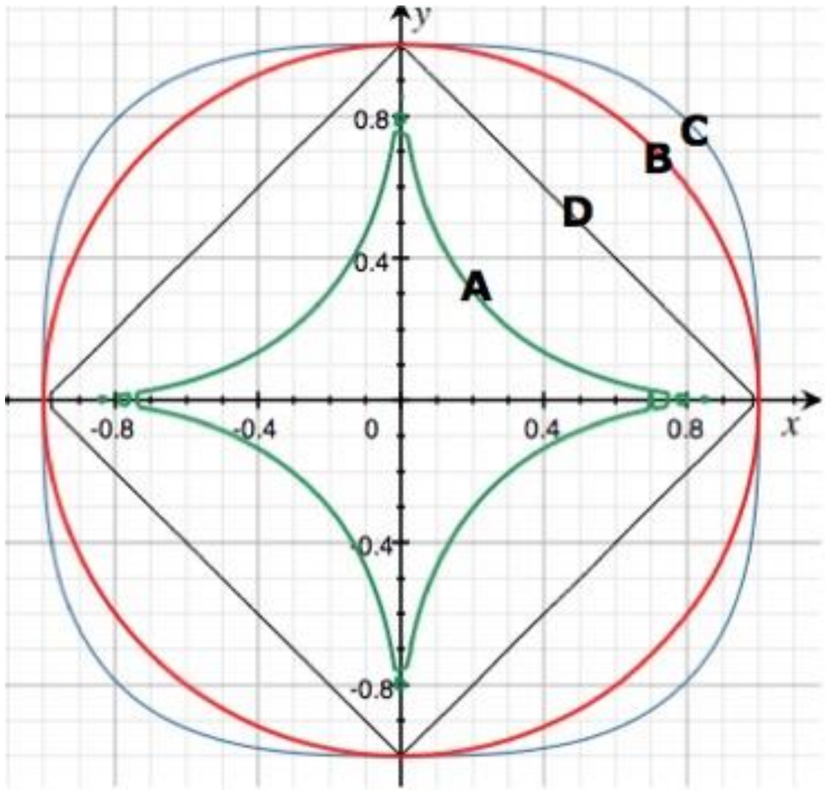
c) $1 \leq x \leq 4$

d) $1 < x < 4$

Answer_________________

7. The four curves shown on the diagram all have equations of the form

$| x |^n + | y |^n = 1$, where n = 0.5, 1, 2 or 3.



All four graphs go through (1, 0), (0, 1), (–1, 0) and (0, –1).

Match the letters for the graphs with the values of n.

a) A: n = 3
   B: n = 2
   C: n = 0.5
   D: n = 1

b) A: n = 0.5
   B: n = 3
   C: n = 2
   D: n = 1

c) A: n = 1
   B: n = 2
   C: n = 3
   D: n = 0.5

d) A: $n = 0.5$
   B: $n = 2$
   C: $n = 3$
   D: $n = 1$

Answer__________________

8. Terri is investigating if $|a + b| \leq |a| + |b|$ for all a and b with $a \geq b$.
The statement is symmetrical in a and b, so Terri assumes that with $a \geq b$and considers only five cases:

(i) $a = 0$, (or $b = 0$)
(ii) $a > 0$ and $b > 0$
(iii) $a < 0$ and $b < 0$
(iv) a > 0 and $b < 0$, $a + b < 0$
(v) $a > 0$ and $b < 0$, $a + b \geq 0$.
Terri writes: LHS = $|a + b|$, RHS = $|a| + |b|$.

In cases (i), (ii) and (iii), LHS = RHS.

What is true about the LHS and the RHS in cases (iv) and (v)?

a) (iv) LHS < RHS; (v) LHS < RHS.

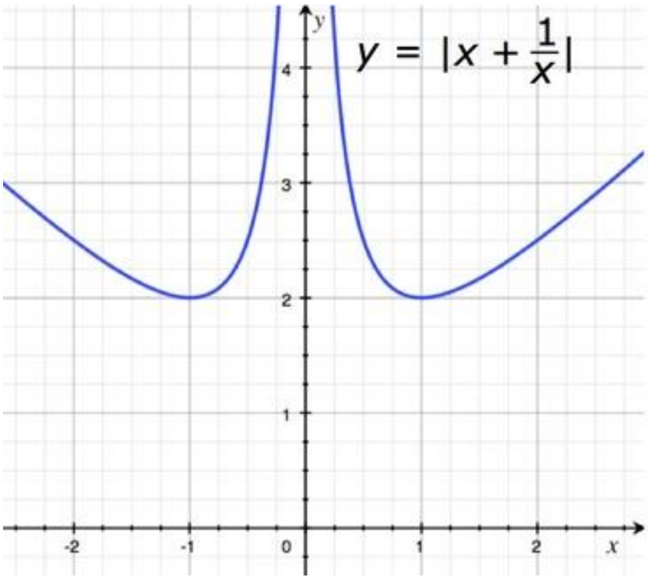
b) iv) LHS = RHS; (v) LHS ≥ RHS.

c) (iv) LHS < RHS; (v) LHS = RHS.

d) (iv) LHS = RHS; (v) LHS = RHS.

Answer___________________

9. Sam, Tom and Una have to find the smallest value of $|x + x^{-1}|$ for all non-zero values of x. Sam decides to use a computer program to draw a graph of $y = |x + x^{-1}|$.



Tom decides to use a spreadsheet to generate values of $|x + x^{-1}|$.

	A	B
1	x	fn(x)
2	-1.5	2.17
3	-1.4	2.11
4	-1.3	2.07
5	-1.2	2.03
6	-1.1	2.01
7	-1	**2.00**
8	-0.9	2.01
9	-0.8	2.05
10	-0.7	2.13
11	-0.6	2.27
12	-0.5	2.50
13	0.5	2.50
14	0.6	2.27
15	0.7	2.13
16	0.8	2.05
17	0.9	2.01
18	1	**2.00**
19	1.1	2.01
20	1.2	2.03
21	1.3	2.07
22	1.4	2.11
23	1.5	2.17

Una uses a calculus technique to find the minimum value of $|x + x^{-1}|$.

Una notes that $f(x) = |x + x^{-1}|$, $x \neq 0$, is an even function

because $f(a) = |a + a^{-1}| = |-(a + a^{-1})| = |-a - a^{-1})| = f(-a)$.

$f(x) > 0$ for all x in the domain.

Una reasons that if there is a minimum turning point in the first quadrant there will be a symmetrically placed minimum turning point in the second quadrant.

Una writes:
If $y = x + x^{-1}$, x >0, then $\frac{dy}{dx} = 1 - x^{-2}$ and $\frac{d^2y}{dx^2} = 2x^{-3}$.

For $x > 0$, the graph is concave up, and there is a turning point at $x = 1$, $y = 2$. This turning point is a minimum.
Therefore the smallest value for $|x + x^{-1}|$ is 2.

Which of the three methods for finding the least value is the most exact?

a) Sam’s method.

b) Tom’s method.

c) Una’s method.

d) They are all approximations.

Answer_______________

10. Zai is asked to solve the problem:

"For what values of m does the equation $|2x - 3| = mx$ have:

(i) no solution
(ii) one solution
(iii) two solutions?”

Zai draws the graph of the equation of $y = |2x - 3|$. On the same diagram he draws the graphs of some members of the family of straight lines $y = mx$ for various values of the slope m.
What should his answers to the question be?

a)
(i) $-2 \le m < 0$,
(ii) $m < -2$, $2 \le m$ and $m = 0$,
(iii) $0 < m < 2$.

b)
(i) $-2 \le m < 0$,
(ii) $0 < m < 2$,
(iii) $m < -2$, $2 \le m$ and $m = 0$.

c)
(i) $-2 \le m \le 0$,
(ii) $0 < m \le 2$,
(iii) $m < -2$, $2 \le m$.

d)
(i) $0 < m < 2$,
(ii) $m < -2$, $2 \le m$ and $m = 0$,
(iii) $-2 \le m < 0$.

Answer__________________

The Answers.

Hey! No peeking until you've finished…

Question 1

Answer: c) (ii), (iii), (vi)

From the number line: $a = -3$, $b = 0$, $c = 3.5$.

(i) $|a| = |-3| = 3 \neq a$

(ii) $|b| = |0| = 0$

(iii) $|-c| = |-3.5| = 3.5 = c$

(iv) LHS = $|3.5 - -3| = |6.5| = 6.5$,

RHS = $a + c = -3 + 3.5 = 0.5 \neq$ LHS.

(v) LHS = $|a + c| = |-3 + 3.5| = |0.5| = 0.5$,

RHS = $|a| + |c| = 3 + 3.5 = 6.5 \neq$ LHS

(vi) $|a| + a = 3 - 3 = 0 = b$.

Question 2

Answer: c) All of them

i) $x = 5$, $y = 2$, distance equals = 3, $|x - y| = |5 - 2| = |3| = 3$.

(ii) $x = 2$, $y = 5$, distance equals = 3, $|x - y| = |2 - 5| = |-3| = 3$.

(iii) $x = 3$, $y = -2$, distance equals = 5, $|x - y| = |3 - (-2)| = |5| = 5$.

(iv) $x = -2$, $y = 3$, distance equals = 5, $|x - y| = |-2 - 3| = |-5| = 5$.

(v) $x = -4$, $y = -2$, distance equals = 2, $|x - y| = |-4 - (-2)| = |-2| = 2$.

(vi) $x = -2$, $y = -4$, distance equals = 2, $|x - y| = |-2 - (-4)| = |2| = 2$.

Question 3

Answer: c)

A: $y = |x| - 1$

B: $y = |x - 1|$

C: $y = |x + 1|$

D: $y = 1 - |x|$

The values of x and y for each function and the matching graphs are shown in the spreadsheet.

Equation	$x =$	−2	−1	0	1	2	Diagram
(i) $y = 1 - \lvert x \rvert$	$y =$	−1	0	1	0	−1	D
(ii) $y = \lvert x \rvert - 1$	$y =$	1	0	−1	0	1	A
(iii) $y = \lvert x + 1 \rvert$	$y =$	1	0	1	2	3	C
(iv) $y = \lvert x - 1 \rvert$	$y =$	3	2	1	0	1	B

Question 4

Answer: a)

$$y = \left.\begin{array}{ll} -2x + 6, & \text{if } x < 3 \\ = 0, & \text{if } x = 3 \\ = 2x - 6 & \text{if } x > 3 \end{array}\right\}$$

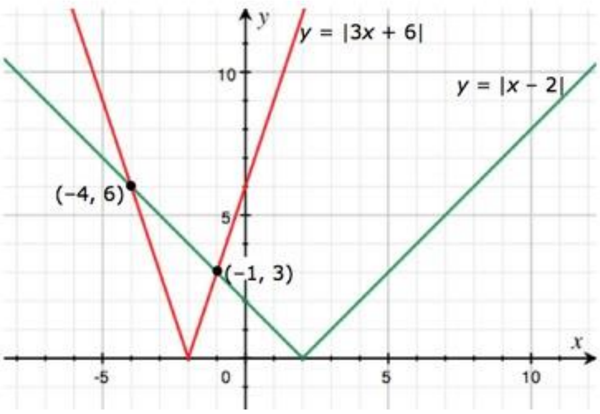
From the definition of absolute value:

$$y = |2x - 6| \left.\begin{array}{ll} = -2x + 6, & \text{if } 2x - 6 < 0, \text{ ie } x < 3 \\ = 0, & \text{if } 2x - 6 = 0, \text{ ie } x = 3 \\ = 2x - 6 & \text{if } 2x - 6 > 0, \text{ ie } x > 3 \end{array}\right\}$$

Question 5

Answer: b) –4 and –1

Draw the graphs and read off the *x*-coordinates of the points of intersection.



Question 6

Answer: c) $1 \le x \le 4$

Simplifying $|x + 2|^2 = |3x - 6|^2$ leads to $x^2 - 5x + 4 = (x - 4)(x - 1) = 0$
From putting $|x + 2|^2 = |3x - 6|^2$ you find that $x = 1$ and $x = 4$ are part of the solution to $|x + 2| - |3x - 6| \ge 0$.
Now try values of x less than 1, between 1 and 4 and greater than 4.

Question 7

Answer: d)

A: $n = 0.5$
B: $n = 2$
C: $n = 3$
D: $n = 1$

From Graph	x	y
A	0.4	≈ 0.1
B	0.4	≈ 0.9
C	0.4	≈ 1
D	0.4	0.6

From function	x	$y = \sqrt[n]{(1 - 0.4^n)}$, by calculator
$n = 0.5$	0.4	$\sqrt[0.5]{(1 - 0.4^{0.5})} \approx 0.1350889359 \ldots$
$n = 1$	0.4	$1 - 0.4 = 0.6$
$n = 2$	0.4	$\sqrt{(1 - 0.4^2)} \approx 0.916515139 \ldots$
$n = 3$	0.4	$\sqrt[3]{(1 - 0.4^3)} \approx 0.9781946493 \ldots$

NOTE: In the table, substituting $n = 0.5$ into the equation led to the unusual notation $\sqrt[0.5]{}$.
$\sqrt[n]{a} = a^{1/n}$. Therefore $\sqrt[0.5]{a} = a^{1/0.5} = a^2$.

Question 8

Answer: a) (iv) LHS < RHS; (v) LHS < RHS.

In (iv) put $a = 1$, $b = -2$.

Note that $a + b = 1 - 2 < 0$.

LHS = $|a + b| = 1$.

RHS = $|a| + |b| = 1 + 2 = 3$.

Therefore LHS < RHS.

In (v) put $a = 2$, $b = -1$.

Note that $a + b = 2 - 1 > 0$.

LHS = $|a + b| = 1$.

RHS = $|a| + |b| = 2 + 1 = 3$.

Therefore LHS < RHS.

In (v) put $a = 2$, $b = -2$.

Note that $a + b = 2 - 2 = 0$.

LHS = $|a + b| = 0$.

RHS = $|a| + |b| = 2 + 2 = 4$.

Therefore LHS < RHS.

Question 9

Answer: c) Unas method

There will be observation errors in reading from a graph.

The spreadsheet does not show all possible values for a function. The values shown for the function depend upon the values chosen for *x*.

The calculus method gives the exact values for the coordinates of the minimum point.

Question 10

Answer: a)

(i) $-2 \le m < 0$,
(ii) $m < -2$, $2 \le m$ and $m = 0$,
(iii) $0 < m < 2$.

Refer to the diagram showing the graphs.

If $-2 \le m < 0$ the lines $y = mx$ do not cut $y = |2x - 3|$.

If $m < -2$ or $m \ge 2$ the lines $y = mx$ cut the branch of $y = |2x - 3|$ with slope -2.

If $0 < m < 2$ the lines $y = mx$ cut both branches of $y = |2x - 3|$.