Math Exchanges

Guiding Young Mathematicians in Small-Group Meetings

Kassia Omohundro Wedekind

Foreword by Suzanne H. Chapin

Stenhouse Publishers
www.stenhouse.com

Credits
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Library of Congress Cataloging-in-Publication Data
Omohundro Wedekind, Kassia
Math exchanges : guiding young mathematicians in small-group meetings / Kassia Omohundro Wedekind.
p. cm.
Includes bibliographical references and index.
ISBN 978-1-57110-826-5 (pbk. : alk. paper) — ISBN 978-1-57110-923-1 (e-book) 1. Mathematics—Study and teaching (Elementary) 2. Mathematics—Study and teaching (Preschool) 3. Group guidance in education. I. Title.
QA20.G76W43 2011
372.7'044—dc23

2011014818

Cover design, interior design, and typesetting by Martha Drury

Manufactured in the United States of America

PRINTED ON 30% PCW
RECYCLED PAPER

17 16 15 14 13 12 11   9 8 7 6 5 4 3 2 1

CHAPTER 2

Math Exchanges: From Students to Mathematicians

JOHN STEINBECK WROTE, "As happens sometimes, a moment settled and hovered and remained for much more than a moment. And sound stopped and movement stopped for much, much more than a moment" (1994 [1937], 91). This is what we are striving for every day as we work with children—meaningful moments that will "hover" and develop in the minds of young mathematicians, that will take on a deeper meaning beyond the moment in which they occur.

Defining and Building Math Exchanges

When I first thought about and established some core beliefs about teaching math, I knew I needed to think more critically about how I was working with small groups of children. No longer could I view our time together as a place to reteach, rehash, or remediate concepts previously taught to the whole class. If I were to truly live and practice my beliefs about mathematics and mathematicians, I knew I needed to change how I viewed the purpose of small-group work, and thus how I worked with small groups of mathematicians.

Math exchanges, the term I use to define the space, place, and time in which teachers work with small groups of young mathematicians, are spaces that nurture joy, rigor, and empowerment and inform the other parts of our math workshops. Mathematicians are created inside and outside of the math workshop, but there is something special about the closeness of working with a few mathematicians to tackle a problem. If we believe that learning math is much more than learning a series of operations and procedures, then the way in which we work with small groups of children must reflect this as well. Math exchanges focus on building number sense and a deep understanding of the big ideas of mathematics, but perhaps most importantly, math exchanges help nurture young mathematicians who will construct their own definitions of, beliefs about, and purposes for math.

As with all journeys, there are few shortcuts or quick fixes and no magic formulas or scripted narratives for success for math exchanges. I offer no glossy promises. I do, however, invite you to share in a process in which there are many joys, frustrations, and ongoing experiments. I offer you a place beside me as a fellow learner in which we will question, doubt, and spend time pouring over student work, wondering what the next step will be for each child. We will work to become experts on the development of each of our young mathematicians. We will facilitate deep growth and understanding. This is the work of a math exchange. It is a space to embrace the unknown, to inquire, to play, to work, to struggle, and to learn and relearn.

Understanding the Work of Math Exchanges

One spring afternoon as my third graders transitioned from the whole-group focus lesson into working independently during math workshop, I asked Marta, Evangeline, Herman, and Alex over to the round table to examine a praying mantis egg sac we found during an exploration of our school's courtyard the previous day. "Oh!" Alex shouted. "This is the thing we found outside yesterday with those bugs inside."

"Yep, this is an egg sac where the praying mantis laid her eggs," I responded, pausing for a moment to let them all touch the egg sac and talk about it.

"Yeah, we found three of those egg sacs outside yesterday!" added Evangeline.

"All hidden in those bushes. I didn't even know what they were before!" Herman followed up.

"Well, neither did I," I reminded them. "Remember? We had to ask Mrs. Rosenbaum [our school's outdoor science teacher] what they were. Yesterday after school I saw Mrs. Rosenbaum again, and she told me something else about the egg sacs that I think you all will really be interested in. She told me that the baby praying mantises won't hatch until the summer, but when they do, up to about 400 little praying mantises can come out of this one little egg sac!"

"Wow! That's a lot!" Marta said with a surprised laugh. We were all completely involved in the moment. The voices and sounds from the rest of the children, working independently and with partners on problem-solving tasks, had faded into the background.

"Today I have a story problem for you about these praying mantis egg sacs," I continued. "Evangeline, you reminded us that we found three egg sacs outside yesterday. We know they won't hatch until the summer, but today we're going to imagine how many praying mantises *could* hatch from the egg sacs. Let's read this story first without the numbers, and see what we can figure out."

I handed each child a piece of paper with the story written on it and blanks where we would write the numbers I'd selected for this group (see Figure 2.1). At the top of the page were pictures of the praying mantis sacs that we took the day before in our outdoor exploration. The four young mathematicians each grabbed one of our special, colored problem-solving pens we use only during math exchanges. Unlike many math classes, we use only pens during math workshop. I teach the children to draw a single line through something if they change their ideas instead of scribbling it out beyond recognition. "All of our thinking is important," I tell them, "even if we change our ideas and answers along the way." As a teacher, having children use pens allows me to view the footprints of their thinking and not just their final answer or idea.

Let's reflect for a moment on the bigger idea of what is happening in small moments such as the one I just described. Someone asked me recently why I use the term *math exchange* to describe working with small groups of students. "Kassia, isn't what you're doing just the same as small-group math?" The answer is yes . . . and no.

Small-group math instruction has traditionally been seen as a form of reaching children who struggle to understand. When I first began teaching math, I, too, thought that if only I could pull all these struggling students

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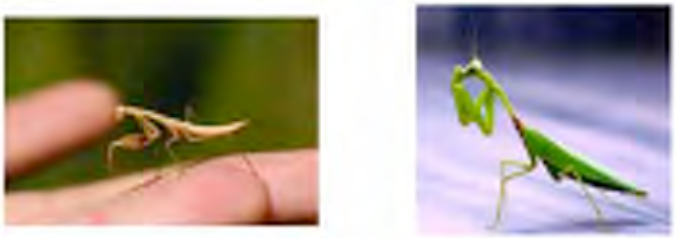

On Wednesday ______ praying mantises hatched out of an egg sac and went looking for food. On Friday ______ praying mantises hatched out of another egg sac and hopped away. How many praying mantises hatched this week?

Figure 2.1 Our Praying Mantis Story Problem

together in a small group I could teach something again, differently, in a better way—that if I only worked hard enough I could *make* these students understand. My intentions were good—but this kind of instruction does not work. Both my struggling students and I ended up frustrated. I ended up doing more and more of the thinking for them in order to *make* them understand at the pace and level where I so desperately wanted them to be. However, when these struggling kids left the small group they went right back to being confused. Our work together had not produced any real change in their thinking.

I knew this kind of small-group instruction was not serving my students' best interests or aligning with my beliefs about teaching math. And so I began the journey of working toward a different kind of small-group math instruction—one that builds deep understanding over time and helps students build on what they know, constructing new meaning and understanding through their interactions with peers and the guidance of the teacher.

There is no single "right" way to work effectively with small groups of mathematicians, but these threads are common to successful math exchanges:

1. Short, focused sessions that bring all mathematical minds together
2. Content and context that are responsive to the needs of the specific group of mathematicians
3. Meaningful guided reflection

Many teachers, even those comfortable with teaching math, struggle to implement meaningful, consistent small-group work within their math workshop format. This book is about understanding the power of math exchanges and finding effective ways to make them work within your classroom. We teach in a time of ever-increasing standardization, and many times the unique role of the classroom teacher to understand and be responsive to each individual child's needs is lost in the search for one program, textbook, or pacing guide that will "solve" everything and everyone in one fell swoop. Regardless of the math program or curriculum you are using, working to understand each individual child's math development and how to responsively guide children to deeper understanding through small-group work remains the strongest way in which we, as teachers, can promote a strong mathematical foundation for our learners.

Much of the philosophy behind teaching and learning through math exchanges comes from the work of those who developed Cognitively Guided Instruction (CGI). In *Children's Mathematics* (1999), Thomas P. Carpenter and his coauthors explain CGI as a philosophy that asserts that "children enter school with a great deal of informal or intuitive knowledge of mathematics that can serve as the basis for developing understanding" of mathematics (4). A classroom rooted in CGI philosophy uses problem solving as the main vehicle of mathematics instruction and understanding. The problem types and development of strategies described in *Children's Mathematics* and the other publications by the developers of CGI are an invaluable resource to any teacher who is working to implement this philosophy through small-group math exchanges.

Also critical to our role as teachers in guiding young mathematicians is understanding how children construct the set of interconnected ideas that form the basis of number sense and the big ideas of mathematics. Catherine Fosnot and Maarten Dolk, in their book *Young Mathematicians at Work* (2001; one book in a series), refer to this as the "landscape of learning comprising the big ideas, strategies, and models" (163). Fosnot and Dolk describe "the landmarks the students pass (collectively and severally) in their journey through this landscape that inform teachers' questions, instructional decisions, and the curriculum" (163). Each student's journey toward mathematical understanding is different, but to teach effectively it is essential to understand how children's ideas develop.

As a teacher, I want children to build an interconnected system of ideas and understanding of math, to work together to problem solve, and to build a community of learners who see math not just as a set of useful skills, but as a creative and useful process. By implementing math exchanges into my math workshop, I have been able to do that. Before we return to the small group working through the praying mantis problem, let's think a little about how groups like this one are formed.

Coming Together: Forming Groups That Make Sense

Groups for math exchanges are flexible. Every time I plan a new group meeting, I choose specific children to be in the group for specific reasons. Children do not remain with an identified "math group" for the entire school year. The groups are *responsive* to a specific need. Children need to interact with all kinds of mathematicians. The idea of grouping kids by ability level is counterproductive to the idea that mathematicians learn from each other. Regie Routman sums up this idea perfectly when she asserts that when readers are grouped by ability, "low-performing students are deprived of the benefits of thinking and discussing with higher-performing peers" (2008, 78). The same is true of mathematics; children should be grouped by their mathematical needs, which can be entirely different than their "ability level."

Although *flexible grouping* has come to be a popular term in education, it is often used to describe grouping that is anything but flexible. Often the "flexible grouping" of children for whole-group and small-group instruction is only a euphemism for ability grouping. Perhaps there are assessments given along the way, and a few children move from group to group, but the great majority of children remain classified by the old crippling practice of separating "those of us who are good at math" from "the rest of us."

So how *should* we form groups for math exchanges? I was reminded of the importance of this question after a teacher came into my room when I was working with a group of students. Later she casually remarked to me, "I was so surprised you put *those* kids together."

Those kids were Alex, Marta, Evangeline, and Herman, the four third graders I wrote about earlier in this chapter, who were preparing to do some problem solving related to praying mantises. Upon first glance, these four may seem like an odd group. Marta is a very strong mathematician who knows how to articulate her solid mathematical understanding. She enjoys attempting and talking about interesting problem-solving tasks that challenge her, even if she isn't able to solve them immediately. Evangeline also embraces challenges but has many fewer tools and less efficient strategies than Marta has. Alex is impulsive. He is eager to take on a task but often loses stamina and interest at the point of difficulty. Herman, in looking for "quick tricks" to solve problems, often tries to use traditional algorithms he only partially understands. As a result, Herman is often unsure whether his ideas and answers make sense. Given a traditional assessment, the scores of these four kids would be all over the map. Despite their apparent differences in mathematical understanding and skills, I chose this group with a specific purpose in mind. I wanted these four students to practice listening to and talking about a specific strategy used by a member of the group, and to work to understand why he or she had chosen it. I wanted the child explaining the strategy to work to justify the use of this strategy for this particular problem.

Let's return to the children working through the praying mantis problem with me. I watched them finishing up the problem solving. Herman was rereading the story problem: "On Wednesday 397 praying mantises hatched out of an egg sac and went looking for food. On Friday 205 praying mantises hatched out of another egg sac and hopped away. How many praying mantises hatched this week?"

I chose the numbers in this problem carefully. During whole-group discussions with the class, some students had begun explaining their ideas using compensation strategies. We had talked about how mathematicians examine the numbers in a problem and base their choice of strategy on efficiency for the particular problem and numbers. In this problem I chose the number 397 because of its proximity to 400. As the students began to work on the problem, I wondered if they would notice this and use it to help them solve the problem efficiently. Of these four students, I had seen Marta and Evangeline use compensation strategies in their work previously. Evangeline had used compensation only with smaller numbers. Marta was comfortable with executing compensation strategies, but I wondered if she could clearly explain *why* they worked.

I watched quietly as the group made sense of the problem. Alex and Herman worked together, carefully constructing the numbers with base ten blocks. Herman took the lead. "Alex, you make 205 and I'll make 397. Then we can see how many praying mantises there were. You know, we'll put them all together." Evangeline had been watching them quietly but hadn't written anything yet.

"What are you thinking, Evangeline?" I asked.

"I'm just thinking that it's a lot of work to make all that with blocks."

"Oh. And are you thinking that there is an easier way to do it?"

"Well, 397 is close to 400, so you could just build it with four hundreds blocks," Evangeline responded.

"But it's *not* 400, it's 397. That's different," responded Alex, satisfied with his strategy.

Marta had already solved the problem using a compensation strategy. On her paper she had written:

397 + 3 = 400
400 + 205 = 605
605 – 3 = 602

Having quickly solved the problem, Marta sat back and considered what the others were building with the base ten blocks. Evangeline, seemingly stumped by what to do once she had changed 397 to 400, solved the problem using her tried-and-true strategy of decomposing the numbers in order to add them according to their place value, a valuable strategy but not the most efficient for this problem. She wrote:

397 + 205 =
300 + 200 = 500
90 + 0 = 90
7 + 5 = 12
90 + 12 = 102
500 + 102 = 602

By this time, Alex and Herman had finished constructing and combining the numbers with base ten blocks. All four students arrived at the correct answer. In this case, I was fairly confident they would each have a strategy that would get them there. However, their strategies could not have been more different in terms of both understanding the math and efficiency of computation. I could have stopped the work there. We'd all arrived at an answer of 602. I could have been satisfied. But this would not have been a math exchange. No change had occurred in the minds of the four young mathematicians.

I carefully crafted my words. "Let's take a look at Marta's strategy. She solved the problem a little differently, but I think there's a connection to what

Evangeline was thinking about with the number 400, and I'm wondering if we can show Marta's strategy using the tools Herman and Alex chose for this problem, the base ten blocks."

Our discussion continued for the next five minutes. Marta explained her strategy, and Evangeline was able to retell it, making the jump to why it was possible to change 397 to 400 if you "made up for it" at the end of the problem by subtracting three. Marta and Evangeline helped Alex and Herman construct the compensation strategy with the base ten blocks. "Four hundred is a lot easier to build. It's just a little bit more than 397," commented Alex. The four mathematicians were all at different points in understanding the strategy I'd hoped to highlight with this problem. And that was okay. My goal for this math exchange was not for everyone to solve 397 + 205 using a compensation strategy. My goal was for students to consider and work to understand the efficiency of problem-solving strategies based on the kind of problem and the numbers they were solving.

Although there are sometimes "aha" moments within a single math exchange that produce great and lasting change within the mind of a child, in most cases true change occurs over time. When someone brings up a compensation strategy at another point during math workshop, Herman might remember that this is similar to what we discussed in the praying mantis problem math exchange. A few days later when adding 49 + 22, Alex or Evangeline might try solving the problem using 50 + 21. Marta's experience explaining her strategy might clarify her thoughts and allow her to continue developing a wide range of sophisticated problem solving for different problems and numbers. The power of math exchanges is in the changes they produce over time.

Math exchanges emphasize *change*. This may sound simple, but the purpose of meeting with small groups of mathematicians is to produce change and growth in their thinking. Too often, teachers meet with groups for the sake of meeting ("It's time to pull these five kids together! What should we talk about?") and without a specific focus. They may have the vague notion that meeting with small groups of mathematicians (or perhaps only meeting with struggling mathematicians) is a good practice. Math exchanges put the focus on planned, purposeful exchanges between mathematicians of different abilities.

I chose those four mathematicians with a purpose, knowing that they would approach the problem with different background knowledge and strategies for solving it. This was exactly what I wanted! Through questioning—by me and other members of group—and conversation, my goal of having these children work to explain and justify more efficient strategies was achieved. As I watched them work and bump into issues of efficiency of problem solving, I guided the conversation toward a discussion of how their different strategies took into account this important concept of efficiency. That doesn't mean that the next time these students encounter this kind of problem-solving situation all of them will all be able or will choose to use such efficient strategies. However, the seeds for deeper understanding have been planted. They have owned the work done in this group session, to which I will now be able to refer as we continue developing and refining our ideas throughout the math workshop.

Nudging the Exchange Toward the Big Ideas

Part of building a strong mathematical community is finding the balance between process and product. When I begin a new school year with students unfamiliar with math exchanges, there is often a tendency among children to focus exclusively on getting the right answer through any means necessary. Our first math exchanges sound a little like this:

"I got twenty-six."

"Yep, the answer is twenty-six!"

"Twenty-six? Me, too. That's right."

"It's twenty-six, Ms. Wedekind. We're done. So, what do we do now?"

Of course, they are a little thrown off when I follow with questions like, "Are you sure?" and "How do you know?" My goal is not to diminish their confidence or enthusiasm for solving a problem correctly but rather to nudge them toward ideas that go beyond the fact that the answer is, indeed, twenty-six.

At my school, like many others, we teach the children to explain their answers, to show their work, and to justify their answers to their peers. Our goal is for students to successfully solve problems while also considering new ideas, strategies, and patterns emerging as they solve. One of the goals of math exchanges is to help children make connections between what they know and what they are just beginning to learn. I choose each problem with a purpose—with an idea of what I want students to discover or build upon. This is something that cannot be accomplished if I allow students to see problem solving as dealing with a string of unrelated problems that we want to hurry up and solve in order to get to the next, more difficult problem.

Luckily, children have some of the most adaptable and incredible mathematical minds out there. Before long, kids have moved on from "Listen, lady, the answer is twenty-six. I know I'm right, now let's all move on" to a deeper level of reflection that sounds more like this:

"Look, when we add ten to sixteen, we get twenty-six. The tens place changes, but the six stays the same."

"That's like when I added thirty-four plus ten. I thought I had to count up with my fingers, but then I remembered a pattern. When you add ten, the tens place changes, but the ones place always stays the same."

And of course, as their teacher I will still always have questions: "That's an interesting pattern you are noticing. Does that always work? How could we test that idea out?"

You've seen how I try to balance process and product in my math exchanges. Let's take a look at a couple of math exchanges across different grade levels to see other teachers balancing process and product while also nudging students toward the big ideas behind the problems.

Katie Keier, a first-grade teacher, sits on the floor with four of her students, who all have a whiteboard in front of them that they are using for problem solving today. Katie has just asked them to solve 10 + 7. A couple of them immediately write down *17*. Others count on from 10 using their fingers to keep track of when to stop. "So we agree that 10 plus 7 is 17," says Katie. "Now I'm going to give you another problem. Think about if 10 plus 7 can help you with our next problem, 9 plus 7." A couple of children use the same counting-on strategy they used with 10 + 7, not seeing any real similarity between the problems that can help them. A couple of students, though, immediately write down *16* on their whiteboards. This is the big idea Katie hoped this group would bump up against. As Katie proceeds to guide the conversation, she'll point out that while everyone got the answer of 16, some kids used derived facts or a compensation strategy to quickly solve 9 + 7 mentally.

"Abdel, how did you solve nine plus seven in your mind? Did ten plus seven help you?" asks Katie.

"Well," explains Abdel, "I already know 10 plus 7 is 17. Nine is 1 less than 10, so the answer to 9 plus 7 will be 1 less than 17. That's 16."

Katie checks in with a couple of other kids in the group to see if they understand what Abdel is thinking. Rebecca responds, "Abdel knows that 10 plus 7 is 17 just like 10 plus 6 is 16 and 10 plus 8 is 18. It's a pattern, so he didn't have to count." It looks like most of the children are making a connection to the plus-10 facts but don't yet understand the connection between plus-10 and plus-9.

"Oh, so that is how Abdel solved that first problem so quickly. Let's write down those facts we know with ten on the chart paper and then see if we can figure out how that helps us when we add numbers to nine."

After the kids have helped Katie construct the chart, they return to looking for patterns and connections between plus-10 and plus-9 facts.

Rachel Knieling's second-grade class has been focusing on problem solving with fractions. Today a group of children sit with Rachel working on this problem:

> Four kids want to share five pancakes so that every kid gets the same amount of pancake. How many pancakes can each kid eat?

In planning for this math exchange, Rachel wanted to know what her small group of students would do with the "extra" fifth pancake. Would they share the pancake equally among the four friends? Would they know the notation $\frac{1}{4}$?

Rachel watches her students working with the problem. They all begin by drawing the five pancakes and four stick figure people, but what they do from that point varies. One student ignores the extra fifth pancake. Another divides the extra pancake into four unequal parts. A couple of students draw a model of four people, each with their whole pancake and one quarter of a pancake, but have not yet figured out the notation of $\frac{1}{4}$. Rachel knows her next move is a discussion with the group about what to do with this extra pancake and a return to the question of how many pancakes each kid can eat.

As students and teachers become more comfortable in the math exchange environment, more meaningful conversations arise. The teacher becomes better at identifying the important ideas around which to center the conversations, and kids become better questioners and wonderers rather than just students in pursuit of the right answer.

The Time Issue: Frequency and Consistency of Math Exchanges

Every teacher I know struggles with issues of time. There is never enough of this precious commodity. In *Teaching Essentials*, Regie Routman writes about "teaching with a sense of urgency." Before reading her words, I had always associated urgency with a feeling of anxiety. However, after reading and thinking further I found that "the expectation that there is not a minute to lose, that every moment must be used for purposeful instruction" rings true not just within the context of teaching literacy, which Routman writes about, but throughout our entire day as teachers (2008, 96).

Time Within Each Math Exchange

Although I do not set hard-and-fast rules for the length of math exchanges, I have found that the most powerful math exchanges are usually between ten and twenty minutes, depending on the age of the mathematicians and the purpose of the group. Kindergarten math exchanges may last only five minutes. This gives the group enough time to retell and talk together about the meaning of the problem-solving work, attempt the task, and listen to and reflect on the strategies of different group members (see Figure 2.2).

When I first began to experiment with math exchanges, I found that I often became so immersed in the ideas of one particular group that I would look up at the clock and realize I had spent thirty minutes with that one group of four students. I knew I wanted to be meeting with more children every day, yet I was finding it hard to find a stopping point. When I find myself falling into a pattern of dissatisfaction with the amount of time each group is spending together, I remind myself of what my math coach, Mimi Granados, often says to me when I ask her questions (to which she always responds with more questions!): "What is your purpose, Kassia? What is it that you want these kids to get out of this time together?"

Figure 2.2 Third-grade teacher Phoebe Markle works with a small group in a math exchange.

Returning to the issue of purpose helps me find the comfortable amount of time to work with each group. A much-repeated idea of Lucy Calkins is "teach the writer, not the writing" (1994, 228). I believe the same is true of how teaching should occur within math exchanges. I often have to remind myself to teach the mathematician, not the math. Within our brief time together in a math exchange, my purpose is not for everyone to arrive quickly and neatly at the "correct" solution. I have to practice resisting the temptation to ask leading questions or to give too much information in an effort to "fix" the errors that come out of the math exchange. This is not my job. My job is to provide many problem-solving situations that are appropriate for the children I work with and to facilitate and guide children as they bump up against challenges, questions, and new ideas.

The more you and your students practice math exchanges, the more you'll find a comfortable pace—a balance between efficiency and depth. There are always many exceptions to any rule of time—a breakthrough in thinking that leads to a longer conversation, an unplanned, spur-of-the-moment two-minute conference with a struggling mathematician. However, knowing that my purpose is not to "fix" the math but to fully use every moment together to guide mathematicians toward greater understanding helps me set my pace.

Math Exchanges over Time

More important than the exact amount of time each exchange lasts is the frequency and consistency with which you meet with students. In order to get the most power out of math exchanges, it is critical for them to be an integral part of the math workshop. Students should come to expect math exchanges on a regular basis. There are some days, some workshops, in which you may decide that your time is better spent observing students, asking probing questions to pairs of students who are playing a math game, or holding longer group discussions. Nevertheless, in any given week, math exchanges should occur three to four days per week.

Within my sixty- to seventy-five minute math workshop (in which about half my time is spent working with small groups) I usually work with two to three groups of four or five students per day. This is a group size with which I feel we can really dive deeply into the math and in which all members are active. There are enough children for there to be a diversity of ideas and background knowledge. I can facilitate a conversation in which students are talking, listening, and responding in meaningful ways. However, you have to find the optimal group size with which you can do these things. I've seen teachers work effectively with anywhere from two to six children at a time.

In deciding how frequently to meet with each child within math exchanges, I have come to believe that the answer lies in equity more than equality. Although I use a calendar to keep track of which children I have worked with most recently, I do not have a set rotation for math exchanges. There are students with whom I work more often than others. They are the struggling students (either in terms of the development of mathematical ideas, the ability to express strategies, or a lack of engagement in other parts of the math workshop). Pat Johnson, in writing about working with struggling readers, reflects upon Marie Clay's idea that "we [teachers] cannot put the reading process into the head of the child; the child must be the one to assemble the working systems" (2006, 4). Again, I believe the same can be said about mathematicians, especially those who struggle the most. As much as we may want to, teachers cannot fill a child's brain with number sense and a solid understanding of math concepts. The child must develop and construct these understandings over time with the *guidance* of the teacher and peers.

Struggling mathematicians need the most time in small groups, working not only with other struggling mathematicians but also with mathematicians who will explain in kid language why they have chosen a particular efficient strategy or what they are doing at the point of difficulty. Mathematicians need many, many opportunities to run into challenging mathematical situations that allow them to build a system of deep understanding.

As a teacher, it has taken me time to work to a place of comfort with math exchanges. I started out having math exchanges with two students and working with only one group per day. Even now at the beginning of the year I may spend so much time focusing on setting up the expectations of a math workshop that I only work with one group per day through October. I've learned to be okay with this. I have learned from experience and practice to trust in teaching deeply within math exchanges. Students are still learning throughout all the parts of the math workshop. It is okay to start small. Give yourself the time to explore how you will best facilitate these math exchanges. When I feel the "I'm-not-getting-to-work-with-enough-children" panic consuming me, I focus on this quote from Jon Muth's *The Three Questions* (2002):

> *Remember then that there is only one important time, and that time is now. The most important one is always the one you are with. And the most important thing is to do good for the one who is standing by your*

side. For these, my dear boy, are the answers to what is most important in this world. This is why we are here.

Joy, Rigor, and Empowerment Through Math Exchanges

Math exchanges involve a struggle and a grasping for deep understanding. Ellin Keene writes about teaching children "to savor the struggle." In reflecting upon struggle, Keene writes about author Reynolds Price, who through his writing "reveals his surprise at not only surviving, but thriving intellectually, and not in spite of, but *because* of his struggle" (2008, 101). Finding joy and empowerment within the rigor and struggle is a critical component and purpose of math exchanges.

Fosnot and Dolk (2001) write about the difference between "mathematics," ideas and procedures created in the past by others, and "mathematizing," constructing one's own ideas and understanding of math. About "mathematizing," they write that "when we define mathematics in this way, and teach accordingly, children will rise to the challenge. They will grapple with mathematical ideas. They will develop and refine strategies as they search for elegance; they will create mathematical models as they attempt to understand and represent their world" (13).

Math exchanges are places where children should learn to "savor the struggle" and "grapple with mathematical ideas." I know many people may be rolling their eyes at the thought of combining math, struggle, and joy. I was one of those people at one time! I understood the connection between rigor and joy in literacy teaching and learning, but math remained disconnected from this idea for me. Only when I stopped seeing mathematics as something separate from my life and the lives of my students, a system imposed on us by others, was I able to find myself as a mathematician, and, in turn, help the young mathematicians I work with view mathematics as one way of understanding our world.