## **Centrifugal Impeller Structural Resonance**

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Engineers have been making use of the Interference Diagram as a tool for assessing potential for impeller structural resonance for decades now. It is also referred to in the literature as the SAFE (Singh's Advanced Frequency Evaluation) Diagram, and the ZZENF (Zigzag Excitation Line in Nodal Diameter vs. Frequency) Diagram. A detailed review is provided here of its construction, along with a case study involving a centrifugal impeller with a vaned diffuser. Also included is a discussion of traveling wave behavior of the impeller response arising from interaction between rotating blades and stationary vanes, the mechanisms for which have received little treatment in related published work.

The centrifugal compressor impeller under study is depicted in Figure 1. It is a covered (shrouded) impeller designed to operate at 14,700 RPM. The original design incorporated 17 impeller blades, and 19 diffuser vanes in close proximity to the impeller outer diameter.



**Figure 1.** Shrouded Centrifugal Compressor Impeller with 17 Blades, 19 Diffuser Vanes (not shown), and Running Speed of 14,700 RPM

The Interference Diagram will be used to determine if a structural resonance can occur given the running speed, the number of rotating blades, the number of stationary vanes, and the modal parameters of the impeller. There are only a few relatively simple conditions that must be met for resonance to occur. The first is that the number of nodal diameters (N) of the resonant mode must equal the sum or difference of the number of rotating blades (B), and number of stationary vanes (V). Integer multiples of the blades and vanes also apply. Mathematically this is expressed as:

$$|jV \pm kB| = N \tag{1}$$

Where j and k are integers, typically 1 or 2 (although k=0 would be a special case of a pure disk mode). The second condition is that the frequency of the resonant vibration must equal the same integer multiple of the stationary vane pass frequency:

$$\boldsymbol{f} = \boldsymbol{j} \boldsymbol{V} \boldsymbol{\omega} \qquad (2)$$

Where f is the resonant frequency, and  $\omega$  is the rotational speed of the impeller in radians/sec. Figure 2 provides a graphical depiction of the blade/vane difference concept. For clarity purposes, 16 rotating blades and 20 stationary vanes are used in the figure. The blade/vane difference number of four in this case results in four locations 90° apart around the periphery where a blade and vane exactly match up, such as at Blade B1 and Vane V1. Ignoring any phase lag between the blade/vane passing and the pressure pulse on the blade, these four locations would then represent an antinode in the sinusoidal vibration response (+ signs in figure). Similarly, 45° from the maximum response would be the minimum response, where the blades and vanes are furthest from matching up ("-" signs in the figure). Drawn between these 8 minimum and maximum locations are the four zero excitation/response nodal diameters (colored lines). Based on this explanation it is clear that the blade/vane difference number corresponds to the number of nodal diameters of the spatial distribution of the pressure loading on the blades. The above does not address the blade/vane sum number. Typically the sum is not considered since it results in a very high number of nodal diameters, and such modes are highly complex, high frequency, and not a resonance concern.



**Figure 2.** Blade/Vane Interaction Spatial Pressure Distribution for 16 Blades and 20 Vanes (blue markings are the stationary vanes)

Higher integer multiples of the blade and vane numbers come into play since the pressure distribution is not a perfect sine wave. Equation 1 indicates that a different nodal diameter pattern results when dealing with higher blade/vane multiples. These higher multiples are typically less energetic than the prime difference number.

Equation 1 leads to the familiar zig-zag lines on the interference diagram. Equation 2 leads to the horizontal lines on the diagram which provide the specific frequencies that would produce resonance if the modal mode shape matches Equation 1. Depicted in Figure 3 is the interference diagram predicted for the case study impeller with 17 blades, 19 vanes, and the running speed of 14,700 RPM. As shown, there are intersections denoted by the blue circles at points where both Equation 1 and 2 are satisfied. One falls at 2 nodal diameters (1V-1B=2) at the 1X vane pass excitation frequency, and the other falls at 4 nodal diameters at 2X vane pass coupled with 2X the number of blades (2V - 2B = 4). Equations for the zig zag and horizontal lines are shown in the figure. Their intersection points satisfy both Equations 1 and 2. The zig zag and horizontal lines are not important, the intersection points are. The lines help to visualize how to make modifications to vane numbers and/or running speed to shift resonance points away from impeller modes. The diagram of Figure 3 is for a constant running speed. At other speeds, the frequency coordinate is scaled proportional to the speed, so that for a variable speed machine, the interference points become vertical lines.



Figure 3. Interference Diagram for B=17 Blades, V=19 Vanes, and  $\omega$ =14,700 RPM Speed.

Figure 3 can be considered an excitation plot. The next step is to determine if the impeller modal response "interferes" with the excitations. A finite element analysis (FEA) is performed which yields the natural frequencies and mode shapes accounting for stress stiffening from centrifugal loading. The black diamonds placed on Figure 4 resulted from a cyclic symmetric finite element analysis (FEA) of a single sector of the impeller. For this analysis the ANSYS Version 19.1 finite element code was used. As shown in the figure, the original design unfortunately had interferences at both excitation points. The mode shape of the 2-nodal diameter that fell at 1X vane pass frequency is depicted in Figure 5. It exhibited significant vibration at the shroud periphery. This is particularly troublesome since the pressure pulsations from the diffuser would be most active in that region. Such two nodal diameter modes are known to be particularly responsive. The other interference point at 4-nodal diameters corresponded to a highly complex mode shape of the entire impeller. Based on the 2-nodal diameter interference, the original design was considered unacceptable. Using the Interference Diagram as a guide, the number of diffuser vanes was changed to 22 before manufacturing the compressor. The diagram for the modified design is shown in Figure 6. As shown, the new excitation points of interest were at 5 nodal diameters (1V-1B) and 7 nodal diameters (2V-3B). The separation margin at the 5 nodal diameter excitation point now exceeded 10% with the modified design.

Note that the impeller modes plotted on the interference diagram do not exceed 8 nodal diameters. This corresponds to one-half of the number of blades minus one:  $\frac{1}{2}(17-1)$ . If the number of

blades had been an even number, then the maximum nodal diameter would be simply one-half the number of blades. According to Singh (2003), modes do not exist beyond this nodal diameter limit. Certainly for a pure disc with no blades there would be no limit to the number of nodal diameters that can exist. If such higher nodal diameter modes did in fact exist in an impeller, they would still be predicted in the FEA analysis. It should be noted however that with a cyclically symmetric model the ANSYS program does not report the mode as a high nodal diameter, but lists it under an aliased lower nodal diameter. For example, if a 10 nodal diameter mode existed for our 17-bladed impeller, then ANSYS would list it as a 7 nodal diameter mode (frequency would be reflected about B/2=8.5). The limitation to half the blade number in the interference diagram therefore does not neglect higher nodal diameter modes, although such modes are typically not responsive.



Figure 4. Interference Diagram for 17 Blades, 19 Vanes and 14,700 RPM Speed. Includes Results from Modal Analysis



Figure 5. Mode Shape for Two-Nodal Diameter Mode Near 1X Diffuser Vane Pass Frequency



**Figure 6.** Modified Interference Diagram for 17 Blades, 22 Vanes and 14,700 RPM Speed. In order to investigate traveling wave behavior, Figure 7 was constructed. It depicts the blade/vane orientations as the 16-bladed impeller rotates a small amount such that the next nearest impeller/vane combination lines up. It is clear that since the impeller is rotating, eventually every blade will have a turn at fully interacting (lining up with) with a stationary vane and experience maximum pressure excitation. This implies that, in the rotating reference frame of the impeller, the resonance manifests as a traveling wave. If it were a standing wave in the rotating frame, then the blades located on nodal diameters would never see high vibration. A 4.5° impeller rotation (blade-to-blade angle minus vane-to-vane angle) would cause Blade B2 to line up with Vane V2 such that the maximum pressure moves from V1 to V2. Since the nodal diameters rotate along with the maximum pressure, this requires that in the stationary reference frame the nodal diameters must rotate one vane pitch clockwise for every 4.5° counterclockwise rotation of the impeller. This clockwise rotation of the nodal diameters would rotate even faster from the reference frame of the counterclockwise rotating impeller.

So it is clear that the nodal diameters rotate with respect to the impeller, and with respect to the stationary frame. As the pressure excitation spatial pattern rotates, it would be expected that the impeller response would follow along. In fact, when animating the predicted impeller mode shapes using ANSYS (Workbench Mechanical Environment), the traveling wave behavior is depicted. The actual speed of the traveling wave can be calculated. It can be shown that the nodal diameter rotational speed in the stationary frame is equal to the blade pass frequency divided by the number of nodal diameters ( $kB\omega/N$ ). Similarly, the rotational speed of the number of nodal diameters (in the vane pass frequency divided by the number of nodal diameters ( $jV\omega/N$ ). The direction of nodal diameter rotation depends on whether there are more vanes (backward) or more blades (forward). Campbell (1924) identifies standing waves setting up in the stationary frame, but this is in relation to pure disk critical speeds excited by multiples of running speed, not by vane/blade interaction. That being said, the  $kB\omega/N$  speed equation is not violated by this observation, since for a pure disk B=0.



Figure 7. Rotation of Nodal Diameters as Impeller Rotates,

16 Blades, 20 Vanes, Right Side Depicts Configuration after a 4.5° Impeller Rotation, Impeller Rotates Counterclockwise while Nodal Diameters Rotate Clockwise

The traveling wave discussion above applies to perfectly cyclic symmetric wheels. If mistuned blades are present then it becomes more complex. Traveling waves can be partially reflected off of the mistuned blades and localize the vibration energy at specific blades. For the case of identical blading, as discussed the nodal diameters (arising from blade/vane interaction) rotate with respect to the impeller so in such a situation no blade is at any higher failure risk than another blade. If there is a consistent pattern to the cracked blades, it may be indicative of mistuning.

This paper reviewed some key aspects of impeller resonance, which are also relevant to bladed disks and blisks. Some other related topics of importance not considered here, in addition to blade mistuning, are stress stiffening, packeted turbine blades, harmonic forced response, damping techniques and analysis, specific FEA techniques and boundary conditions, and the relationship between wheel resonance and rotordynamics. An impeller or bladed disk designer should be well versed in these topics to prevent wheel fatigue failure.

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