

# Study Guide

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# NUMBER SENSE ROUTINES

Building Numerical  
Literacy Every Day  
in Grades **K-3**

Foreword by Lucy West

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## Summary

Get ready to immerse yourself in number sense routines and to be challenged to reignite the way in which you envision the development of number sense in your students. The ideas in Jessica Shumway’s book take on life as you enter the classrooms of teachers who are using a myriad of routines in ways, which are not “routinized.” This book invites you to sit beside the author, try out math tasks, listen to children and teachers, and consider strategies for planning and facilitating your own number sense routines. This is not just another book with a list of activities. Jessica journeys with us as we learn how to respond to our students’ needs with purposeful and intentional planning of number sense routines.

The first section of the book, *Building Number Sense Through Routines*, consists of two chapters. In this part of the book, Jessica clearly lays out the importance of number sense development and offers us a rationale for the use of number sense routines. With accessible and lively writing, we are asked to consider the importance of students “building understanding from within and taking an active part in constructing their number sense” (11). A clear and handy table of routines and questioning strategies provides a reference for you as you continue your journey through the book.

The second section of the book is a treasure box of chapters, which introduces number sense routines such as Counting Routines and Quick Images. These routines provide high leverage opportunities that target the number sense ideas that Kindergarten, first, second and third grade students are developing. The visual images of student work, and the “What’s the Math” boxes, provides support as you analyze the mathematics underlying the routines. The student cases and classroom episodes give us a glimpse into the practical applications of the routines as well as uncover the nuances that teachers need to pay attention to. There are a variety of suggested resources, questioning strategies, and support for getting started and making number choices.

The third section of the book, *More Than Just the Routine*, invites you to consider the role of classroom community. In this section you will explore the importance of math talk, mistakes as learning opportunities, and the process of reflection. You will be asked to join Jessica as she connects the number sense routines to the building of a strong and supportive community of learners. In the final chapter, *Planning Responsive Number Sense Routines*, there is an opportunity to dig into the challenging work of being responsive to individual student needs and how to plan for the whole class as well.

This guide is structured for a study group. While there are many opportunities for collegial conversations, individuals will also find the study guide to be a useful tool for reflection. The following questions and activities will help readers to extend the ideas of the book and to apply them to their own classrooms. You will find questions that will pull you back into thinking further about a teacher’s moves or the ideas of specific students. There are quotes to consider that will stimulate conversations around beliefs and practices, and there are suggestions for classroom applications.

So grab a highlighter, plenty of sticky notes, and a few colleagues! Then get ready to join Jessica and a team of teachers and students as they immerse you in gathering a wealth of ideas for number sense routines, tools for planning and instruction, and effective strategies for supporting your students’ progress toward numerical literacy.



## Introduction

### Discussion (15 minutes)

- At the beginning of the introduction, there are two students whose challenges in mathematics are described. Think of a student you have or have had that you were concerned about. What struggles did that student encounter? What are some ways that you or others intervened? What was challenging about supporting the student's progress toward developing number sense?

### Quotes Worth Discussing (10 minutes)

- Discuss this statement on page 2: *"This book is about tapping into every child's innate sense of number."*

This statement sets the premise that all students have an innate sense of number. What reactions do you and your colleagues have toward this statement?

## Chapter 1

### Number Sense: What Does It Mean?

#### Let's Do Some Math! (10 minutes)

- Solve this problem on page 5 in two different ways.

*600 Monarch butterflies were overwintering in an oyamel tree in Mexico.*

*One spring day, 378 of them flew away. How many are still in the oyamel tree?*

- Compare your methods with a partner. What ideas about numbers and number relationships did you call upon?

#### Discussion/Sharing (15 Minutes)

- Discuss the work of the students (Anita, Carlos, Sumayah and Brandon) on pages 6 through 7. What does each student seem to understand? What ideas may each student still be developing? Connect their ideas to the strategies you used in “Let’s Do Some Math!”
- Examine the *Early Number Sense Learning Trajectory Chart* (Box 1.1). How do these big ideas and strategies relate to the current mathematics topic you are teaching? How did these ideas and strategies show up in the work of the students on pages 6 through 7?

**Quotes Worth Discussing (15 Minutes)** Discuss your interpretation of these quotes and the impact on your understanding of number sense.

- *“It is difficult to compute without number sense” (8).*
- *“Before children even come to school, their intuitive sense of number begins to develop” (11).*
- *“Teaching for number sense involves students building understanding from within and taking an active part in constructing their number sense” (11).*

#### Putting Ideas into Practice (15 Minutes)

- Revisit the list of number sense understandings and skills on pages 8 through 10. Think of a student you would describe as having number sense. What evidence do you have that the student has many components of number sense? How might you assess the student to get more evidence?



- An important part of building number sense is knowing that there are multiple ways to solve problems. How do you support the development and sharing of multiple solution paths? Examine an upcoming lesson, task or routine with which you will engage your students. How can you support students in encountering multiple ways to solve the problem?



## Chapter 2

### Improving Number Sense: Routines That Are Not Routinized

#### Let's Do Some Math! (10 minutes)

- Start at 188 and count backwards by tens. Discuss patterns you noticed and strategies you used as you counted.
- Start 188 and count backwards by 15. What new ideas did you need to think about? How did counting by tens help you count by 15?
- Share your ideas with a partner or a small group. What was similar about your observations? What was different?

#### Discussion/Sharing (15 Minutes)

- Choose one of the students featured on pages 18 through 21 (Jamie, Margaret, or Andy). What mathematical idea(s) is the student working with (you can refer back to pages 8 through 10)? How does the routine(s) mentioned in the vignette support the students in developing number sense?
- Adam (15) gets “stuck” when it is his turn to count back by tens from the previous number in the class count (108). How does the teacher support his sense making? Why do you think the teacher then has the class count back by twenty?

**Quotes Worth Discussing (15 Minutes)** Discuss your interpretation of these quotes. How did the vignette “A Routine in Action: Count Around the Circle” offer opportunities for connecting to the quotes below?

- *“These number sense routines are not ‘auto pilot’ activities, but opportunities for meaningful practice” (14).*
- *“The number sense routines in this explored in this book are ‘responsive’ routines- they are responsive to students’ discussions, understandings, and learning needs” (17).*
- *“Routines provide a comfortable predictability, but at the same time, we plan routines that will keep students challenged, provide opportunities to practice using their numbers sense, and reteach when necessary” (17).*

**Putting Ideas into Practice (15 Minutes)**

- What routines do you already have in place that support a classroom culture of inquiry and risk taking? How will the ideas in this chapter help you refine the routines you may already have in place? Examine your math block and develop a plan for including the routines described in Chapter Two.
- On page 17 there is a list of six ways in which number sense routines are responsive and support students' discussions, understandings, and learning needs. Choose one of the ideas on the list and discuss how the idea will support your mathematical goals for your students.



## Chapter 3

### Visual Routines: Seeing and Conceptualizing Quantities

#### Let's Do Some Math! (10 minutes)

- Mental Math: Solve the following problems and then discuss with your group the methods each of you used. Relate your strategies to the work with visual routines in this chapter. What number relationships did you use? What understandings about ten played out in your strategies?

$$26 + 14$$

$$98 + 122$$

$$79 + 63$$

- Analyze Michael's and Kelly's strategies on page 48. How are they making use of an understanding of "ten" and the idea of decomposing numbers?

#### Discussion/Sharing (15 Minutes)

- On page 41 the author states, "Informal and ongoing assessment of students' math strategies drove our instructional plans for this routine." Revisit the work of the students on pages 40 through 42. How do the teachers engage the students in number sense routines that support their current understandings and observed needs?
- Describe the difference between perceptual and conceptual subitizing. Share with a partner or a small group two or three dot cards or ten frame images from the chapter and explain how they support subitizing.
- Examine the resources in Box 3.7 on page 52. Check one or two of them out and share with your book study group.

**Quotes Worth Discussing (15 Minutes)** Discuss your interpretation of these quotes. Reflect on the routines in this chapter and connect them to the quotes.

- *"Counting does not mean much to young children if they do not have the visual images to go with the number words or written numerals" (53).*
- *"Becoming more efficient is a natural part of getting familiar with the routine and the ideas and concepts it presents" (50).*



**Putting Ideas into Practice (15 Minutes)**

- Choose one of the visual routines in the chapter. Think of a student in your class who would benefit from some further work with ten as a unit. Prepare a set of experiences with the chosen routine. Reflect and share: What did you learn about how the student is visualizing and making use of ten? What are your next steps for this student? You can use the “Case Study on Layla” in Box 3.2 as a guide, along with a planning template on page 167.
- Appendix A has Dot Sequences you can try. (Refer to page 40 where the author discusses planning with a particular goal in mind and the need to adjust.) Try out a sequence and reflect on adjustments you had to make, or might have made, based on the students’ responses.



## Chapter 4

### Counting Routines: Understanding Place Value and the Number System

#### Let's Do Some Math! (10 minutes)

- Analyze the three options for visual support for counting by ones to 97 (63). What does each option highlight about the counting sequence?
- Work with a partner or a small group to create three visual supports for counting by threes starting at 98. Discuss the three options you created and how they support making estimates, using tens or landmark numbers, or make use of other number relationships.

#### Discussion/Sharing (15 Minutes)

- On page 56, the four objectives for routines that build counting proficiency are listed.
  - Analyze the work that Jessica and Carrie engaged in with the second grad-er students (58 through 64). Discuss how the two teachers planned count-ing routines and posed questions that would support those four objectives. Share specific examples with a partner.
  - What evidence do you see in the students' responses that the counting routines are supporting their number sense development? Share specific examples with a partner.

**Quotes Worth Discussing (15 Minutes)** Discuss your interpretation of these quotes. Relate the quotes to a recent experience you had with your students around one of the routines in Chapters 3 or 4.

- *“Students who struggle with mathematics often lack counting skills” (56).*
- *“Just as with any math routine, although it is important to be well prepared, you also need to be flexible with your plans and adjust based on your observations, ongoing formative assessments, and students' needs” (60).*

#### Putting Ideas into Practice (15 Minutes)

- Use current information you have about the needs of your class in terms of de-veloping more efficient strategies for adding numbers (the range of numbers ac-cording to your grade level). Develop a three-day sequence of counting routines

based on those observations. Refer to Boxes 4.2 and 4.3 (pages 58 and 59) for ideas to get you started. Keep notes on how the students responded to the routines and how you made adjustments when needed. Share your learning with a partner.

- Analyze the use of the Organic Number Line (72 through 77). Try out the use of the Organic Number Line with a counting routine. What do you notice about your students' ability to recognize where to place common benchmarks? Adjust the work to include either whole numbers or fractions depending on the grade level.
- Rich math discussions are supported by both the use of number sense routines and by establishing norms for Math Talk. Examine Box 4.4 on page 65. What protocols have you taught your students? Which are working well and which need further attention?



## Chapter 5

### Playing with Quantities: Making Sense of Numbers and Relationships

#### Let's Do Some Math! (10 minutes)

- Mental Math: Solve each of these problems using mental math. Then examine the strategies and number choices in Box 5.4. What do you notice about the strategies that you used? How do they connect to the ones offered in Box 5.4? Try showing two or more of your strategies on a number line.

$18 + 6$

$39 + 12$

$119 + 29$

$64 - 9$

$200 - 39$

#### Discussion/Sharing (15 Minutes)

- On pages 93 through 95, Jessica uses informal assessments to guide her work with Andy. Analyze the vignette and discuss the following questions:
  - How does Jessica build on Andy's current understanding as he is solving the story problem about Samia and the pencils?
  - How does she press Andy to consider his mental math strategies when she notices he is using a counting all strategy on the paper?
  - How does Jessica and Andy's teacher use their assessment of students' current thinking to plan a series of mental math routines?
- On pages 96 through 98, Jessica plans a mental math routine for a class of third grade students. Examine the choices she makes for the subtraction problems. Also examine the questions she poses and the moves she chooses to make. Discuss how the questions, moves, and number choices caused students to reflect on their strategies and the discussions that emerged.
- Discuss the differences and similarities between the two routines "Today's Number" and "Ways to Make a Number." In what ways do these routines support an understanding of place value ideas and the operations?

**Quotes Worth Discussing (15 Minutes)** Discuss your interpretation of these quotes. Reflect on the routines in this chapter and connect them to the quotes.

- *"When students understand the way numbers work in our base ten number system, the way they look at and use numbers changes" (80).*



- *Moreover, you may be surprised to find that your students use more efficient strategies when they unravel a problem in their heads rather than on paper” (91).*

### **Putting Ideas into Practice (15 Minutes)**

- On page 92, the author states, “Our job is to help them (the students) make explicit what is going on in their heads.” With this specific teacher move in mind, choose a mental math problem with which to engage your class (or a small group). Refer to Box 5. 4 on page 95 for ideas. Anticipate strategies your students might offer, and jot down questions you can pose to assist them in making their ideas more explicit.
- Ways to Make a Number is introduced on page 83. Examine the student work on pages 84 and 85. Select a number for your class to work on and bring the student work to share with your small group. Discuss what you learn about the students’ numbers sense. Develop plans for a new Ways to Make a Number with specific constraints that will address the students’ needs.

## Chapter 6

### Calendar and Data Routines: Using Numbers Every Day

#### Let's Do Some Math! (10 minutes)

- It is the 26th day of school! Use Box 6.4 to develop strategies for the Counting the Days of School Routine.
  - Develop questions using a Linear Model.
  - Develop questions using an Array Model.
  - Develop questions using Manipulatives (tiles, cubes).

#### Discussion/Sharing (15 Minutes)

- Counting the Days in School offers a myriad of opportunities for looking at patterns, making use of place value ideas and number relationships and learning how to represent quantities. This routine can include different ways to represent the number of days in school. Discuss how each of these might effectively support students. (Note pages 109 through 110 outline some ineffective use of specific models in particular grades.)
- On page 102, first grade teacher Mary Anne shares her reflections on her use of calendar time. Discuss how the group of teachers tackle the problem together.

**Quotes Worth Discussing (15 Minutes)** Discuss your interpretation of these quotes. What connections can you make to the Early Number Sense Learning Trajectory on page 9 (Box 1.1).

- *“Very young children need to “see” the number of days represented by Unifix cubes, because the numeral 4 might not mean much to them yet” (102).*
- *“A good routine, along with the questions you ask, will give you the opportunity to get inside the students’ heads and be aware of their thinking, understandings, and misconceptions” (116).*

#### Putting Ideas into Practice (15 Minutes)

- Work with your grade level team to develop a long-term data collection investigation that supports your grade level science standards. Consider and plan for (Refer to Box 6.3 on page 108):

- Where will you display the data?
- How will you involve students in the collection and recording of the data?
- What questions and problems will you pose to allow students to interact with mathematical ideas related to the data?
- Try out at least two of the Calendar Routine Prompts and Questions in Box 6.1. What are some responses that your students offered? What surprised you?
- Develop a set of new prompts and questions to use during Calendar Time for the upcoming month. Think about areas your students need additional practice in or ideas that you want them to expand upon (consider returning to Box 1.1, page 9).
- On page 116, the author poses the following questions to help us step back and reflect on our current calendar and data routines. Discuss these with a partner and develop at least one action step.
  - What kind of questions can you ask to ratchet up the mathematics discussions?
  - Which models will help you better differentiate your routines?
  - What are some ways you can better organize long-term data collection?
  - How can you organize the data to best provide students with opportunities to apply their number sense understandings to analyzing data?

## Chapter 7

### Learning From Each Other: Building a Strong Community of Learners Through Math Talk, Mistakes, and Reflections

#### Let's Do Some Math! (10 minutes)

- Choose a routine from one of the previous chapters. Take turns doing a “rehearsal” with your team. Use the Math Talk Tips on pages 160 and 161 to plan the questions you will ask. Reflect as a team on how the conversation supported the number sense goals.

#### Discussion/Sharing (15 Minutes)

- On page 122 the author lists four skills that are critical in developing a community in which mathematical discourse is central. Discuss how these four skills have been present in the vignettes you have read in previous chapters. Then examine the classroom episode “Talking About Visual Quantities” on page 120. How does the teacher support mathematical discourse? What evidence do you see in the students’ responses that show the impact of the teacher’s purposeful support?
- On pages 130 through 132, mistakes are discussed as opportunities for learning. Choose a routine with which to engage your class, and plan a three-day sequence. Anticipate misconceptions or mistakes that might surface, and prepare for them using the phrases and questions on page 131. Over the course of the three days, keep track of mistakes that students made and how you responded to them. Share your reflections and next steps with a partner.

**Quotes Worth Discussing (15 Minutes)** Discuss your interpretation of these quotes. Reflect on the steps you have taken to develop a classroom community that support sense making.

- *“Building a strong and supportive community of learners is essential in making number sense routines beneficial and productive for everyone” (120).*
- *“Discussion among students is an absolutely critical component in their mathematics development” (120).*
- *“It is important to have students explain their thinking both when their answer is correct and when it is incorrect” (124).*

**Putting Ideas into Practice (15 Minutes)**

- Choose one area of Math Talk that you wish to focus on (see pages 160 through 161). Prepare a routine and focus on intentional teacher moves to support your focus. Record one of your Classroom Routine sessions and analyze how you supported your goal for supporting mathematical discourse. How did your students respond? What would you like to improve upon? Try a second routine and repeat the process. What do you notice? What are areas you would like to focus on next?



## Chapter 8

### Planning Responsive Number Sense Routines

#### Let's Do Some Math! (10 minutes)

- Solve the problem on page 137 in two ways. Discuss what number sense ideas you called upon to solve the problem and then share out specific routines that would support students in developing those ideas.

Somia, Kelsey, and Adam made 26 cookies for their friends.

They put five cookies in each box.

How many boxes did they need for all their cookies?

#### Discussion/Sharing (15 Minutes)

- Appendix D on page 164 contains a detailed two-week plan to address a group of students' needs. Examine the plan and discuss how the sequence of routines builds over time.
- Return to Malaak's understanding before and after the first five days of focused routines (137 through 138). Describe how her ideas about multiplication, division, and counting by groups have been impacted by the experiences.

**Quotes Worth Discussing (15 Minutes)** Discuss your interpretation of these quotes. Reflect on your own practice and identify a goal to work on based on the ideas in these quotes.

- *"It is often difficult to figure out what students know, because it is common for teachers to focus too much on what students do not know" (137).*
- *"Assessment and instruction are symbiotic" (140).*

#### Putting Ideas into Practice (15 Minutes)

- Choose three pieces of student work that you have collected related to the routine you have been implementing. Use the "Analyzing Students' Work, Thinking and Learning" template in Appendix D as an analysis tool. Discuss your analysis with your study group. What commonalities or trends do you see in the students' responses? What different needs are evidenced in the work samples?
- Identify a small group of students who have similar needs. Plan a five-day se-

quence of routines to address their needs. Use formative assessment strategies to monitor their progress. At the end of the five days, reassess and examine their current understanding.



## Conclusion

### A Place to Begin

#### Discussion/Sharing (15 Minutes)

- On page 142, the author encourages us to “start small.” Take time with your study group to reflect on the small steps, challenges and accomplishments that have been part of your journey to implement Number Sense Routines.

**Quotes Worth Discussing (15 Minutes)** Discuss your interpretation of these quotes. Reflect on the routines in this chapter and connect them to the quotes.

- *“Developing one’s own number sense is never ‘done;’ therefore, enjoy the voyage of the continuous development . . .” (143).*

#### Putting Ideas into Practice (15 Minutes)

- This last chapter begins with quotes from two different students who are sharing how they feel about the impact of routines on their number sense. Choose a few students in your class to interview or prepare a survey to gather comments from your students. Reflect on how your students
  - Feel about mathematics; what is their disposition
  - Connect the work with routines to their own ability to solve word problems
  - Think about efficient strategies and the use of multiple strategies
  - See the role of visual images (IE number lines, arrays, charts)
  - View the role of mathematical discourse
- Look back over the work you have done in this study group. What changes do you see in your own understanding of the term “Number Sense?” What impact has this work had on your students?
- Make a plan for how you will continue to share and build on your study group’s ideas! Keep the collaboration going.